

1. Simplify

$$\frac{7 + \sqrt{5}}{\sqrt{5} - 1}$$

giving your answer in the form $a + b\sqrt{5}$, where a and b are integers.

(4)

$$1) \quad \frac{7 + \sqrt{5}}{\sqrt{5} - 1}$$

$$\frac{(7 + \sqrt{5})(\sqrt{5} + 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)}$$

$$\frac{7\sqrt{5} + 7 + 5 + \sqrt{5}}{5 + \sqrt{5} - \sqrt{5} - 1}$$

$$\frac{8\sqrt{5} + 12}{4}$$

$$2\sqrt{5} + 3$$

$$\underline{\underline{3 + 2\sqrt{5}}}$$

(Total 4 marks)

Q1



2. Find

$$\int \left(10x^4 - 4x - \frac{3}{\sqrt{x}} \right) dx$$

giving each term in its simplest form.

(4)

$$2) \int \left(10x^4 - 4x - \frac{3}{\sqrt{x}} \right) dx$$

$$\int 10x^4 - 4x - 3x^{-\frac{1}{2}} dx$$

$$\frac{10x^5}{5} - \frac{4x^2}{2} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\underline{\underline{2x^5 - 2x^2 - 6x^{\frac{1}{2}} + C}}$$

Q2

(Total 4 marks)



3. (a) Find the value of $8^{\frac{5}{3}}$

(2)

(b) Simplify fully $\frac{(2x^{\frac{1}{2}})^3}{4x^2}$

(3)

$$3a) \quad 8^{\frac{5}{3}} = 2^5 = \underline{\underline{32}}$$

$$3b) \quad \frac{(2x^{\frac{1}{2}})^3}{4x^2}$$
$$\frac{8x^{\frac{3}{2}}}{4x^2}$$
$$\underline{\underline{2x^{-\frac{1}{2}}}}$$



4. A sequence a_1, a_2, a_3, \dots is defined by

$$\begin{aligned} a_1 &= 4 \\ a_{n+1} &= k(a_n + 2), \quad \text{for } n \geq 1 \end{aligned}$$

where k is a constant.

(a) Find an expression for a_2 in terms of k .

(1)

Given that $\sum_{i=1}^3 a_i = 2$,

(b) find the two possible values of k .

(6)

$$\begin{aligned} 4a) \quad a_{n+1} &= k(a_n + 2) \\ a_2 &= k(a_1 + 2) \\ a_1 = 4 \quad a_2 &= k(4 + 2) \\ a_2 &= \underline{\underline{6k}} \end{aligned}$$

$$\begin{aligned} 4b) \quad a_3 &= k(a_2 + 2) \\ &= k(6k + 2) \\ &= 6k^2 + 2k \end{aligned}$$

$$\begin{aligned} \sum_{i=1}^3 a_i = 2 \quad \therefore \quad 4 + 6k + 6k^2 + 2k &= 2 \\ 6k^2 + 8k + 4 &= 2 \\ 6k^2 + 8k + 2 &= 0 \\ 3k^2 + 4k + 1 &= 0 \\ (3k + 1)(k + 1) &= 0 \\ \underline{\underline{k = -\frac{1}{3}}} \quad \underline{\underline{k = -1}} \end{aligned}$$



5. Find the set of values of x for which

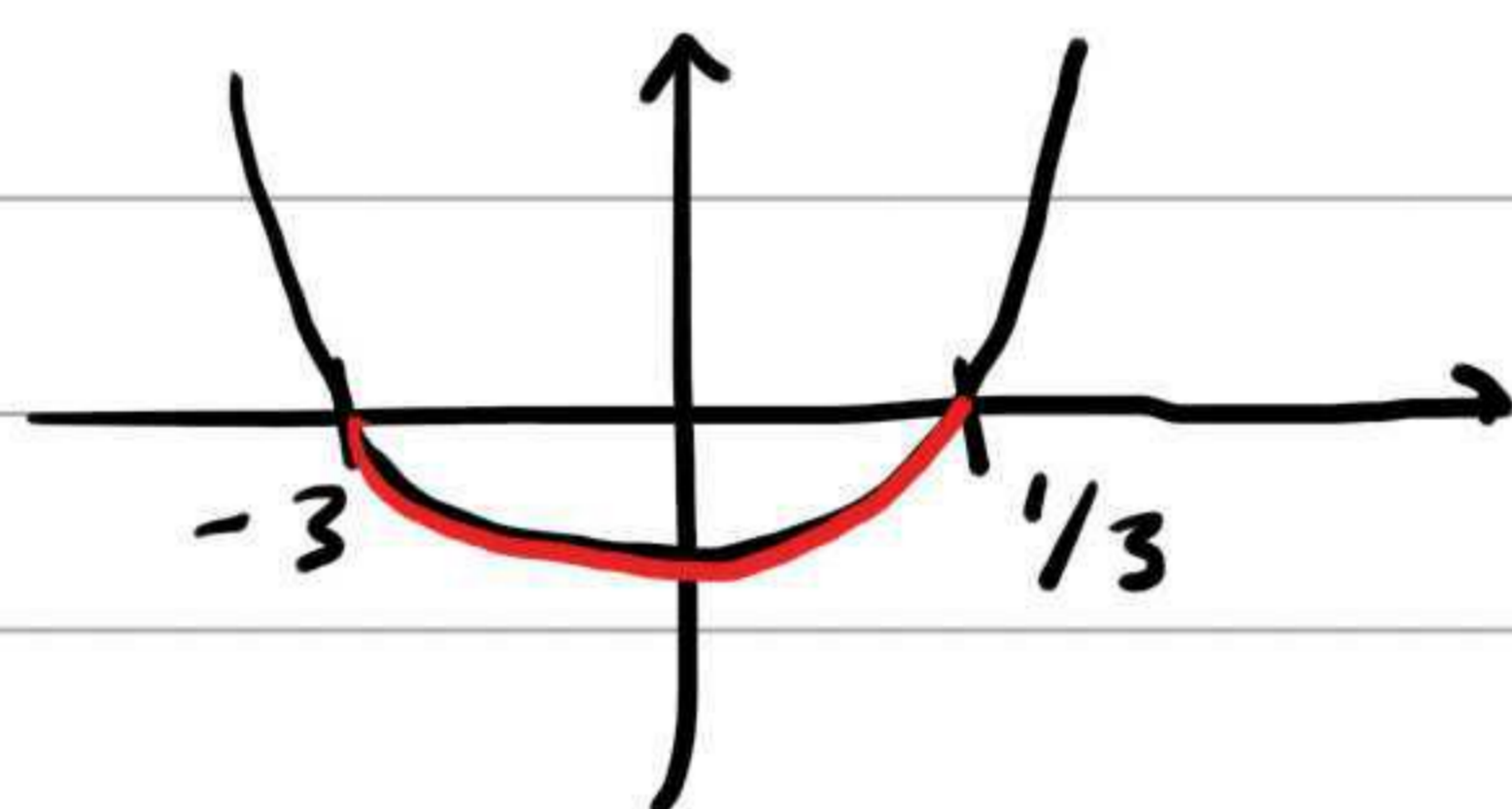
$$(a) \quad 2(3x + 4) > 1 - x \quad (2)$$

$$(b) \quad 3x^2 + 8x - 3 < 0 \quad (4)$$

$$\begin{aligned} 5a) \quad & 2(3x + 4) > 1 - x \\ & 6x + 8 > 1 - x \\ & 7x + 8 > 1 \\ & 7x > -7 \\ & \underline{\underline{x > -1}} \end{aligned}$$

$$\begin{aligned} 5b) \quad & 3x^2 + 8x - 3 < 0 \\ & (3x - 1)(x + 3) < 0 \end{aligned}$$

$f(x) = 0$ at $x = \frac{1}{3}$ and $x = -3$



$$\underline{\underline{-3 < x < \frac{1}{3}}}$$



6. The straight line L_1 passes through the points $(-1, 3)$ and $(11, 12)$.

(a) Find an equation for L_1 in the form $ax + by + c = 0$,

where a , b and c are integers.

(4)

The line L_2 has equation $3y + 4x - 30 = 0$.

(b) Find the coordinates of the point of intersection of L_1 and L_2 .

(3)

$$6a) \quad \begin{array}{cc} (-1, 3) & (11, 12) \\ x_1, y_1 & x_2, y_2 \end{array}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 3}{11 - (-1)} = \frac{9}{12} = \frac{3}{4}$$

$$y = \frac{3}{4}x + c$$

$$\begin{array}{l} (-1, 3) \\ \text{"y"} \end{array} \quad \begin{array}{l} 3 = \frac{3}{4}(-1) + c \\ 3 = -\frac{3}{4} + c \end{array}$$

$$3\frac{3}{4} = c$$

$$c = \frac{15}{4}$$

$$y = \frac{3}{4}x + \frac{15}{4}$$

$$4y = 3x + 15$$

$$0 = 3x - 4y + 15$$

$$6b) \quad \begin{array}{l} 3x - 4y + 15 = 0 \\ 3y + 4x - 30 = 0 \end{array}$$

$$3x - 4y = -15 \quad \times 4$$

$$4x + 3y = 30 \quad \times 3$$

$$12x - 16y = -60$$

$$12x + 9y = 90$$

$$-25y = -150$$

$$y = 6$$



Question 6 continued

$$4x + 3y = 30$$

$$y = 6 \quad 4x + 3(6) = 30$$

$$4x + 18 = 30$$

$$4x = 12$$

$$x = 3$$

The lines intersect at $(3, 6)$.

Q6

(Total 7 marks)



7. A company, which is making 200 mobile phones each week, plans to increase its production.

The number of mobile phones produced is to be increased by 20 each week from 200 in week 1 to 220 in week 2, to 240 in week 3 and so on, until it is producing 600 in week N .

- (a) Find the value of N . (2)

The company then plans to continue to make 600 mobile phones each week.

- (b) Find the total number of mobile phones that will be made in the first 52 weeks starting from and including week 1. (5)

$$7a) \quad a=200 \quad d=20$$

$$\begin{aligned}
 U_n &= 600 & U_n &= a + (n-1)d \\
 600 &= 200 + (N-1)20 \\
 400 &= (N-1)20 \\
 400 &= 20N - 20 \\
 420 &= 20N \\
 N &= 21
 \end{aligned}$$

7b) In the first 21 weeks:

$$\begin{aligned}
 S_n &= \frac{n}{2}(2a + (n-1)d) \\
 S_{21} &= \frac{21}{2}(2(200) + 20(20)) \\
 &= \frac{21}{2}(400 + 400) \\
 &= \frac{21}{2}(800) \\
 &= 21 \times 400 \\
 &= 8400
 \end{aligned}$$

$$\begin{aligned}
 \text{In the following 31 weeks: } & 31 \times 600 \\
 &= 18600
 \end{aligned}$$

$$\begin{array}{r}
 18600 \\
 + 8400 \\
 \hline
 27000
 \end{array}$$

27000 phones



8.

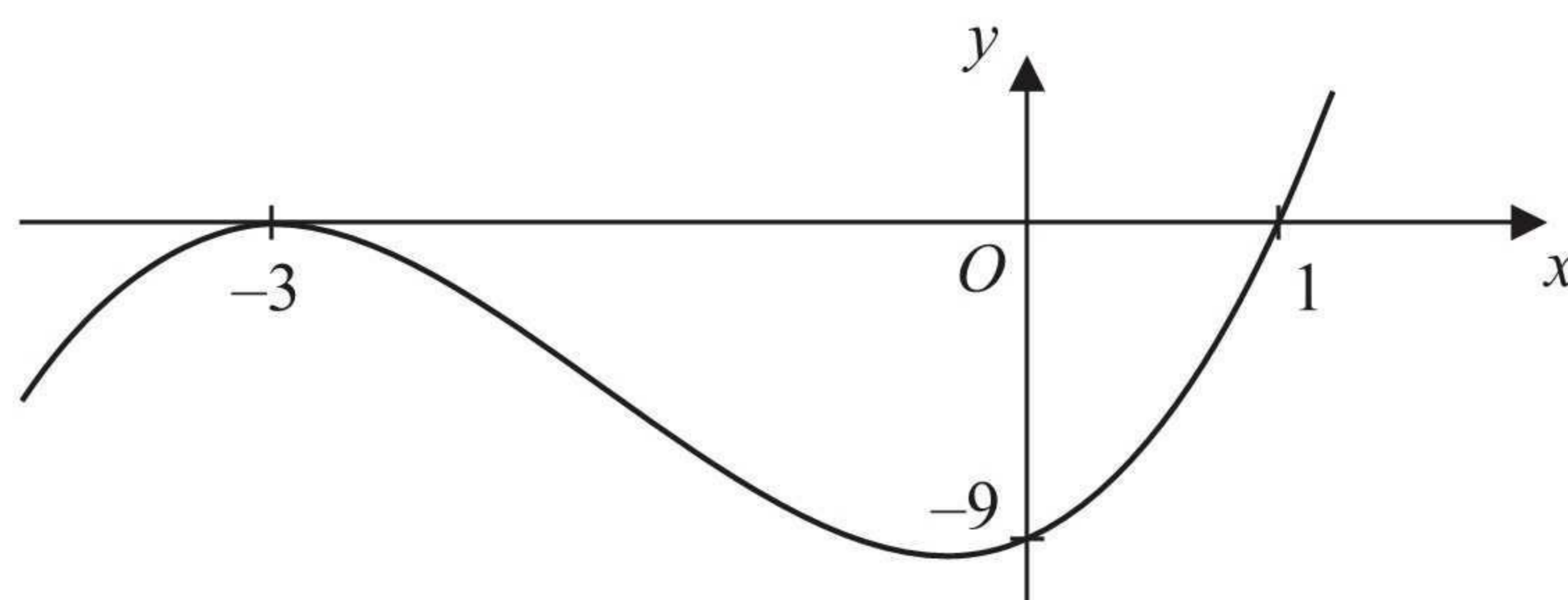


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = (x + 3)^2(x - 1), \quad x \in \mathbb{R}.$$

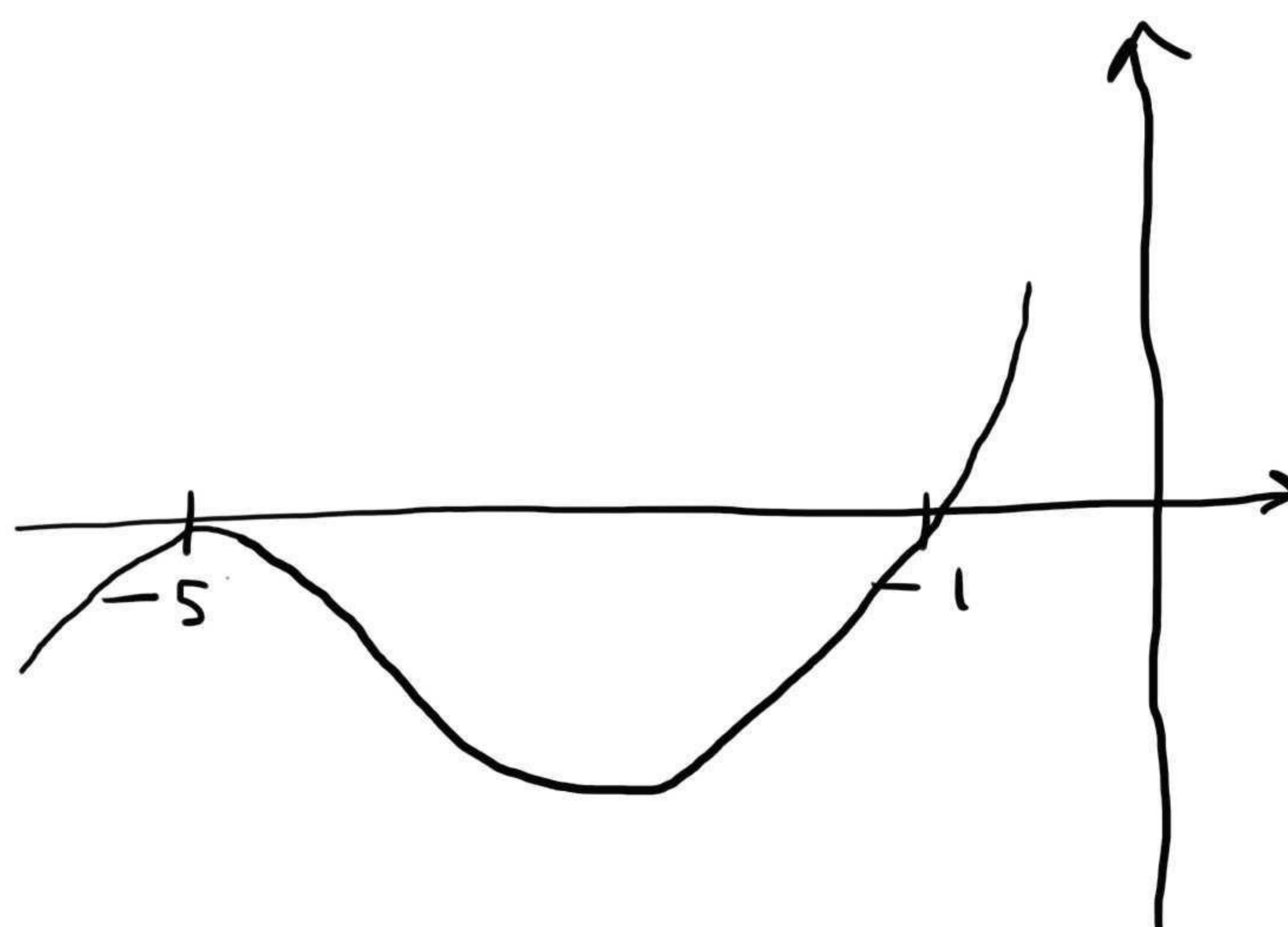
The curve crosses the x -axis at $(1, 0)$, touches it at $(-3, 0)$ and crosses the y -axis at $(0, -9)$

(a) In the space below, sketch the curve C with equation $y = f(x + 2)$ and state the coordinates of the points where the curve C meets the x -axis. (3)

(b) Write down an equation of the curve C . (1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C meets the y -axis. (2)

8a)



Question 8 continued

$$8b) \quad f(x) = (x+3)^2(x-1)$$

$$f(x+2) = (x+2+3)^2(x+2-1)$$

$$\underline{f(x+2) = (x+5)^2(x+1)}$$

8c) curve crosses the y axis when $x=0$

$$y = (5)^2(1)$$

$$y = \underline{\underline{25}}$$

$$\underline{\underline{(0, 25)}}$$



Question 8 continued

Q8

(Total 6 marks)



P 4 1 8 0 2 A 0 1 7 2 8

9.

$$f'(x) = \frac{(3-x^2)^2}{x^2}, \quad x \neq 0$$

(a) Show that $f'(x) = 9x^{-2} + A + Bx^2$,

where A and B are constants to be found.

(3)

(b) Find $f''(x)$.

(2)

Given that the point $(-3, 10)$ lies on the curve with equation $y = f(x)$,

(c) find $f(x)$.

(5)

$$\begin{aligned} 9a) \quad f'(x) &= \frac{(3-x^2)^2}{x^2} \\ &= \frac{(3-x^2)(3-x^2)}{x^2} \\ &= \frac{9 - 3x^2 - 3x^2 + x^4}{x^2} \\ &= \frac{9 - 6x^2 + x^4}{x^2} \\ &= 9x^{-2} - 6 + x^2 \end{aligned}$$

$$A = -6 \quad B = 1$$

$$9b) \quad f'(x) = 9x^{-2} - 6 + x^2$$

$$f''(x) = -18x^{-3} + 2x$$

$$9c) \quad f'(x) = 9x^{-2} - 6 + x^2$$

$$f(x) = \frac{9x^{-1}}{-1} - 6x + \frac{x^3}{3} + C$$

$$y = -9x^{-1} - 6x + \frac{x^3}{3} + C$$

$$\begin{matrix} (-3, 10) \\ \underbrace{x} \quad \underbrace{y} \end{matrix} \quad 10 = -9(-3)^{-1} - 6(-3) + \frac{(-3)^3}{3} + C$$



Question 9 continued

$$10 = \frac{-9}{-3} + 18 - \frac{27}{3} + C$$

$$10 = 3 + 18 - 9 + C$$

$$10 = 12 + C$$

$$C = -2$$

$$\therefore f(x) = -9x^{-1} - 6x + \frac{x^3}{3} - 2$$

(Total 10 marks)

Q9



10. Given the simultaneous equations

$$\begin{aligned} 2x + y &= 1 \\ x^2 - 4ky + 5k &= 0 \end{aligned}$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0 \quad (2)$$

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k . (3)

(c) For this value of k , find the solution of the simultaneous equations. (3)

$$\begin{aligned} 10a) \quad 2x + y &= 1 \\ y &= 1 - 2x \end{aligned}$$

$$\begin{aligned} y = 1 - 2x \quad x^2 - 4ky + 5k &= 0 \\ x^2 - 4k(1 - 2x) + 5k &= 0 \\ x^2 - 4k + 8kx + 5k &= 0 \\ x^2 + k + 8kx &= 0 \\ x^2 + 8kx + k &= 0 \end{aligned}$$

$$10b) \quad \text{equal roots means } b^2 - 4ac = 0$$

$$a = 1 \quad b = 8k \quad c = k$$

$$(8k)^2 - 4(1)(k) = 0$$

$$64k^2 - 4k = 0$$

$$4k(16k - 1) = 0$$

$$k = 0 \quad k = \frac{1}{16}$$

k is non-zero $\therefore k = \frac{1}{16}$

$$10c) \quad x^2 + 8kx + k = 0$$

$$k = \frac{1}{16} \quad x^2 + 8\left(\frac{1}{16}\right)x + \frac{1}{16} = 0$$

$$16x^2 + 8x + 1 = 0$$

$$(4x + 1)(4x + 1) = 0$$



Question 10 continued

$$x = -\frac{1}{4}$$

$$y = 1 - 2x$$

$$x = -\frac{1}{4} \quad y = 1 - 2\left(-\frac{1}{4}\right)$$

$$y = 1 + \frac{1}{2}$$

$$y = \frac{3}{2}$$

$$\underline{x = -\frac{1}{4}, y = \frac{3}{2}}$$



11.

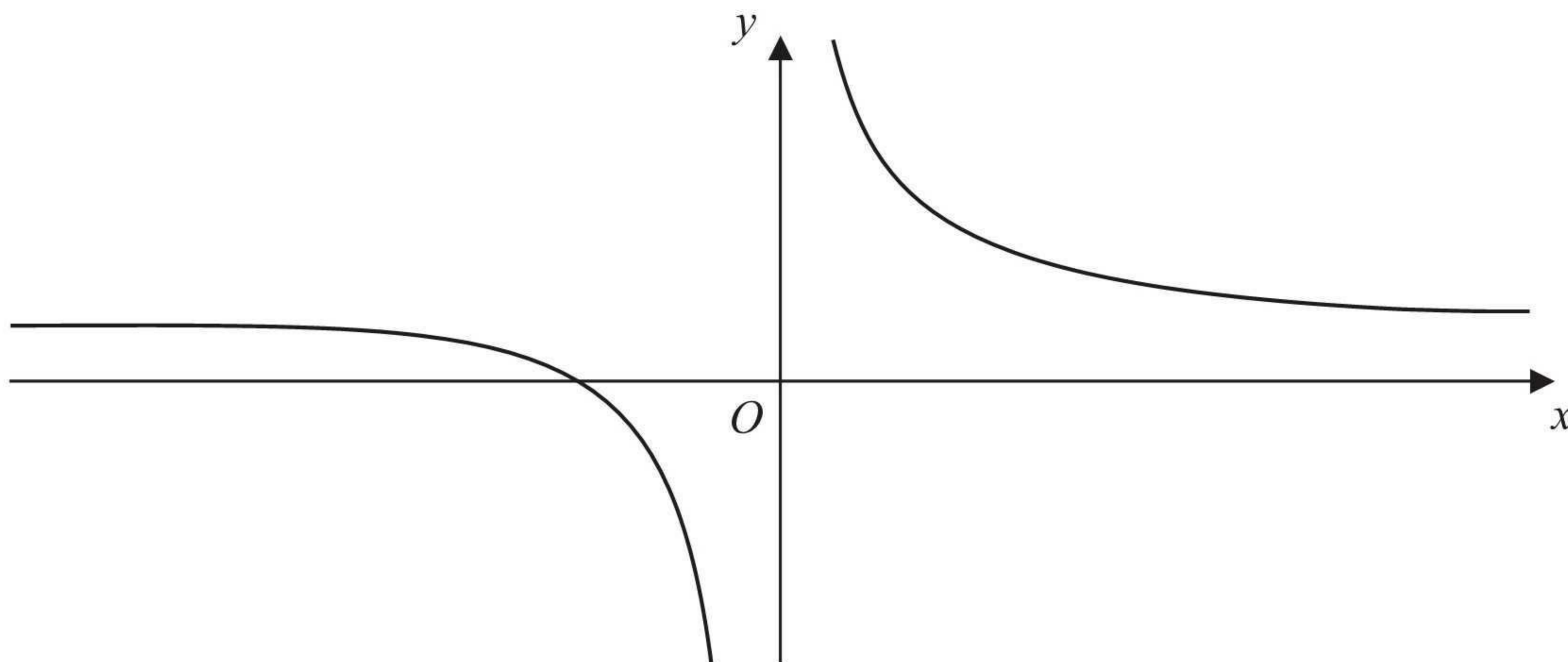


Figure 2

Figure 2 shows a sketch of the curve H with equation $y = \frac{3}{x} + 4$, $x \neq 0$.

(a) Give the coordinates of the point where H crosses the x -axis. (1)

(b) Give the equations of the asymptotes to H . (2)

(c) Find an equation for the normal to H at the point $P(-3, 3)$. (5)

This normal crosses the x -axis at A and the y -axis at B .

(d) Find the length of the line segment AB . Give your answer as a surd. (3)

$$11a) \quad y = \frac{3}{x} + 4$$

crosses x axis when $y = 0$

$$0 = \frac{3}{x} + 4$$

$$-4 = \frac{3}{x}$$

$$x = -\frac{3}{4}$$

$$11b) \quad x = 0 \quad y = 4$$

$$11c) \quad y = 3x^{-1} + 4$$

$$\frac{dy}{dx} = -3x^{-2}$$

$$(-3, 3) \text{ when } x = -3 \quad \frac{dy}{dx} = -3(-3)^{-2} = -\frac{3}{9} = -\frac{1}{3}$$



Question 11 continued

$$m = 3 \quad (-3, 3)$$

$x \quad y$

$$y = 3x + c$$

$$3 = 3(-3) + c$$

$$3 = -9 + c$$

$$c = 12$$

Equation of normal : $y = 3x + 12$

11d) A) crosses x axis when $y = 0$

$$0 = 3x + 12$$

$$-12 = 3x$$

$$-4 = x$$

$$(-4, 0)$$

B) crosses y axis when $x = 0$

$$y = 12$$

$$(0, 12)$$

Length of line segment.

$$\sqrt{4^2 + 12^2}$$

$$\sqrt{16 + 144}$$

$$\sqrt{160}$$

$$\sqrt{16 \times 10}$$

$$4\sqrt{10}$$

