Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 13 May 2020

Morning (Time: 2 hours) | Paper Reference **8MA0/01**

Mathematics

Advanced Subsidiary

Paper 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1/1/1/1/1/
Pearson

1. A curve has equation

$$y = 2x^3 - 4x + 5$$

Find the equation of the tangent to the curve at the point $P(2, 13)$.

Write your answer in the form $y = mx + c$, where m and c are integers to be found.

Solutions relying on calculator technology are not acceptable.

(5)

$\frac{dy}{dx} = 6x^2 - 4$

when $x = 2$ $\quad \frac{dy}{dx} = 6(2)^2 - 4$

$= 20$

$m = 20 \quad (2, 13)$ with x_1, y_1

$y - y_1 = m(x - x_1)$

$y - 13 = 20(x - 2)$

$y - 13 = 20x - 40$

$y = 20x - 27$

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Question 1 continued

(Total for Question 1 is 5 marks)

2. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A coastguard station O monitors the movements of a small boat.

At 10:00 the boat is at the point $(4\mathbf{i} - 2\mathbf{j})$ km relative to O.

At 12:45 the boat is at the point $(-3\mathbf{i} - 5\mathbf{j})$ km relative to O.

The motion of the boat is modelled as that of a particle moving in a straight line at constant speed.

(a) Calculate the bearing on which the boat is moving, giving your answer in degrees to one decimal place.

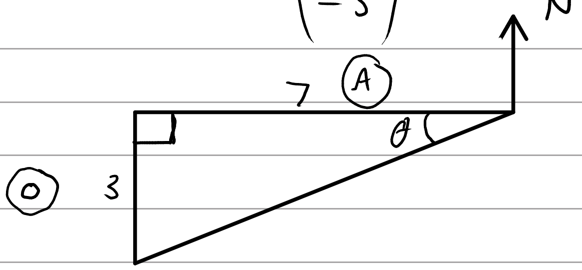
(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

a) Displacement $= \begin{pmatrix} -3 \\ -5 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ -3 \end{pmatrix}$



$\tan(\theta) = \frac{O}{A}$

$\tan(\theta) = \frac{3}{7}$

$\theta = \tan^{-1}\left(\frac{3}{7}\right)$

$= 23.2°$

Bearing $= 360 - 90 - 23.2$

$= \underline{246.8°}$

b) Distance $= \sqrt{3^2 + 7^2}$

$= \sqrt{58}$ km

Speed $= \frac{\text{distance}}{\text{time}} = \frac{\sqrt{58}}{2\text{hrs } 45\text{mins}}$

$= \frac{\sqrt{58}}{2.75}$

$= \underline{2.77 \text{ km/h}}$ (3sf)

P 6 2 6 4 5 R A 0 4 4 0

Question 2 continued

(Total for Question 2 is 6 marks)

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(i) Solve the equation

$$x\sqrt{2} - \sqrt{18} = x$$

writing the answer as a surd in simplest form.

(3)

(ii) Solve the equation

$$4^{3x-2} = \frac{1}{2\sqrt{2}}$$

(3)

i/ $\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

$x\sqrt{2} - 3\sqrt{2} = x$

$x\sqrt{2} = x + 3\sqrt{2}$

$x\sqrt{2} - x = 3\sqrt{2}$

$x(\sqrt{2} - 1) = 3\sqrt{2}$

$x = \dfrac{3\sqrt{2}\,(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}$

$= \dfrac{6 + 3\sqrt{2}}{2 + \sqrt{2} - \sqrt{2} - 1}$

$= 6 + 3\sqrt{2}$

ii/ $4^{3x-2} = \dfrac{1}{2\sqrt{2}}$

$2^{2(3x-2)} = \dfrac{1}{2^1 \times 2^{\frac{1}{2}}}$

$2^{6x-4} = \dfrac{1}{2^{3/2}}$

$2^{6x-4} = 2^{-\frac{3}{2}}$

$6x - 4 = -\frac{3}{2}$

$6x = \frac{5}{2}$

$x = \frac{5}{12}$

Question 3 continued

(Total for Question 3 is 6 marks)

4. In 1997 the average CO_2 emissions of new cars in the UK was 190 g/km.

In 2005 the average CO_2 emissions of new cars in the UK had fallen to 169 g/km.

Given A g/km is the average CO_2 emissions of new cars in the UK n years after 1997 and using a linear model,

(a) form an equation linking A with n.

(3)

In 2016 the average CO_2 emissions of new cars in the UK was 120 g/km.

(b) Comment on the suitability of your model in light of this information.

(3)

a) $(0, 190)$ $(8, 169)$ $x = n$, $y = A$

x_1 y_1 x_2 y_2

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{169 - 190}{8 - 0} = \frac{-21}{8}$

$y - y_1 = m(x - x_1)$

$y - 190 = \frac{-21}{8}(x - 0)$

$y - 190 = -\frac{21}{8}x$

$y = -\frac{21}{8}x + 190$

$A = -\frac{21}{8}n + 190$

b) $n = 19$ $A = 120$

using the model : $A = \frac{-21}{8}(19) + 190$

$= 140.125$ g/km

The model's predection is too high, therefore it may not be suitable

Question 4 continued

(Total for Question 4 is 6 marks)

5.

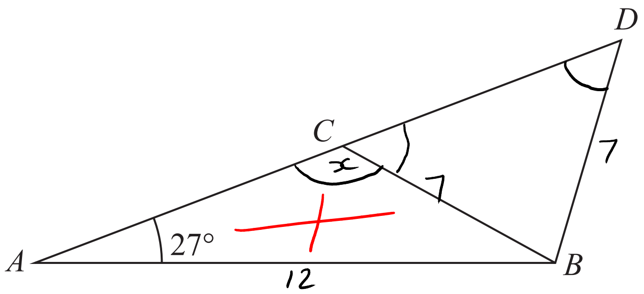


Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four steel beams, AB, BD, BC and AD.

Given $AB = 12$ m, $BC = BD = 7$m and angle $BAC = 27°$

(a) find, to one decimal place, the size of angle ACB.

(3)

The steel beams can only be bought in whole metre lengths.

(b) Find the minimum length of steel that needs to be bought to make the complete structure.

(3)

a/ $\frac{\sin x}{12} = \frac{\sin(27)}{7}$

$\sin x = \frac{\sin(27)}{7} \times 12$

$\sin x = 0.778\ldots$

$x = \sin^{-1}(0.778)$

$= 51.1°$

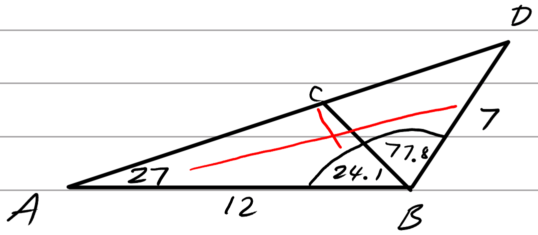
ACB is obtuse $ACB = 180 - 51.1° = \underline{128.9°}$

b/ $BCD = 180 - 128.9 = 51.1°$

$BDC = 51.1°$

$CBD = 180 - 2(51.1)$

$= 77.8°$



$ABC = 180 - 128.9 - 27$

$= 24.1°$

$ABD = 24.1 + 77.8 = \underline{101.9°}$

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Question 5 continued

$$\frac{AD}{\sin(101.9)} = \frac{7}{\sin(27)}$$

$$AD = \frac{7}{\sin(27)} \times \sin(101.9)$$

$$= 15.1 \text{ m}$$

Total length $= 15.1 + 12 + 7 + 7$

$= 41.1$ m

The steel can only be bought in metres $\therefore$ 42m

(Total for Question 5 is 6 marks)

6. (a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a non-zero constant. Write each coefficient as simply as possible.

(3)

Given that in the expansion of $(1 + kx)^{10}$ the coefficient x^3 is 3 times the coefficient of x,

(b) find the possible values of k.

(3)

a/ 1 10 45 120

$1(1)^{10} + 10(1)^9(kx) + 45(1)^8(kx)^2 + 120(1)^7(kx)^3$

$1 + 10kx + 45k^2x^2 + 120k^3x^3$

b/ $120k^3 = 3(10k)$

$120k^3 = 30k$

$4k^3 = k$

$4k^3 - k = 0$

$k(4k^2 - 1) = 0$ k is not zero

$4k^2 - 1 = 0$

$4k^2 = 1$

$k^2 = \frac{1}{4}$

$k = \pm\sqrt{\frac{1}{4}}$

$k = \pm\frac{1}{2}$

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Question 6 continued

(Total for Question 6 is 6 marks)

7. Given that k is a positive constant and $\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$

(a) show that $3k + 5\sqrt{k} - 12 = 0$

(4)

(b) Hence, using algebra, find any values of k such that

$$\int_1^k \left(\frac{5}{2\sqrt{x}} + 3\right) dx = 4$$

(4)

a/ $\int_1^k \left(\frac{5}{2}x^{-\frac{1}{2}} + 3\right) dx = 4$

$\left[5x^{\frac{1}{2}} + 3x + c\right]_1^k = 4$

$\left(5k^{\frac{1}{2}} + 3k + c\right) - \left(5(1)^{\frac{1}{2}} + 3(1) + c\right) = 4$

$\left(5k^{\frac{1}{2}} + 3k + c\right) - \left(5 + 3 + c\right) = 4$

$5k^{\frac{1}{2}} + 3k - 8 = 4$

$5\sqrt{k} + 3k - 12 = 0$

$3k + 5\sqrt{k} - 12 = 0$

let $\sqrt{k} = x$ $\qquad 3x^2 + 5x - 12 = 0$

$(3x - 4)(x + 3)$

$x = \frac{4}{3} \qquad x = -3$

$\sqrt{k} = \frac{4}{3} \qquad \sqrt{k} = -3$

$k = \frac{16}{9} \qquad \times$

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Question 7 continued

(Total for Question 7 is 8 marks)

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Turn over

8. The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,
(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35 °C.
(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15 °C.
(1)

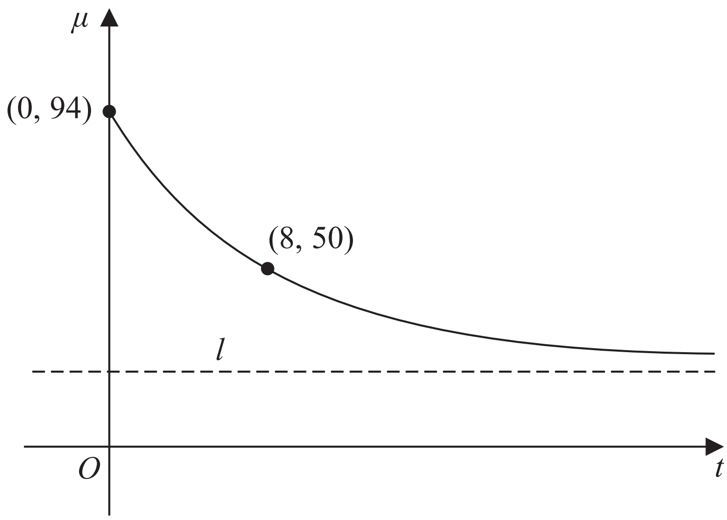

Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.
(4)

a) when $t=0$ $\quad \theta = 18 + 65$

$= 83°$

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Question 8 continued

b/ $35 = 18 + 65e^{-\frac{t}{8}}$

$17 = 65e^{-\frac{t}{8}}$

$\frac{17}{65} = e^{-\frac{t}{8}}$

$\ln\left(\frac{17}{65}\right) = -\frac{t}{8}$

$8\ln\left(\frac{17}{65}\right) = -t$

$t = -8\ln\left(\frac{17}{65}\right)$

$t = 10.7$ mins

c/ As t increases (getting closer to ∞) θ gets closer to 18°C. (The temperature cannot drop below 18°C)

d/ $\mu = A + Be^{-\frac{t}{8}}$

when $t=0$ $\mu = 94$ $94 = A + B$

when $t=8$ $\mu = 50$ $50 = A + Be^{-1}$

$44 = B - Be^{-1}$

$44 = B(1 - e^{-1})$

$B = \frac{44}{1-e^{-1}}$

$= 69.6$

$A + B = 94$

$A = 94 - 69.6$

$= 24.4$

$\mu = 24.4$

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 9 marks)

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9.

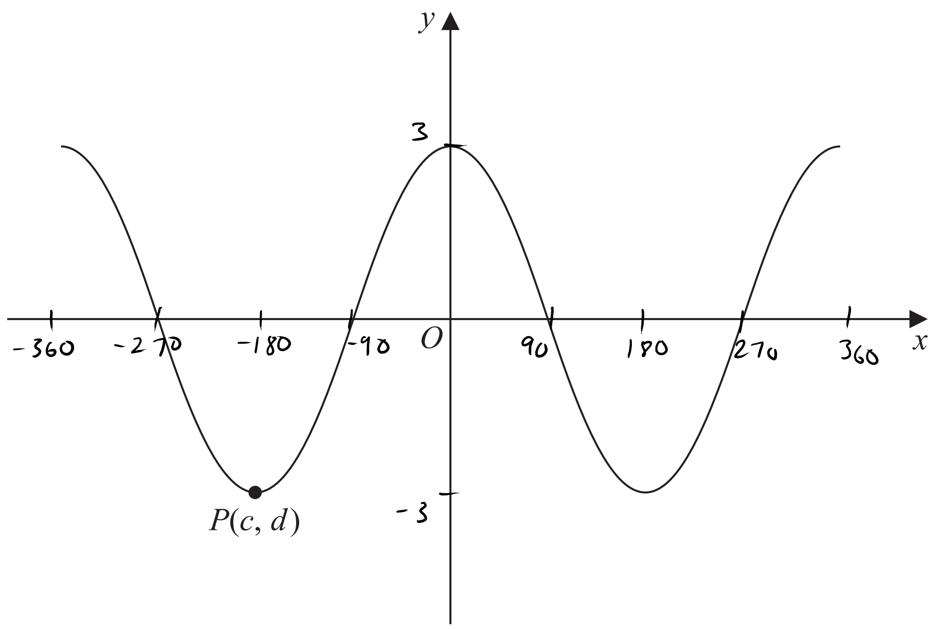

Figure 3

Figure 3 shows part of the curve with equation $y = 3\cos x°$.

The point $P(c, d)$ is a minimum point on the curve with c being the smallest negative value of x at which a minimum occurs.

(a) State the value of c and the value of d.

(1)

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 3\cos x°$ to the curve with equation

(i) $y = 3\cos\left(\frac{x°}{4}\right)$

(ii) $y = 3\cos(x - 36)°$

(2)

(c) Solve, for $450° \leqslant \theta < 720°$,

$$3\cos\theta = 8\tan\theta$$

giving your solution to one decimal place.

In part (c) you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(5)

Question 9 continued

a/ $(-180, -3)$

b/ $(-720, -3)$

ii/ $(-144, -3)$

c/ $3\cos\theta = 8\tan\theta$

$3\cos\theta = 8\dfrac{\sin\theta}{\cos\theta}$

$3\cos^2\theta = 8\sin\theta$ $\qquad \cos^2\theta + \sin^2\theta = 1$

$3(1-\sin^2\theta) = 8\sin\theta$ $\qquad \cos^2\theta = 1 - \sin^2\theta$

$3 - 3\sin^2\theta = 8\sin\theta$

$0 = 3\sin^2\theta + 8\sin\theta - 3$

$0 = (3\sin\theta - 1)(\sin\theta + 3)$

$\sin\theta = \frac{1}{3}$ $\quad \sin\theta = -3$ ✗

$\theta = 19.5°, 160.5°, 379.5°, 520.5°$

$160.5 \leftarrow 180 - 19.5$; $379.5 \leftarrow 19.5 + 360$; $520.5 \leftarrow 160.5 + 360$

$\underline{\underline{\theta = 520.5°}}$

Question 9 continued

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Question 9 continued

(Total for Question 9 is 8 marks)

10. $$g(x) = 2x^3 + x^2 - 41x - 70$$

(a) Use the factor theorem to show that g(x) is divisible by (x – 5). **(2)**

(b) Hence, showing all your working, write g(x) as a product of three linear factors. **(4)**

The finite region R is bounded by the curve with equation $y = g(x)$ and the x-axis, and lies below the x-axis.

(c) Find, using algebraic integration, the exact value of the area of R. **(4)**

a) $g(5) = 2(5)^3 + (5)^2 - 41(5) - 70$

$= 0$

b)

	$2x^2$	$+11x$	$+14$
x	$2x^3$	$+11x^2$	$+14x$
-5	$-10x^2$	$-55x$	-70

$(x-5)(2x^2 + 11x + 14)$

$(x-5)(2x+7)(x+2)$

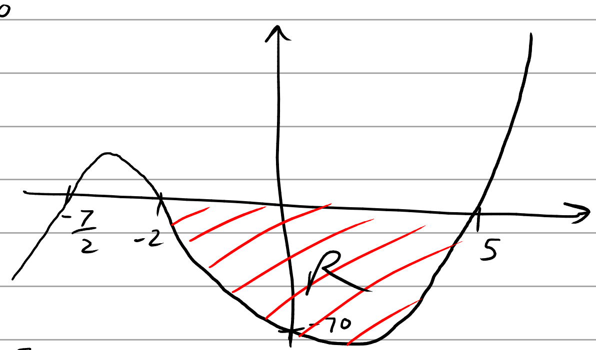
c) $y = (x-5)(2x+7)(x+2)$

when $y = 0$ $(x-5)(2x+7)(x+2) = 0$

$x = 5$ $x = -\frac{7}{2}$ $x = -2$

when $x = 0$ $y = (-5)(7)(2)$

$= -70$



$\int_{-2}^{5} 2x^3 + x^2 - 41x - 70 \; dx$

$\left[\frac{1}{2}x^4 + \frac{1}{3}x^3 - \frac{41}{2}x^2 - 70x\right]_{-2}^{5}$

Question 10 continued

$$\left(\frac{1}{2}(5)^4 + \frac{1}{3}(5)^3 - \frac{41}{2}(5)^2 - 70(5)\right) - \left(\frac{1}{2}(-2)^4 + \frac{1}{3}(-2)^3 - \frac{41}{2}(-2)^2 - 70(-2)\right)$$

$$-\frac{1525}{3} - \left(\frac{190}{3}\right)$$

$$= -\frac{1715}{3}$$

$$\text{Area} = \frac{1715}{3} \text{ units}^2$$

Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

11. (i) A circle C_1 has equation

$$x^2 + y^2 + 18x - 2y + 30 = 0$$

The line l is the tangent to C_1 at the point $P(-5, 7)$.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 8x + 12y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the 4th quadrant, find the range of possible values for k.

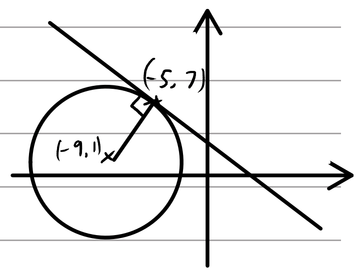
(4)

a/ $x^2 + 18x + y^2 - 2y + 30 = 0$

$(x+9)^2 - 81 + (y-1)^2 - 1 + 30 = 0$

$(x+9)^2 + (y-1)^2 = 52$

centre $(-9, 1)$ [x_1, y_1] $P(-5, 7)$ [x_2, y_2]



gradient of radius $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{7-1}{-5--9} = \dfrac{6}{4} = \dfrac{3}{2}$

gradient of tangent $= -\dfrac{2}{3}$ $P(-5, 7)$ [x, y]

$y - y_1 = m(x - x_1)$

$y - 7 = \dfrac{-2}{3}(x+5)$

$3(y-7) = -2(x+5)$

$3y - 21 = -2x - 10$

$2x + 3y - 11 = 0$

Question 11 continued

b/ $x^2 - 8x + y^2 + 12y + k = 0$

$(x-4)^2 - 16 + (y+6)^2 - 36 + k = 0$

$(x-4)^2 + (y+6)^2 = 52 - k$

The radius must be less than 4

$\sqrt{52-k} < 4$

$52 - k < 16$

$36 < k$

$52 - k$ must also be positive $\therefore k < 52$

$\underline{\underline{36 < k < 52}}$

Question 11 continued

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Question 11 continued

(Total for Question 11 is 9 marks)

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12. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30,\ t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live. Give your answer to 2 significant figures.

(2)

a) $\log_{10} V = 0.072t + 2.379$

$V = 10^{0.072t + 2.379}$

$V = 10^{0.072t} \times 10^{2.379}$

$= 10^{2.379}(10^{0.072})^t$

$= 239(1.18)^t$

b) ab is the number of views when t=1 (after 1 day)

c) $239(1.18)^{20} = \underline{\underline{6600}}$ (2sf)

P 6 2 6 4 5 R A 0 3 2 4 0

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Question 12 continued

Question 12 continued

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Question 12 continued

(Total for Question 12 is 7 marks)

13. (a) Prove that for all positive values of a and b

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b.

(1)

$$\frac{4a}{b} + \frac{b}{a} \geqslant 4$$

$$\frac{4a^2}{ab} + \frac{b^2}{ab} \geqslant 4$$

$$\frac{4a^2+b^2}{ab} \geqslant 4$$

$$4a^2 + b^2 \geqslant 4ab$$

$$4a^2 - 4ab + b^2 \geqslant 0$$

$$(2a-b)(2a-b) \geqslant 0$$

$$(2a-b)^2 \geqslant 0$$

when $(2a-b)$ is squared it will be greater than (or equal to) zero.

b/ $\frac{4a}{b} + \frac{b}{a} \geqslant 4$

when $a=-1$ $b=1$ $\frac{4(-1)}{1} + \frac{1}{-1}$

$-4 - 1 = -5$

-5 is less than 4

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Question 13 continued

(Total for Question 13 is 5 marks)

P 6 2 6 4 5 R A 0 3 7 4 0

14. A curve has equation $y = \mathrm{g}(x)$.

Given that

- $\mathrm{g}(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = \mathrm{g}(x)$ passes through the origin   when $x=0$ $y=0$
- the curve with equation $y = \mathrm{g}(x)$ has a stationary point at (2, 9)

(a) find $\mathrm{g}(x)$,

(7)

(b) prove that the stationary point at (2, 9) is a maximum.

(2)

14a) $y = ax^3 + bx^2 + cx + d$

$y = ax^3 + bx^2 + ax + d$

$0 = a(0)^3 + b(0)^2 + a(0) + d$

$d = 0$

$y = ax^3 + bx^2 + ax$

(2, 9)
x y

$9 = a(2)^3 + b(2)^2 + a(2)$

$9 = 8a + 4b + 2a$

$9 = 10a + 4b$

stationary point where $\frac{dy}{dx} = 0$ $\frac{dy}{dx} = 3ax^2 + 2bx + a$

when $x = 2$ $0 = 3a(2)^2 + 2b(2) + a$

$0 = 12a + 4b + a$

$0 = 13a + 4b$

$9 = 10a + 4b$

$0 = 13a + 4b$

$9 = -3a$

$a = -3$

$9 = 10(-3) + 4b$

$9 = -30 + 4b$

$39 = 4b$

$b = 39/4$

Question 14 continued

$\frac{dy}{dx} = 3ax^2 + 2bx + a$

$g(x) = -3x^3 + \frac{39}{4}x^2 - 3x$

b/ $\frac{dy}{dx} = 3(-3)x^2 + 2\left(\frac{39}{4}\right)x - 3$

$= -9x^2 + \frac{39}{2}x - 3$

$\frac{d^2y}{dx^2} = -18x + \frac{39}{2}$

when $x = 2$ $\quad \frac{d^2y}{dx^2} = -18(2) + \frac{39}{2}$

$= \frac{-33}{2}$

$\frac{d^2y}{dx^2} < 0 \quad \therefore$ MAXIMUM

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Question 14 continued

(Total for Question 14 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS