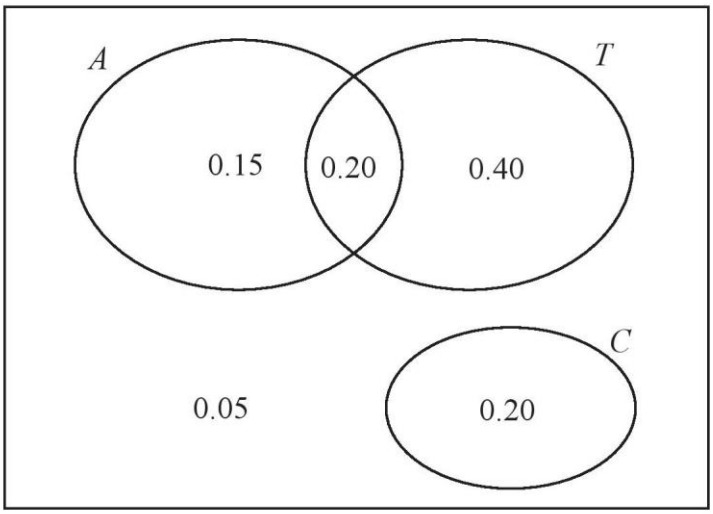
Paper 2: Statistics and Mechanics Mark Scheme

Question	Scheme	Marks	AOs
1 (a)	Systematic (sample)	B1cao	1.2
(b)	In LDS some days have gaps because the data was not recorded	B1	2.4
(c)	$\left[\bar{t} = \frac{374}{20} = 18.7\right]$ $\sigma_t = \sqrt{\frac{7600}{20} - \bar{t}^2}$ $[= \sqrt{30.31}]$	M1	1.1a
	= 5.5054… awrt **5.51** (Accept use of $s_t = \sqrt{\frac{7600 - 20\bar{t}^2}{19}}$ = 5.6484…)	A1	1.1b
			(4 marks)

Part	Notes
(b)	B1 a correct explanation
(c)	M1 for a correct expression for $\bar{t}$ and σ_t or s_t . Ft an incorrect evaluation of $\bar{t}$
	A1 for σ_t = awrt 5.51 or s_t = awrt 5.65

Question	Scheme	Marks	AOs
2	$17 + 45 + \frac{1}{3}\times 9$ [= 65]	M1	2.2a
	(7 – 8) **14** or (16 – 20) **5** [Values may be seen in the table]	M1 A1	3.1a 1.1b
	Percentage of motorists is $\frac{"65"}{6+"14"+17+45+9+"5"}\times 100$	M1	3.1b
	= **67.7%**	A1	1.1b
			(5 marks)

Part	Notes
	1st M1 for a fully correct expression for the number of motorists in the interval
	2nd M1 for clear use of frequency density in (4-6) or (13-15) cases to establish the fd scale. Then use of area to find frequency in one of the missing cases.
	1st A1 for both correct values seen
	3rd M1 for realising that total is required and attempting a correct expression for %
	2nd A1 for awrt 67.7%

Question	Scheme	Marks	AOs
3 (a)	p = [1 – 0.75 – 0.05 =] **0.20**	B1	1.1b
		(1)	
(b)	q = **0.15**	B1ft	1.1b
	P(A) = 0.35 P(T) = 0.6 P(A and T) = 0.20 P(A)×P(T) = 0.21	M1	2.1
	Since 0.20 ≠ 0.21 therefore A and T are **not** independent	A1	2.4
		(3)	
			
(c)	P(not [A or C]) = **0.45**	B1	1.1b
		(1)	
			(5 marks)

Part	Notes
(a)	B1cao for p = 0.20
(b)	B1ft for use of their p and P(A or T) to find q i.e. 0.75 – "p" – 0.40 or q = 0.15
	M1 for the statement of all probabilities required for a suitable test and sight of any appropriate calculations required.
	A1 All probabilities correct, correct comparison and suitable comment.
(c)	B1cao for 0.45

Question	Scheme	Marks	AOs
4(a)	IQR = 2.3 and 20.6 $\gg$ 2.4 + 1.5$\times$2.3 (= 5.85) (Compare correct values)	B1	1.1b
		(1)	
(b)(i)	e.g. it is a piece of data and we should consider all the data (o.e.)	B1	2.4
(ii)	e.g. it is an extreme value and could unduly influence the analysis or it could be a mistake	B1	2.4
		(2)	
(c)	e.g. "as humidity increases rainfall increases"	B1	2.2b
		(1)	
(d)	e.g. a 10% increase in humidity gives rise to a 1.5 mm increase in rainfall or represents 0.15mm of rainfall per percentage of humidity	B1	3.4
		(1)	
(e)(i)	Not a good method since only uses 11 days from one location in one month.	B1	2.4
(ii)	e.g. She should use data from more of the UK locations and more of the months or using a spreadsheet or computer package she could use all of the available UK data	B1	2.4
		(2)	
		(7 marks)	

Part	Notes
(a)	B1 for sight of the correct calculation and suitable comparison with 20.6
(b)(i)	B1 for a suitable reason for including the data point
(ii)	B1 for a suitable reason for excluding the data point
(c)	B1 for a suitable interpretation of positive correlation mentioning humidity and rainfall
(d)	B1 for a suitable description of the rate: rainfall per percentage of humidity including reference to values.
(e)(i)	B1 for a comment that supports the idea that her sampling method was not a good one
(ii)	B1 for some sensible suggestions that would give a better representation of the data across the UK. Must show some awareness of the fact that LDS has different locations and more months of data available but must be clear they are NOT using any overseas locations. NB B0 for a comment that says use more than one location without specifying that only UK locations are required.

Question	Scheme	Marks	AOs
5(a)	$P(X \geqslant 16) = 1 - P(X \leqslant 15)$	M1	1.1b
	$= 1 - 0.949077\ldots$ = awrt **<u>0.0509</u>**	A1	1.1b
		(2)	
(b)	$H_0 : p = 0.3$ $H_1 : p \neq 0.3$ (Both correct in terms of p or π)	B1	2.5
		(1)	
(c)	[$Y \sim B(20, 0.3)$] sight of $P(Y \leqslant 2) = 0.0355$ or $P(Y \leqslant 9) = 0.9520$	M1	2.1
	Critical region is $\{Y \leqslant \mathbf{2}\}$ or (o.e.)	A1	1.1b
	$\{Y \geqslant \mathbf{10}\}$ (o.e.)	A1	1.1b
		(3)	
(d)	[0.0355 + (1 – 0.9520)] = 0.0835 <u>or</u> **<u>8.35%</u>**	B1ft	1.1b
		(1)	
(e)	(Assuming that the 20 customers represent a random sample then) 12 is in the CR so the manager's suspicion is supported	B1ft	3.2a
		(1)	
(f)	e.g. (e) requires the 20 customers to be a random sample or independent and the members of the scout group may invalidate this so binomial distribution would not be valid (and conclusion in (e) is probably not valid)	B1	3.5a
		(1)	
			(9 marks)

Part	Notes
(a)	M1 for dealing with $P(X \geqslant 16)$ – they need to use cumulative prob. function on calc.
	A1 awrt 0.0509 (from calculator)
(b)	B1 for both hypotheses in terms of p or π and H_1 must be 2-tail
(c)	M1 for correct use of tables to find probability associated with critical value.
	1st A1 for the correct lower limit of the CR. Do not award for $P(Y \leqslant 2)$
	2nd A1 for the correct upper limit.
(d)	B1ft ft on their 0.0355 and (1 – their 0.9520) provided each probability is less than 0.05
(e)	B1ft for a comment that relates 12 to their CR and makes a consistent comment relating this to the manager's suspicion
(f)	B1 for a comment that: gives a suitable reason based on lack of independence <u>or</u> the sample not being random <u>so</u> the binomial model is not valid

Question	Scheme	Marks	AOs
6.	Using distance = total area under graph (e.g. area of rectangle + triangle or trapezium or rectangle – triangle)	M1	2.1
	e.g. $D = UT + \frac{1}{2} Th$, where h is height of triangle	A1	1.1b
	Using gradient = acceleration to substitute $h = aT$	M1	1.1b
	$D = UT + \frac{1}{2} aT^2$ *	A1 *	1.1b
		4	
			(4 marks)

Notes

1st M1 for use of distance = total area to give an equation in D, U, T and one other variable
1st A1 for a correct equation
2nd M1 for using gradient = a to eliminate other variable to give an equation in D, U, T and a *only*
2nd A1* for a correct **given answer**

Question	Scheme	Marks	AOs
7(i)(ii)	Using a correct strategy for solving the problem by setting up two equations in a and u only and solving for either	M1	3.1b
	Equation in a and u only	M1	3.1b
	$22 = 2u + \frac{1}{2} a 2^2$	A1	1.1b
	Another equation in a and u only	M1	3.1b
	$126 = 6u + \frac{1}{2} a 6^2$	A1	1.1b
	5 m s^{-2}	A1	1.1b
	6 m s^{-1}	A1 **ft**	1.1b
			(7 marks)

Notes

1st M1 for solving the problem by setting up two equations in a and u only and solving for either
2nd M1 use of (one or more) *suvat* formulae to produce equation in u and a only
1st A1 for a correct equation
3rd M1 use of (one or more) *suvat* formulae to produce another equation in u and a only
2nd A1 for a correct equation
3rd A1 for correct accln 5 m s^{-2}
4th A1 for correct speed 6 m s^{-1} (The second of these A marks is an **ft** mark, following an incorrect value for u or a, depending on which has been found first)
N.B. Do not award the **ft** mark for absurd answers e.g. $a > 15$, $u > 50$

See alternative on next page

ALTERNATIVE

Question	Scheme	Marks	AOs
7(i)(ii)	Using a correct strategy for solving the problem by obtaining actual speeds at two times and using a = change in speed / time taken.	M1	3.1b
	Actual speed at $t = 1$ = Average speed over interval	M1	3.1b
	$22/2 = 11$	A1	1.1b
	Actual speed at $t = 4$ = Average speed over interval	M1	3.1b
	$104/4 = 26$	A1	1.1b
	$5\ m\ s^{-2}$	A1	1.1b
	$6\ m\ s^{-1}$	A1 **ft**	1.1b
			(7 marks)

Notes

1st M1 for solving the problem by obtaining two actual speeds and use of $a = (v - u)/ t$
2nd M1 use of speed at half-time = av speed over interval to produce a speed at $t = 1$
1st A1 for a correct speed
3rd M1 use of speed at half-time = av speed over interval to produce a speed at $t = 4$
2nd A1 for a correct speed
3rd A1 for correct accln $5\ m\ s^{-2}$
4th A1 **ft** for correct speed $6\ m\ s^{-1}$ (This is an ft mark, following an incorrect value of a)
N.B. Do not award the **ft** mark for absurd answers e.g. $a > 15$, $u > 50$

Question	Scheme	Marks	AOs
8(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t - 10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t - 5)(t - 10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance = 62.5 m	A1 ft	1.1b
		(6)	
			(9 marks)

Notes

(a) M1 for substituting $t = 0$ and $t = 10$ into s expression
A1 for noting that $s = 0$ at both times
A1 Since s is a perfect square, $s > 0$ for all other t- values.
(b) 1st M1 for differentiating s w.r.t. t to give v (powers of t reducing by 1)
1st A1 for a correct v expression in any form
2nd M1 for equating v to 0 and factorising
2nd A1 for correct t values
3rd M1 for substituting their intermediate t value into s
3rd A1 **ft** following an incorrect t-value.

Question	Scheme	Marks	AOs
9(a) (i)	Equation of motion for A	M1	3.3
	$T - 12.7 = 2.5a$	A1	1.1b
(ii)	Equation of motion for B	M1	3.3
	$1.5g - T = 1.5a$	A1	1.1b
		(4)	
(b)	Solving two equations for a	M1	1.1b
	$a = 0.5$	A1	1.1b
		(2)	
(c)	$1 = \frac{1}{2} \times 0.5\, t^2$	M1	3.4
	$t = 2$ seconds	A1ft	1.1b
		(2)	
(d)	(i) Not very appropriate for valid reason, see below in notes	**B1**	3.5a
	(ii) Valid improvement in model, see below in notes.	**B1**	3.5c
		(2)	
			(10 marks)

Notes

(a) (i) 1st M1 for resolving horizontally for A
1st A1 for a correct equation
(ii) 2nd M1 for resolving vertically for B
2nd A1 for a correct equation

(b) M1 for complete correct strategy for solving the problem, setting up **two** equations in a, and then solving them for a
A1 for $a = 0.5$

(c) M1 for a complete method (which could involve use of more than one *suvat* formula) to give an equation in t only
A1ft from their a to get time in seconds

(d) (i) B1 for model is inappropriate, with valid reason
e.g. the ball has taken longer to reach the floor because the model
- does not include air resistance
- does not include the roughness of the pulley

or any other appropriate comment

(ii) B1 for e.g. Do not model ball B as a particle but give its dimensions so distance it falls changes
e.g. Do not model pulley as being small so string not parallel to table
e.g. Do not model resistance as being constant