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Surname	Other names	
Pearson Edexcel	Centre Number	Candidate Number
Level 3 GCE	<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>	<input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/> <input type="text"/>
Mathematics		
Advanced		
Paper 2: Pure Mathematics 2		
Sample Assessment Material for first teaching September 2017 Time: 2 hours		Paper Reference 9MA0/02
You must have: Mathematical Formulae and Statistical Tables, calculator		Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer cross it out and put your new answer and any working out underneath.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 2x^3 - 5x^2 + ax + a$$

Given that $(x + 2)$ is a factor of $f(x)$, find the value of the constant a .

(3)

$$f(-2) = 0$$

$$2(-2)^3 - 5(-2)^2 + a(-2) + a = 0$$

$$-36 - 2a + a = 0$$

$$-36 = a$$

$$\underline{\underline{a = -36}}$$

(Total for Question 1 is 3 marks)

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2. Some A level students were given the following question.

Solve, for $-90^\circ < \theta < 90^\circ$, the equation

$$\cos \theta = 2 \sin \theta$$

The attempts of two of the students are shown below.

<u>Student A</u>
$\cos \theta = 2 \sin \theta$ $\tan \theta = 2$ $\theta = 63.4^\circ$

<u>Student B</u>
$\cos \theta = 2 \sin \theta$ $\cos^2 \theta = 4 \sin^2 \theta$ $1 - \sin^2 \theta = 4 \sin^2 \theta$ $\sin^2 \theta = \frac{1}{5}$ $\sin \theta = \pm \frac{1}{\sqrt{5}}$ $\theta = \pm 26.6^\circ$

- (a) Identify an error made by student A.

(1)

Student B gives $\theta = -26.6^\circ$ as one of the answers to $\cos \theta = 2 \sin \theta$.

- (b) (i) Explain why this answer is incorrect.

- (ii) Explain how this incorrect answer arose.

(2)

a) they have divided $\cos \theta$ by $\sin \theta$ and this should be $\cot \theta$ not $\tan \theta$.

b) $\cos(-26.6) \neq 2 \sin(-26.6)$

$\cos(-26.6)$ is positive
 $2 \sin(-26.6)$ is negative

c) when both sides were squared everything became positive.

(Total for Question 2 is 3 marks)

3. Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n(Ax + B)$$

where n , A and B are constants to be found.

(4)

$$u = x \quad v = (2x + 1)^4$$

$$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = 8(2x + 1)^3$$

$$\begin{aligned} \frac{dy}{dx} &= v \frac{du}{dx} + u \frac{dv}{dx} \\ &= (2x + 1)^4 + 8x(2x + 1)^3 \\ &= (2x + 1)^3 ((2x + 1) + 8x) \\ &= (2x + 1)^3 (2x + 1 + 8x) \\ &= (2x + 1)^3 (10x + 1) \end{aligned}$$

$$n = 3 \quad A = 10 \quad B = 1$$

(Total for Question 3 is 4 marks)

4. Given

$$f(x) = e^x, \quad x \in \mathbb{R}$$

$$g(x) = 3 \ln x, \quad x > 0, x \in \mathbb{R}$$

(a) find an expression for $gf(x)$, simplifying your answer.

(2)

(b) Show that there is only one real value of x for which $gf(x) = fg(x)$

(3)

$$\begin{aligned} \text{a/} \quad gf(x) &= 3 \ln(e^x) \\ &= 3x \ln e \\ &= 3x \end{aligned}$$

$$\begin{aligned} \text{b/} \quad fg(x) &= e^{3 \ln x} \\ &= \cancel{e^3} \\ &= e^{\ln x^3} \\ &= x^3 \end{aligned}$$

$$3x = x^3$$

$$0 = x^3 - 3x$$

$$0 = x(x^2 - 3)$$

$$x = 0 \quad x = \pm\sqrt{3}$$

$$x = 0 \quad x = \sqrt{3} \quad x = -\sqrt{3}$$

x has to be bigger than zero $\therefore x = \underline{\underline{\sqrt{3}}}$
(The domain of $g(x)$ is $x > 0$)

(Total for Question 4 is 5 marks)

5. The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

- (a) find the mass of the radioactive substance six months after it was first observed, (2)
- (b) show that $\frac{dm}{dt} = km$, where k is a constant to be found. (2)

a/ $t = 0.5$

$$m = 25e^{-0.05(0.5)}$$

$$= \underline{24.4 \text{ g}} \quad (3 \text{ sf})$$

b/ $\frac{dm}{dt} = -\frac{5}{4}e^{-0.05t}$

$$\boxed{\begin{aligned} m &= 25e^{-0.05t} \\ \frac{m}{25} &= e^{-0.05t} \end{aligned}}$$

$$\therefore \frac{dm}{dt} = -\frac{5}{4} \cdot \frac{m}{25}$$

$$= \underline{\underline{-\frac{1}{20} m}}$$

(Total for Question 5 is 4 marks)

6. Complete the table below. The first one has been done for you.

For each statement you must state if it is always true, sometimes true or never true, giving a reason in each case.

Statement	Always True	Sometimes True	Never True	Reason
The quadratic equation $ax^2 + bx + c = 0$, ($a \neq 0$) has 2 real roots.		✓		It only has 2 real roots when $b^2 - 4ac > 0$. When $b^2 - 4ac = 0$ it has 1 real root and when $b^2 - 4ac < 0$ it has 0 real roots.
(i) When a real value of x is substituted into $x^2 - 6x + 10$ the result is positive.	✓			$x^2 - 6x + 10$ $(x - 3)^2 + 1$ min point at (3, 1) ∴ always positive
(ii) If $ax > b$ then $x > \frac{b}{a}$		✓		when a is positive it is true $2x > b$ $x > \frac{b}{2}$ when a is negative not true $-2x > b$ $x < \frac{b}{-2}$
(iii) The difference between consecutive square numbers is odd.	✓			$(n + 1)^2 - n^2$ $n^2 + 2n + 1 - n^2$ $2n + 1$ even + 1 = odd.

(Total for Question 6 is 6 marks)

7. (a) Use the binomial expansion, in ascending powers of x , to show that

$$\sqrt{4-x} = 2 - \frac{1}{4}x + kx^2 + \dots$$

where k is a rational constant to be found.

(4)

A student attempts to substitute $x = 1$ into both sides of this equation to find an approximate value for $\sqrt{3}$.

- (b) State, giving a reason, if the expansion is valid for this value of x .

(1)

$$\begin{aligned} \text{a/ } & (4-x)^{\frac{1}{2}} \\ & 4^{\frac{1}{2}} \left(1 - \frac{x}{4}\right)^{\frac{1}{2}} \\ & 4^{\frac{1}{2}} \left(1 + \frac{1}{2} \left(\frac{-x}{4}\right) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2} \left(\frac{-x}{4}\right)^2 + \dots\right) \\ & 2 \left(1 - \frac{1}{8}x - \frac{1}{8} \cdot \frac{x^2}{16}\right) \\ & 2 \left(1 - \frac{1}{8}x - \frac{1}{128}x^2\right) \\ & 2 - \frac{1}{4}x - \frac{1}{64}x^2 \\ & k = \frac{1}{64} \end{aligned}$$

b/ The expansion is valid when $\left|\frac{x}{4}\right| < 1$

$$|x| < 4$$

\therefore The expansion is valid for $x=1$

8.

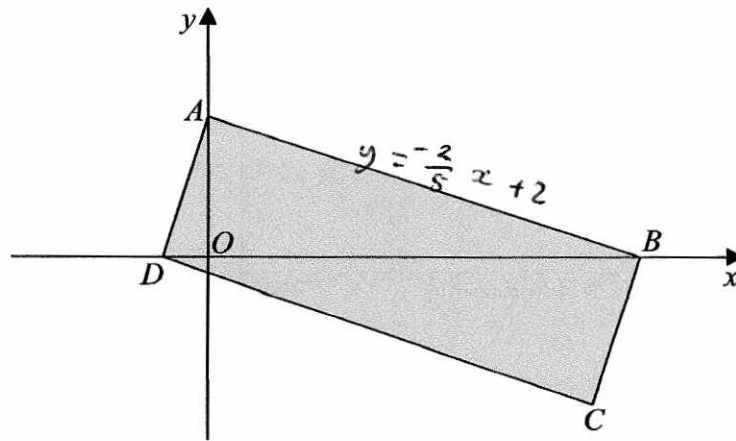


Figure 1

Figure 1 shows a rectangle $ABCD$.

The point A lies on the y -axis and the points B and D lie on the x -axis as shown in Figure 1.

Given that the straight line through the points A and B has equation $5y + 2x = 10$

(a) show that the straight line through the points A and D has equation $2y - 5x = 4$ (4)

(b) find the area of the rectangle $ABCD$. (3)

$$\begin{aligned} \text{a/} \quad 5y + 2x &= 10 \\ 5y &= -2x + 10 \\ y &= -\frac{2}{5}x + 2 \end{aligned}$$

$$y \text{ intercept} = 2 \quad \text{perp gradient} = \frac{5}{2}$$

$$\therefore AD \quad y = \frac{5}{2}x + 2$$

$$2y = 5x + 4$$

$$\underline{\underline{2y - 5x = 4}}$$

Question 8 continued

b/ AB crosses x when $y=0$

$$5(0) + 2x = 10$$

$$2x = 10$$

$$x = 5$$

$$A: (0, 2) \quad B: (5, 0)$$

$$\begin{aligned} \text{Length } AB &= \sqrt{5^2 + 2^2} \\ &= \sqrt{29} \end{aligned}$$

AD crosses x when $y=0$

$$2(0) - 5x = 4$$

$$-5x = 4$$

$$x = -\frac{4}{5}$$

$$A: (0, 2) \quad D: \left(-\frac{4}{5}, 0\right)$$

$$\begin{aligned} \text{Length } AD &= \sqrt{\left(-\frac{4}{5}\right)^2 + 2^2} \\ &= \frac{2\sqrt{29}}{5} \end{aligned}$$

$$\begin{aligned} \text{Area} &= AB \cdot AD \\ &= \sqrt{29} \cdot \frac{2\sqrt{29}}{5} \\ &= \frac{58}{5} \\ &= 11.6 \end{aligned}$$

(Total for Question 8 is 7 marks)

9. Given that A is constant and

$$\int_1^4 (3\sqrt{x} + A) dx = 2A^2$$

show that there are exactly two possible values for A .

(5)

$$\int_1^4 3x^{\frac{1}{2}} + A dx = 2A^2$$

$$\left[2x^{\frac{3}{2}} + Ax \right]_1^4 = 2A^2$$

$$(16 + 4A) - (2 + A) = 2A^2$$

$$16 + 4A - 2 - A = 2A^2$$

$$3A + 14 = 2A^2$$

$$0 = 2A^2 - 3A - 14$$

$$0 = (2A - 7)(A + 2)$$

$$\underline{\underline{A = \frac{7}{2}}} \quad \underline{\underline{A = -2}}$$

\therefore There are exactly two possible values of A .

(Total for Question 9 is 5 marks)

10. In a geometric series the common ratio is r and sum to n terms is S_n

Given

$$S_{\infty} = \frac{8}{7} \times S_6$$

show that $r = \pm \frac{1}{\sqrt{k}}$, where k is an integer to be found.

(4)

$$S_n = \frac{a(1-r^n)}{1-r} \qquad S_{\infty} = \frac{a}{1-r}$$

$$\frac{a}{1-r} = \frac{8}{7} \cdot \frac{a(1-r^6)}{1-r}$$

$$1 = \frac{8}{7} (1-r^6)$$

$$1 = \frac{8}{7} (1-r^6)$$

\rightarrow

$$\frac{7}{8} = 1 - r^6$$

$$r^6 = \frac{1}{8}$$

$$r^2 = \frac{1}{2}$$

$$r = \pm \frac{1}{\sqrt{2}}$$

$$\underline{\underline{k = 2}}$$

(Total for Question 10 is 4 marks)

11.

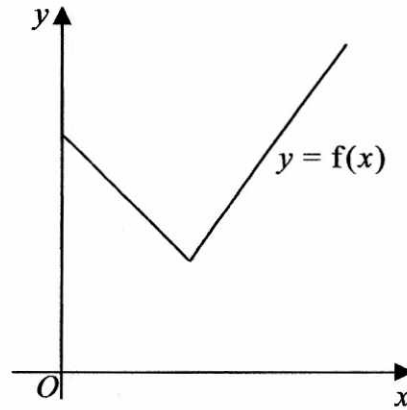


Figure 2

Figure 2 shows a sketch of part of the graph $y = f(x)$, where

$$f(x) = 2|3 - x| + 5, \quad x \geq 0$$

(a) State the range of f

(1)

(b) Solve the equation

$$f(x) = \frac{1}{2}x + 30$$

(3)

Given that the equation $f(x) = k$, where k is a constant, has two distinct roots,

(c) state the set of possible values for k .

(2)

$$a) \quad f(x) \geq 5$$

$$b) \quad 2|3 - x| + 5 = \frac{1}{2}x + 30$$

$$2|3 - x| = \frac{1}{2}x + 25$$

$$2(3 - x) = \frac{1}{2}x + 25$$

$$-2(3 - x) = \frac{1}{2}x + 25$$

$$6 - 2x = \frac{1}{2}x + 25$$

$$-6 + 2x = \frac{1}{2}x + 25$$

$$12 - 4x = x + 50$$

$$-12 + 4x = x + 50$$

$$-38 = 5x$$

$$3x = 62$$

$$x = -7.6$$

$$x = \frac{62}{3}$$

$$x \geq 0 \quad \therefore \text{not a solution}$$

$$\underline{\underline{\frac{62}{3}}}$$

Question 11 continued

c/ $f(x)$ intersects y when $x = 0$

$$2|3 - 0| + 5 = \underline{\underline{11}}$$

$$\underline{\underline{5 < k \leq 11}}$$

(Total for Question 11 is 6 marks)

12. (a) Solve, for $-180^\circ \leq x < 180^\circ$, the equation

$$3 \sin^2 x + \sin x + 8 = 9 \cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3 \sin^2(2\theta - 30^\circ) + \sin(2\theta - 30^\circ) + 8 = 9 \cos^2(2\theta - 30^\circ)$$

giving your answer to 2 decimal places.

(2)

a/ $3 \sin^2 x + \sin x + 8 = 9(1 - \sin^2 x)$

$$3 \sin^2 x + \sin x + 8 = 9 - 9 \sin^2 x$$

$$12 \sin^2 x + \sin x - 1 = 0$$

$$(4 \sin x - 1)(3 \sin x + 1) = 0$$

$$\sin x = \frac{1}{4} \quad \sin x = -\frac{1}{3}$$

$$x = \underline{14.48^\circ}, \underline{165.52^\circ} \quad x = \underline{-19.47^\circ}, \underline{-160.53^\circ}$$

b/ $x = 2\theta - 30$

$$\frac{x + 30}{2} = \theta$$

$$\frac{-19.47 + 30}{2} = \theta$$

$$\theta = \underline{5.26^\circ}$$

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13. (a) Express $10 \cos \theta - 3 \sin \theta$ in the form $R \cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$.
Give the exact value of R and give the value of α , in degrees, to 2 decimal places. (3)

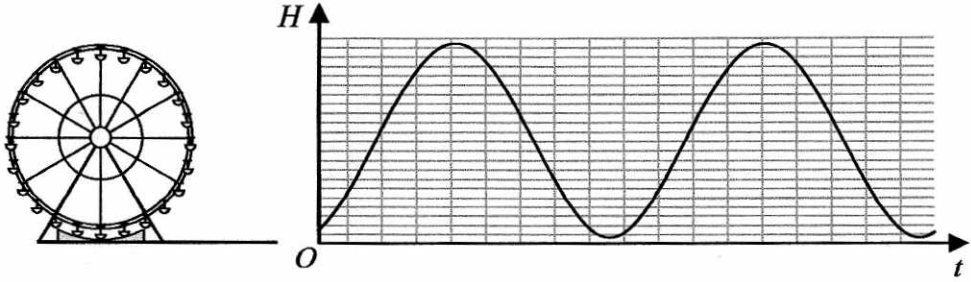


Figure 3

The height above the ground, H metres, of a passenger on a Ferris wheel t minutes after the wheel starts turning, is modelled by the equation

$$H = a - 10 \cos(80t)^\circ + 3 \sin(80t)^\circ$$

where a is a constant.

Figure 3 shows the graph of H against t for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

- (b) (i) find a complete equation for the model,
(ii) hence find the maximum height of the passenger above the ground. (2)

- (c) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.
(Solutions based entirely on graphical or numerical methods are not acceptable.) (3)

It is decided that, to increase profits, the speed of the wheel is to be increased.

- (d) How would you adapt the equation of the model to reflect this increase in speed? (1)

a/ $\cos(A+B) = \cos A \cos B - \sin A \sin B$
 $R \cos(\theta + \alpha) = R \cos \theta \cos \alpha - R \sin \theta \sin \alpha$
 $10 \cos \theta - 3 \sin \theta$

$R \cos \alpha = 10 \quad R \sin \alpha = 3$

$R = \sqrt{10^2 + 3^2}$
 $= \sqrt{109}$

$\tan \alpha = \frac{3}{10}$
 $\alpha = \tan^{-1}\left(\frac{3}{10}\right)$
 $= 16.70^\circ$

Question 13 continued

$$\sqrt{109} \cos(\theta + 16.70)$$

b) $H = a - (10 \cos(80t) - 3 \sin(80t))$

$$H = a - \sqrt{109} \cos(80t + 16.70)$$

when $t = 0$ $H = 1$

$$1 = a - 10$$

$$a = 11$$

$$\therefore H = 11 - \sqrt{109} \cos(80t + 16.70)$$

ii) max height when $\cos(80t + 16.70) = -1$

$$H = 11 - \sqrt{109}(-1)$$

$$= 11 + \sqrt{109} \text{ m}$$

$$= 21.44 \text{ m}$$

c) $\cos(80t + 16.70) = -1$

$$80t + 16.70 = 180, 540$$

$$~~80t = 163.3~~$$

$$~~t =~~$$

On 2nd cycle!

$$80t + 16.70 = 540$$

$$80t = 523.3$$

$$t = 6.54 \text{ mins}$$

$$= 6 \text{ mins } 32 \text{ secs.}$$

d) increase the coefficient of t

eg. $a - 10 \cos(100t) + 3 \sin(100t)$

(Total for Question 13 is 9 marks)

14. A company decides to manufacture a soft drinks can with a capacity of 500 ml.

The company models the can in the shape of a right circular cylinder with radius r cm and height h cm.

In the model they assume that the can is made from a metal of negligible thickness.

(a) Prove that the total surface area, S cm², of the can is given by

$$S = 2\pi r^2 + \frac{1000}{r} \quad (3)$$

Given that r can vary,

(b) find the dimensions of a can that has minimum surface area. (5)

(c) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area. (1)

$$\begin{aligned} 1 \text{ litre} &= 1000 \text{ cm}^3 \\ 500 \text{ ml} &= 500 \text{ cm}^3 \quad (V) \end{aligned}$$

$$\begin{aligned} V &= \pi r^2 h \\ 500 &= \pi r^2 h \\ h &= \frac{500}{\pi r^2} \end{aligned}$$

$$\begin{aligned} S &= 2\pi r^2 + 2\pi r h \\ &= 2\pi r^2 + 2\pi r \cdot \frac{500}{\pi r^2} \\ &= 2\pi r^2 + \frac{1000\pi r}{\pi r^2} \\ &= 2\pi r^2 + \frac{1000}{r} \end{aligned}$$

$$\text{b/} \quad \frac{dS}{dr} = 4\pi r - 1000r^{-2}$$

$$\text{minimum where } \frac{dS}{dr} = 0$$

Question 14 continued

$$4\pi r - \frac{1000}{r^2} = 0$$

$$4\pi r^3 - 1000 = 0$$

$$\pi r^3 - 250 = 0$$

$$\pi r^3 = 250$$

$$r = \sqrt[3]{\frac{250}{\pi}}$$

$$= 4.30 \text{ cm (3sf)}$$

$$h = \frac{500}{\pi(4.30)^2}$$

$$= 8.60 \text{ cm (3sf)}$$

c/ The radius is much bigger than a regular drink can.
People with small hands may struggle to hold it!

(Total for Question 14 is 9 marks)

15.

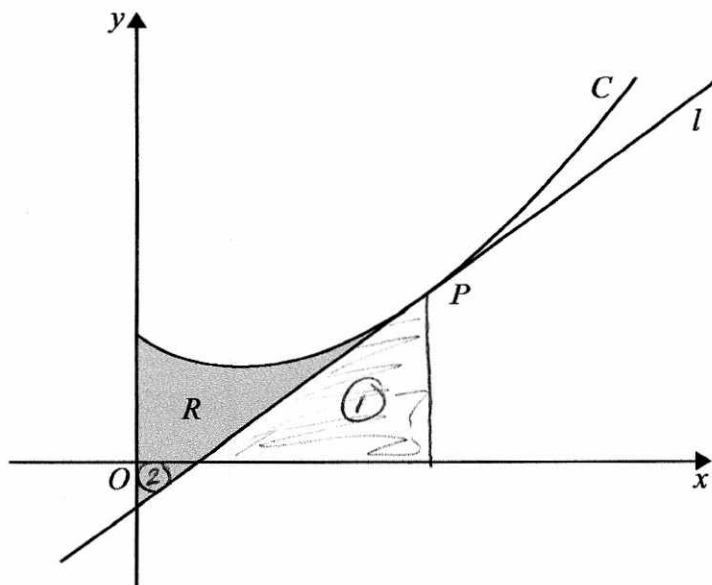


Figure 4

Figure 4 shows a sketch of the curve C with equation

$$y = 5x^{\frac{3}{2}} - 9x + 11, x \geq 0$$

The point P with coordinates $(4, 15)$ lies on C .

The line l is the tangent to C at the point P .

The region R , shown shaded in Figure 4, is bounded by the curve C , the line l and the y -axis.

Show that the area of R is 24, making your method clear.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(10)

$$\frac{dy}{dx} = \frac{15}{2}x^{\frac{1}{2}} - 9$$

$$\text{when } x = 4 \quad \frac{dy}{dx} = \frac{15}{2}(4)^{\frac{1}{2}} - 9$$

$$= 6$$

$$y = 6x + c \quad (4, 15)$$

$$15 = 6(4) + c$$

$$c = -9$$

$$\underline{\underline{y = 6x - 9}}$$

Question 7 continued

crosses x when $y=0$

$$0 = 6x - 9$$

$$9 = 6x$$

$$x = \frac{3}{2}$$

$$\text{Area of } \textcircled{1} = \frac{1}{2} \left(4 - \frac{3}{2} \right) (15)$$

$$= \frac{75}{4} \text{ units}^2$$

$$\text{Area of } \textcircled{2} = \frac{1}{2} \left(\frac{3}{2} \right) (9)$$

$$= \frac{27}{4} \text{ units}^2$$

$$\text{Area under curve} = \int_0^4 5x^{\frac{3}{2}} - 9x + 11 \, dx$$

$$= \left[2x^{\frac{5}{2}} - \frac{9}{2}x^2 + 11x \right]_0^4$$

$$\left(2(4)^{\frac{5}{2}} - \frac{9}{2}(4)^2 + 11(4) \right) - (0)$$

$$= 36 \text{ units}^2$$

$$\text{Area of } R = 36 + \textcircled{2} - \textcircled{1}$$

$$= 36 + \frac{27}{4} - \frac{75}{4}$$

$$= \underline{\underline{24 \text{ units}^2}}$$

16. (a) Express $\frac{1}{P(11-2P)}$ in partial fractions.

(3)

A population of meerkats is being studied.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{22}P(11-2P), \quad t \geq 0, \quad 0 < P < 5.5$$

where P , in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that there were 1000 meerkats in the population when the study began,

(b) determine the time taken, in years, for this population of meerkats to double,

(6)

(c) show that

$$P = \frac{A}{B + Ce^{-\frac{1}{2}t}}$$

where A , B and C are integers to be found.

(3)

$$a) \quad \frac{1}{P(11-2P)} = \frac{A}{P} + \frac{B}{11-2P}$$

$$1 = A(11-2P) + B(P)$$

$$\text{Let } p=0 \quad 1 = 11A$$

$$A = \frac{1}{11}$$

$$\text{Let } p = \frac{11}{2} \quad 1 = \frac{11}{2}B$$

$$B = \frac{2}{11}$$

$$\frac{\frac{1}{11}}{P} + \frac{\frac{2}{11}}{11-2P}$$

$$\frac{1}{11P} + \frac{2}{11(11-2P)}$$

Question 16 continued

$$b) \quad \frac{dP}{dt} = \frac{1}{22} P(11-2P)$$

$$\int \frac{22}{11-2P} dP = \int 1 dt$$

$$\int \frac{2}{P} + \frac{4}{11-2P} dP = \int 1 dt$$

$$2 \ln|P| - 2 \ln|11-2P| = t + c$$

$$2 \ln P - 2 \ln(11-2P) = t + c$$

when $t=0$, $P=1$

$$2 \ln 1 - 2 \ln 9 = c$$

$$-2 \ln 9 = c$$

$$2 \ln P - 2 \ln(11-2P) = t - 2 \ln 9$$

pop. doubles when $P=2$

$$2 \ln 2 - 2 \ln 7 = t - 2 \ln 9$$

$$2 \ln 2 - 2 \ln 7 + 2 \ln 9 = t$$

$$t = \underline{\underline{1.89 \text{ years (3sf)}}}$$

$$c) \quad 2 \ln P - 2 \ln(11-2P) + 2 \ln 9 = t$$

$$2 (\ln P + \ln 9 - \ln(11-2P)) = t$$

Question 16 continued

$$2 \left(\ln \left(\frac{9P}{11-2P} \right) \right) = t$$

$$\ln \left(\frac{9P}{11-2P} \right) = \frac{1}{2} t$$

$$\frac{9P}{11-2P} = e^{\frac{1}{2}t}$$

$$9P = e^{\frac{1}{2}t} (11 - 2P)$$

$$9P = 11e^{\frac{1}{2}t} - 2Pe^{\frac{1}{2}t}$$

$$9P + 2Pe^{\frac{1}{2}t} = 11e^{\frac{1}{2}t}$$

$$P(9 + 2e^{\frac{1}{2}t}) = 11e^{\frac{1}{2}t}$$

$$P = \frac{11e^{\frac{1}{2}t}}{9 + 2e^{\frac{1}{2}t}}$$

divide top
and bottom
by $e^{\frac{1}{2}t}$

$$= \frac{11}{9e^{-\frac{1}{2}t} + 2}$$

$$= \frac{11}{2 + 9e^{-\frac{1}{2}t}}$$

$$A = 11 \quad B = 2 \quad C = 9$$

(Total for Question 16 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

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