

1. Simplify

(a) $(2\sqrt{5})^2$ (1)

(b) $\frac{\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$ giving your answer in the form $a + \sqrt{b}$, where a and b are integers. (4)

1a)

$$\begin{aligned} & 2\sqrt{5} \times 2\sqrt{5} \\ &= 2 \times 2 \times \sqrt{5} \times \sqrt{5} \\ &= 4 \times 5 \\ &= \underline{\underline{20}} \end{aligned}$$

b/

$$\frac{\sqrt{2} (2\sqrt{5} + 3\sqrt{2})}{(2\sqrt{5} - 3\sqrt{2})(2\sqrt{5} + 3\sqrt{2})}$$

$$\frac{2\sqrt{10} + 6}{20 - 18}$$

$$\frac{2\sqrt{10} + 6}{2}$$

$$\underline{\underline{3 + \sqrt{10}}}$$



2. Solve the simultaneous equations

$$y - 2x - 4 = 0 \quad (1)$$

$$4x^2 + y^2 + 20x = 0 \quad (2)$$

(7)

$$(1) \quad y = 2x + 4$$

sub (1) into (2)

$$4x^2 + (2x + 4)^2 + 20x = 0$$

$$4x^2 + (2x + 4)(2x + 4) + 20x = 0$$

$$4x^2 + 4x^2 + 8x + 8x + 16 + 20x = 0$$

$$8x^2 + 36x + 16 = 0$$

$$2x^2 + 9x + 4 = 0$$

$$(2x + 1)(x + 4) = 0$$

$$\underline{\underline{x = -\frac{1}{2}}} \quad \underline{\underline{x = -4}}$$

$$y = 2(-\frac{1}{2}) + 4$$

$$\underline{\underline{= 3}}$$

$$y = 2(-4) + 4$$

$$\underline{\underline{= -4}}$$



3. Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$

(3)

(b) $\int y dx$

(3)

a/ $y = 4x^3 - 5x^{-2}$

$$\frac{dy}{dx} = \underline{12x^2 + 10x^{-3}}$$

b/ $\int y dx = \frac{4x^4}{4} - \frac{5x^{-1}}{-1} + c$

$$= \underline{x^4 + 5x^{-1} + c}$$



4. (i) A sequence U_1, U_2, U_3, \dots is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geq 1$$

$$U_1 = 4 \text{ and } U_2 = 4$$

Find the value of

(a) U_3

(1)

(b) $\sum_{n=1}^{20} U_n$

(2)

- (ii) Another sequence V_1, V_2, V_3, \dots is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 1$$

$$V_1 = k \text{ and } V_2 = 2k, \text{ where } k \text{ is a constant}$$

- (a) Find V_3 and V_4 in terms of k .

(2)

Given that $\sum_{n=1}^5 V_n = 165$,

- (b) find the value of k .

(3)

i/ a/ $U_3 = 2U_2 - U_1$
 $= 2(4) - 4$
 $= 4$

b/ $20 \times 4 = \underline{80}$

ii a/ $V_3 = 2V_2 - V_1$
 $= 2(2k) - k$
 $= 3k$
 $V_4 = 2V_3 - V_2$
 $= 2(3k) - 2k$
 $= \underline{4k}$



Question 4 continued

$$\begin{aligned}V_5 &= 2V_4 - V_3 \\ &= 2(4k) - 3k \\ &= 5k\end{aligned}$$

$$\begin{aligned}b) \quad k + 2k + 3k + 4k + 5k &= 165 \\ 15k &= 165 \\ \underline{\underline{k}} &= \underline{\underline{11}}\end{aligned}$$

(Total 8 marks)

Q4



5. The equation

$$(p-1)x^2 + 4x + (p-5) = 0, \text{ where } p \text{ is a constant}$$

has no real roots.

(a) Show that p satisfies $p^2 - 6p + 1 > 0$

(3)

(b) Hence find the set of possible values of p .

(4)

no real roots $\therefore b^2 - 4ac < 0$

$$a = p - 1$$

$$b = 4$$

$$c = p - 5$$

$$(4)^2 - 4(p-1)(p-5) < 0$$

$$16 - 4(p^2 - 5p - p + 5) < 0$$

$$16 - 4(p^2 - 6p + 5) < 0$$

$$16 - 4p^2 + 24p - 20 < 0$$

$$-4p^2 + 24p - 4 < 0$$

$$4p^2 - 24p + 4 > 0$$

$$p^2 - 6p + 1 > 0$$

b/ $(p-3)^2 - 9 + 1 > 0$

$$(p-3)^2 - 8 > 0$$

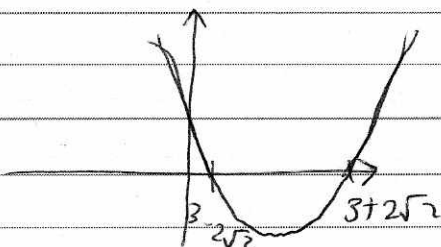
$$(p-3)^2 > 8$$

$$p - 3 > \pm\sqrt{8}$$

$$p > 3 \pm 2\sqrt{2}$$

roots at $3 + 2\sqrt{2}$

$3 - 2\sqrt{2}$



$$p < 3 - 2\sqrt{2}$$

or $p > 3 + 2\sqrt{2}$



6. The curve C has equation

$$y = \frac{(x^2 + 4)(x - 3)}{2x}, \quad x \neq 0$$

(a) Find $\frac{dy}{dx}$ in its simplest form.

(5)

(b) Find an equation of the tangent to C at the point where $x = -1$

Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

6 /

$$y = \frac{x^3 - 3x^2 + 4x - 12}{2x}$$

$$= \frac{1}{2}x^2 - \frac{3}{2}x + 2 - 6x^{-1}$$

$$\frac{dy}{dx} = x - \frac{3}{2} + 6x^{-2}$$

b/

when $x = -1$

$$\frac{dy}{dx} = (-1) - \frac{3}{2} + \frac{6}{(-1)^2}$$

$$= \frac{7}{2}$$

$$y = \frac{7}{2}x + c$$

when $x = -1$ $y = \frac{(5)(-4)}{-2}$

$$= 10$$

10

$$10 = \frac{7}{2}(-1) + c$$

$$c = \frac{27}{2}$$

$$y = \frac{7}{2}x + \frac{27}{2}$$

$$2y = 7x + 27$$

$$7x - 2y + 27 = 0$$



7. Given that $y = 2^x$,

(a) express 4^x in terms of y .

(1)

(b) Hence, or otherwise, solve

$$8(4^x) - 9(2^x) + 1 = 0$$

(4)

7a/

$$\begin{aligned}4^x &= 2^{2x} \\ &= 2^x \times 2^x \\ &= y^2\end{aligned}$$

b/ $8y^2 - 9y + 1 = 0$

$$(8y - 1)(y - 1) = 0$$

$$y = \frac{1}{8} \quad y = 1$$

$$2^x = \frac{1}{8} \quad 2^x = 1$$

$$\underline{\underline{x = -3}} \quad \underline{\underline{x = 0}}$$



8. (a) Factorise completely $9x - 4x^3$

(3)

(b) Sketch the curve C with equation

$$y = 9x - 4x^3$$

Show on your sketch the coordinates at which the curve meets the x -axis.

(3)

The points A and B lie on C and have x coordinates of -2 and 1 respectively.

(c) Show that the length of AB is $k\sqrt{10}$ where k is a constant to be found.

(4)

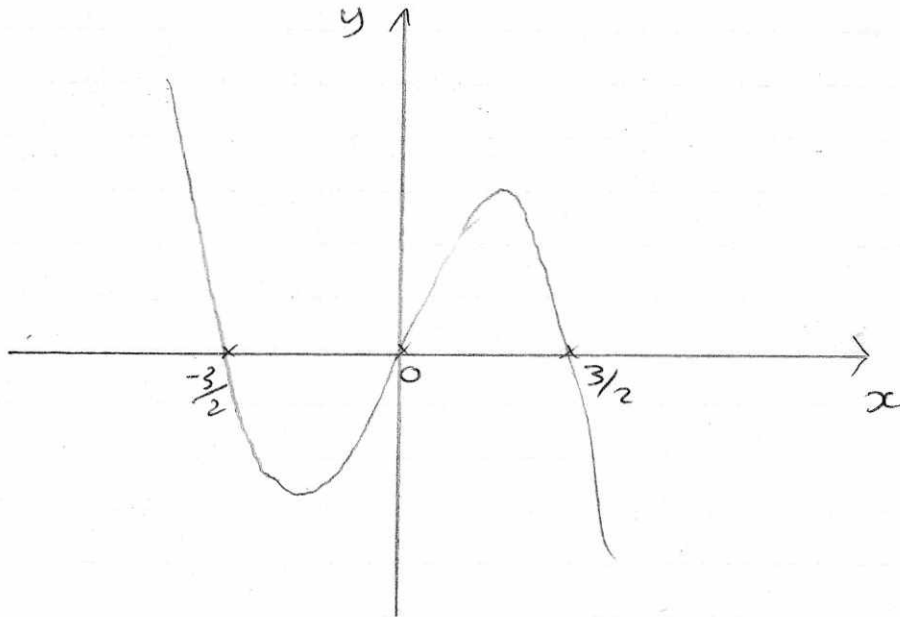
$$\begin{aligned} a) \quad & x(9 - 4x^2) \\ & x(3 + 2x)(3 - 2x) \end{aligned}$$

$$x=0 \quad x = -\frac{3}{2} \quad x = \frac{3}{2}$$

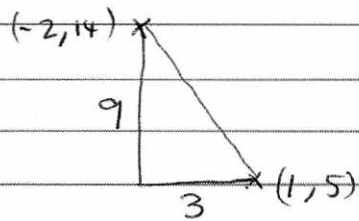


Question 8 continued

b/



c/	$A: x = -2$ $y = 9(-2) - 4(-2)^3$ $= 14$	$B: x = 1$ $y = 9(1) - 4(1)^3$ $= 5$
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$$\begin{aligned} \text{Distance} &= \sqrt{3^2 + 9^2} \\ &= \sqrt{90} \\ &= \underline{\underline{3\sqrt{10}}} \end{aligned}$$



9. Jess started work 20 years ago. In year 1 her annual salary was £17000. Her annual salary increased by £1500 each year, so that her annual salary in year 2 was £18500, in year 3 it was £20000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32000 in year k . Her annual salary then remained at £32000.

(a) Find the value of the constant k .

(2)

(b) Calculate the total amount that Jess has earned in the 20 years.

(5)

$$a/ \quad a = 17000$$

$$d = 1500$$

$$u_k = 32000$$

$$32000 = 17000 + (k-1)(1500)$$

$$15000 = (k-1)(1500)$$

$$10 = k-1$$

$$\underline{k = 11}$$

$$b/ \quad S_{11} = \frac{n}{2}(a+L)$$

$$= \frac{11}{2}(17000 + 32000)$$

$$= \frac{11}{2}(49000)$$

$$= 11(24500)$$

$$= \underline{269500}$$

$$10 \times 24500 = 245000$$

$$1 \times 24500 = +24500$$

$$\hline 269500$$

$$9 \times 32000$$

$$= 288000$$

$$10 \times 32000 = \cancel{2}26000$$

$$1 \times 32000$$

$$32000$$

$$288000$$

$$269500$$

$$+ 288000$$

$$\hline 557500$$

$$\pounds 557500$$



10. A curve with equation $y = f(x)$ passes through the point $(4, 9)$.

Given that

$$f'(x) = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2, \quad x > 0$$

(a) find $f(x)$, giving each term in its simplest form.

(5)

Point P lies on the curve.

The normal to the curve at P is parallel to the line $2y + x = 0$

(b) Find the x coordinate of P .

(5)

$$\begin{aligned} a/ \quad f'(x) &= \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2 \\ &= \frac{3}{2}x^{1/2} - \frac{9}{4}x^{-1/2} + 2 \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{\frac{3}{2}x^{3/2}}{3/2} - \frac{\frac{9}{4}x^{1/2}}{1/2} + 2x + C \\ &= x^{3/2} - \frac{9}{2}x^{1/2} + 2x + C \end{aligned}$$

x	y	
$(4, 9)$		$9 = (4)^{3/2} - \frac{9}{2}(4)^{1/2} + 2(4) + C$

$$9 = 8 - 9 + 8 + C$$

$$C = 2$$

$$y = x^{3/2} - \frac{9}{2}x^{1/2} + 2x + 2$$

$$\begin{aligned} b/ \quad 2y &= -x \\ y &= -\frac{1}{2}x \end{aligned}$$

$$\underline{m = -1/2} \quad \therefore \text{normal } \underline{m = 2}$$



Question 10 continued

$$2 = \frac{3\sqrt{x}}{2} - \frac{9}{4\sqrt{x}} + 2$$

$$2\sqrt{x} = \frac{3}{2}x - \frac{9}{4} + 2\sqrt{x}$$

$$0 = \frac{3}{2}x - \frac{9}{4}$$

$$\frac{9}{4} = \frac{3}{2}x$$

$$\frac{18}{12} = x$$

$$x = \underline{\underline{\frac{3}{2}}}$$

