Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 7 October 2020

Afternoon (Time: 2 hours) | Paper Reference **9MA0/01**

Mathematics

Advanced

Paper 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

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1. (a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use $x = \dfrac{1}{32}$ in the expansion to find an approximation for $\sqrt{5}$

There is no need to carry out the calculation.

(2)

a) $1 + \left(\frac{1}{2}\right)(8x) + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(8x)^2 + \frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{6}(8x^3)$

$\underline{\underline{1 + 4x - 8x^2 + 32x^3}}$

b) $\sqrt{1 + 8\left(\frac{1}{32}\right)}$

$= \sqrt{1 + \frac{1}{4}}$

$= \sqrt{\frac{5}{4}}$

$= \frac{\sqrt{5}}{\sqrt{4}}$

$= \frac{\sqrt{5}}{2}$

substituting $\frac{1}{32}$ would find $\frac{\sqrt{5}}{2}$, we could then double the answer to find an estimate for $\sqrt{5}$

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Question 1 continued

(Total for Question 1 is 5 marks)

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2. By taking logarithms of both sides, solve the equation

$$4^{3p-1} = 5^{210}$$

giving the value of p to one decimal place.

(3)

$\ln 4^{3p-1} = \ln 5^{210}$

$(3p-1)\ln 4 = 210 \ln 5$

$3p - 1 = \dfrac{210 \ln 5}{\ln 4}$

$3p = \dfrac{210 \ln 5}{\ln 4} + 1$

$3p = 244.80245$

$p = \underline{\underline{81.6}}$ 1dp

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Question 2 continued

(Total for Question 2 is 3 marks)

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3. Relative to a fixed origin O

- point A has position vector $2\mathbf{i} + 5\mathbf{j} - 6\mathbf{k}$
- point B has position vector $3\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$
- point C has position vector $2\mathbf{i} - 16\mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(2)

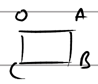
(b) Show that quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

(2)

a/ $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix} = \begin{pmatrix} 1 \\ -8 \\ 2 \end{pmatrix}$

b/ OABC is a trapezium if it has one set of parallel sides

AB and OC
or OA and CB



$\overrightarrow{OC} = \begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix}$ $\qquad \overrightarrow{OC} = 2\overrightarrow{AB}$

$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 5 \\ -6 \end{pmatrix} \qquad \overrightarrow{CB} = \begin{pmatrix} 3 \\ -3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ -16 \\ 4 \end{pmatrix} = \begin{pmatrix} 1 \\ 13 \\ -8 \end{pmatrix}$

$\overrightarrow{OA}$ and $\overrightarrow{CB}$ are not parallel.
$\overrightarrow{OC}$ and $\overrightarrow{AB}$ are parallel.
$\therefore$ OABC is a trapezium.

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Question 3 continued

(Total for Question 3 is 4 marks)

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4. The function f is defined by

$$f(x) = \frac{3x-7}{x-2} \qquad x \in \mathbb{R}, x \neq 2$$

(a) Find $f^{-1}(7)$ Input for inverse is output for original function **(2)**

(b) Show that $ff(x) = \dfrac{ax+b}{x-3}$ where a and b are integers to be found. **(3)**

a/ $7 = \dfrac{3x-7}{x-2}$

$7(x-2) = 3x-7$

$7x - 14 = 3x - 7$

$4x - 14 = -7$

$4x = 7$

$x = \dfrac{7}{4}$

b/ $f(x) = \dfrac{3x-7}{x-2}$

$ff(x) = \dfrac{3\left(\frac{3x-7}{x-2}\right) - 7}{\frac{3x-7}{x-2} - 2}$

$= \dfrac{\frac{9x-21}{x-2} - 7}{\frac{3x-7}{x-2} - 2}$

$= \dfrac{\frac{9x-21}{x-2} - \frac{7(x-2)}{x-2}}{\frac{3x-7}{x-2} - \frac{2(x-2)}{x-2}}$

$= \dfrac{\frac{9x-21-7x+14}{x-2}}{\frac{3x-7-2x+4}{x-2}}$

$= \dfrac{9x-21-7x+14}{3x-7-2x+4}$

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Question 4 continued

$$= \frac{2x - 7}{x - 3}$$

(Total for Question 4 is 5 marks)

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5. A car has six forward gears.

The fastest speed of the car

- in 1st gear is 28 km h^{-1}
- in 6th gear is 115 km h^{-1}

Given that the fastest speed of the car in successive gears is modelled by an **arithmetic sequence**,

$U_n = a + (n-1)d$

(a) find the fastest speed of the car in 3rd gear.

(3)

Given that the fastest speed of the car in successive gears is modelled by a **geometric sequence**,

(b) find the fastest speed of the car in 5th gear.

(3)

a) $U_n = a + (n-1)d$

$a = 28$

$115 = 28 + 5d$

$87 = 5d$

$d = 17.4$

$U_3 = 28 + 2(17.4)$

$= \underline{\underline{62.8 \text{ km/h}}}$

b) $U_n = ar^{n-1}$

$115 = 28r^5$

$\frac{115}{28} = r^5$

$r = \sqrt[5]{\frac{115}{28}}$

$= 1.3265$

$U_5 = 28(1.3265)^4$

$= \underline{\underline{86.7 \text{ km/h}}}$

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Question 5 continued

(Total for Question 5 is 6 marks)

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6. (a) Express $\sin x + 2\cos x$ in the form $R\sin(x + \alpha)$ where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and give the value of α in radians to 3 decimal places.

(3)

The temperature, θ°C, inside a room on a given day is modelled by the equation

$$\theta = 5 + \sin\left(\frac{\pi t}{12} - 3\right) + 2\cos\left(\frac{\pi t}{12} - 3\right) \qquad 0 \leqslant t < 24$$

where t is the number of hours after midnight.

Using the equation of the model and your answer to part (a),

(b) deduce the maximum temperature of the room during this day,

(1)

(c) find the time of day when the maximum temperature occurs, giving your answer to the nearest minute.

(3)

a/ $\sin(A+B) = \sin A\cos B + \cos A\sin B$

$R\sin(x+\alpha) = R\underline{\sin x}\cos\alpha + R\underline{\cos x}\sin\alpha$

$\underline{\sin x} + 2\underline{\cos x}$

$R\cos\alpha = 1 \qquad R\sin\alpha = 2$

$\frac{R\sin\alpha}{R\cos\alpha} = \frac{2}{1}$

$\tan\alpha = 2$

$\alpha = \underline{\underline{1.107}}$

$R = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$\underline{\underline{\sqrt{5}\sin(x + 1.107)}}$

b/ $\theta = 5 + \sqrt{5}\sin\left(\frac{\pi t}{12} - 3 + 1.107\right)$

Max temp when $\sin\left(\frac{\pi t}{12} - 3 + 1.107\right) = 1$

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Question 6 continued

$\theta = 5 + \sqrt{5}\ ^\circ C \quad = 7.24^\circ C$

c/ $\sin\left(\frac{\pi t}{12} - 3 + 1.107\right) = 1$

$\sin\theta = 1$ when $\theta = \frac{\pi}{2}$

$\frac{\pi t}{12} - 3 + 1.107 = \frac{\pi}{2}$

$\frac{\pi t}{12} - 1.893 = \frac{\pi}{2}$

$\pi t - 22.716 = 6\pi$

$\pi t = 6\pi + 22.716$

$t = \frac{6\pi + 22.716}{\pi}$

$= 13.23$ hrs

$= 13$ hrs 14 mins

$= \underline{\underline{13:14}}$

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 7 marks)

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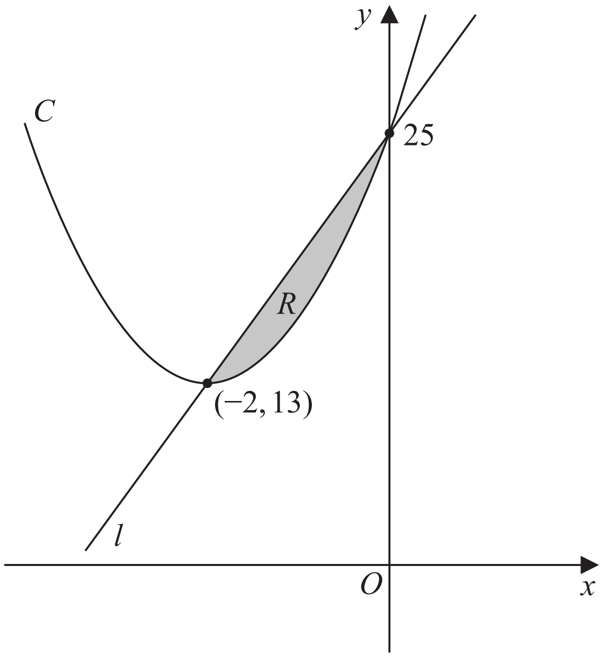

Figure 1

Figure 1 shows a sketch of a curve C with equation $y = \mathrm{f}(x)$ and a straight line l.

The curve C meets l at the points $(-2, 13)$ and $(0, 25)$ as shown.

The shaded region R is bounded by C and l as shown in Figure 1.

Given that

- $\mathrm{f}(x)$ is a quadratic function in x
- $(-2, 13)$ is the minimum turning point of $y = \mathrm{f}(x)$

use inequalities to define R.

(5)

$f(x) = a(x+b)^2 + c$

min point $= (-2, 13)$

$\therefore f(x) = a(x+2)^2 + 13$

$f(x)$ passes through $(0, 25)$

$25 = a(0+2)^2 + 13$

$25 = 4a + 13$

$12 = 4a$

$a = 3$

$f(x) = 3(x+2)^2 + 13$

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Question 7 continued

l passes through $\underset{x_1 \quad y_1}{(-2, 13)}$ and $\underset{x_2 \quad y_2}{(0, 25)}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{25 - 13}{0 - -2}$

$= \dfrac{12}{2} = 6$ $\quad m = 6 \quad c = 25$

$y = \underline{\underline{6x + 25}}$

$y = 3(x+2)^2 + 13$

$3(x+2)^2 + 13 \leqslant y \leqslant 6x + 25$

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 5 marks)

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8. A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation.*)

(2)

$n = Ae^{kt}$

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Question 8 continued

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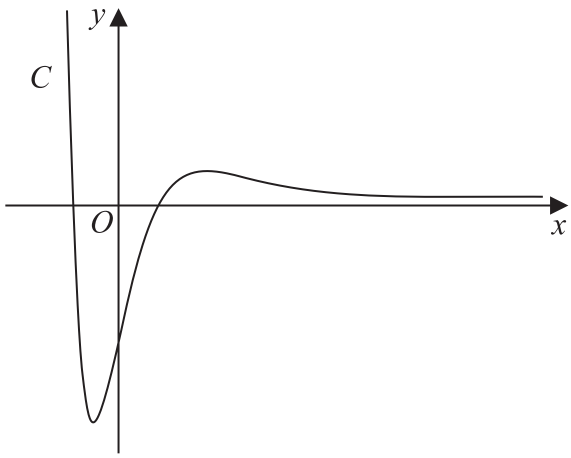

Figure 2

Figure 2 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 4(x^2 - 2)\mathrm{e}^{-2x} \qquad x \in \mathbb{R}$$

(a) Show that $\mathrm{f}'(x) = 8(2 + x - x^2)\mathrm{e}^{-2x}$

(3)

(b) Hence find, in simplest form, the exact coordinates of the stationary points of C.

(3)

The function g and the function h are defined by

$$\mathrm{g}(x) = 2\mathrm{f}(x) \qquad x \in \mathbb{R}$$

$$\mathrm{h}(x) = 2\mathrm{f}(x) - 3 \qquad \underline{x \geqslant 0}$$

(c) Find (i) the range of g

(ii) the range of h

(3)

a/ $f(x) = 4(x^2 - 2)e^{-2x}$

$u = 4x^2 - 8 \qquad v = e^{-2x}$

$\frac{du}{dx} = 8x \qquad \frac{dv}{dx} = -2e^{-2x}$

$f'(x) = 8xe^{-2x} + (4x^2 - 8)(-2e^{-2x})$

$= 8xe^{-2x} - 8x^2e^{-2x} + 16e^{-2x}$

$= 8e^{-2x}(x - x^2 + 2)$

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Question 9 continued

$= 8(2+x-x^2)e^{-2x}$

b/ stationary points where $f'(x)=0$

$8(2+x-x^2)e^{-2x} = 0$

$8(1+x)(2-x)e^{-2x} = 0$

$x=-1 \quad x=2$

when $x=-1 \quad y = 4((-1)^2-2)e^{-2(-1)}$
$= -4e^2$

when $x=2 \quad y = 4((2)^2-2)e^{-2(2)}$
$= 8e^{-4}$

$(-1, -4e^2)$ and $(2, 8e^{-4})$

c/ i/ $f(x) \geqslant -4e^2$

$g(x) \geqslant -8e^2$

ii/ $f(x) = 4(x^2-2)e^{-2x}$ crosses y when $x=0$

$y = 4(0^2-2)e^0$

$= -8$

$2(-8)-3 \leqslant h(x) \leqslant 2(8e^{-4})-3$
$-19 \leqslant h(x) \leqslant 16e^{-4}-3$

Question 9 continued

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Question 9 continued

(Total for Question 9 is 9 marks)

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10. (a) Use the substitution $x = u^2 + 1$ to show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \int_p^q \frac{6\,\mathrm{d}u}{u(3+2u)}$$

where p and q are positive constants to be found.

(4)

(b) Hence, using algebraic integration, show that

$$\int_5^{10} \frac{3\,\mathrm{d}x}{(x-1)\left(3+2\sqrt{x-1}\right)} = \ln a$$

where a is a rational constant to be found.

(6)

$$\int_5^{10} \frac{3}{(x-1)(3+2\sqrt{x-1}} \, dx \qquad x = u^2 + 1$$

$$10 = u^2 + 1$$

$$\int_2^3 \frac{3}{(x-1)(3+2\sqrt{x-1})} \frac{dx}{du} \, du \qquad u = 3$$

$$5 = u^2 + 1$$

$$u = 2$$

$$\frac{dx}{du} = 2u$$

$$\int_2^3 \frac{3}{u^2(3+2u)} 2u \, du \qquad x - 1 = u^2$$

$$\int_2^3 \frac{6u}{3u^2 + 2u^3} \, du$$

$$\int_2^3 \frac{6}{3u + 2u^2} \, du$$

$$\int_2^3 \frac{6}{u(3+2u)} \, du$$

b/ $$\frac{6}{u(3+2u)} = \frac{A}{u} + \frac{B}{3+2u}$$

$$6 = A(3+2u) + Bu$$

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Question 10 continued

when $u=0$ $\quad 6 = 3A$

$A = 2$

when $u = \frac{-3}{2}$ $\quad 6 = -\frac{3}{2}B$

$B = -4$

$$\int_2^3 \frac{2}{u} - \frac{4}{3+2u} \, du$$

$$\left[2\ln|u| - \frac{4}{2}\ln|3+2u| \right]_2^3$$

$$(2\ln 3 - 2\ln 9) - (2\ln 2 - 2\ln 7)$$

$$2\ln 3 - 2\ln 3^2 - 2\ln 2 + 2\ln 7$$

$$2\ln 3 - 4\ln 3 - 2\ln 2 + 2\ln 7$$

$$-2\ln 3 - 2\ln 2 + 2\ln 7$$

$$2\ln 7 - 2\ln 3 - 2\ln 2$$

$$\ln 49 - \ln 9 - \ln 4$$

$$\ln\left(\frac{49}{36}\right)$$

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Question 10 continued

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Question 10 continued

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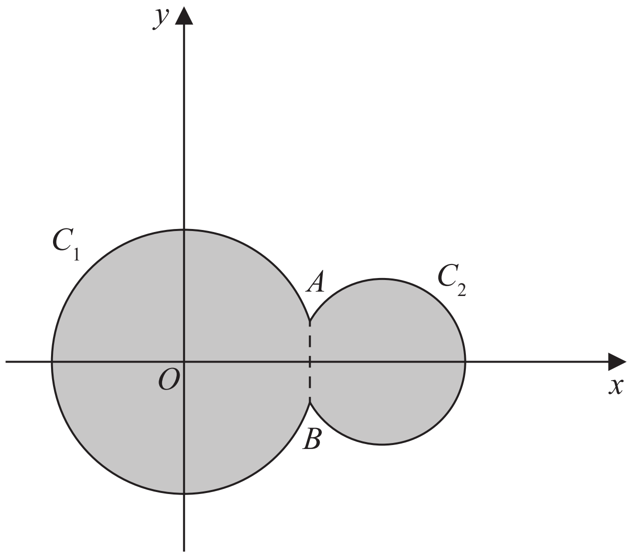

Figure 3

Circle C_1 has equation $x^2 + y^2 = 100$

Circle C_2 has equation $(x - 15)^2 + y^2 = 40$

The circles meet at points A and B as shown in Figure 3.

(a) Show that angle $AOB = 0.635$ radians to 3 significant figures, where O is the origin.

(4)

The region shown shaded in Figure 3 is bounded by C_1 and C_2

(b) Find the perimeter of the shaded region, giving your answer to one decimal place.

(4)

a/ $x^2 + y^2 = 100$ $\quad$ $(x-15)^2 + y^2 = 40$

$y^2 = 100 - x^2$

circles meet when:

$(x-15)^2 + 100 - x^2 = 40$

$(x-15)(x-15) + 100 - x^2 = 40$

$x^2 - 15x - 15x + 225 + 100 - x^2 = 40$

$325 - 30x = 40$

$285 = 30x$

$x = 9.5$

when $x = 9.5$ $\quad$ $(9.5)^2 + y^2 = 100$

$y^2 = \frac{39}{4}$

$y = \pm\sqrt{\frac{39}{4}}$

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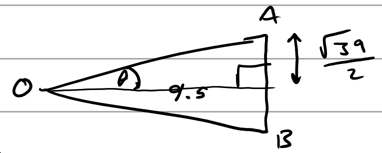
Question 11 continued

$A: \left(9.5, \frac{\sqrt{39}}{2}\right)$ $B: \left(9.5, -\frac{\sqrt{39}}{2}\right)$

$O: (0,0)$



$\tan\theta = \dfrac{\sqrt{39}/2}{9.5}$

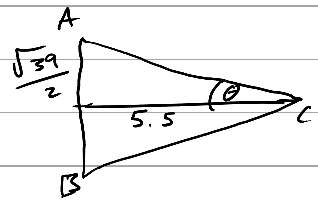
$\theta = \tan^{-1}\left(\frac{\sqrt{39}}{19}\right)$

$= 0.31756...$

$OAB = 2\theta = \underline{\underline{0.635}}$

b) centre of C_2 $(15,0)$

$15 - 9.5 = 5.5$



$\tan\theta = \dfrac{\sqrt{39}/2}{5.5}$

$\theta = \tan^{-1}\left(\frac{\sqrt{39}}{11}\right)$

$= 0.516$

$ACB = 2\theta = 1.0327$ radians

Arc length $= r\theta$

For $C_1 = 10(2\pi - 0.635) = 56.5$

For $C_2 = \sqrt{40}(2\pi - 1.0327) = 33.2$

Total perimeter $= 56.5 + 33.2$

$= \underline{\underline{89.7}}$

Question 11 continued

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Question 11 continued

(Total for Question 11 is 8 marks)

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12. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\operatorname{cosec}\theta - \sin\theta \equiv \cos\theta\cot\theta \qquad \theta \neq (180n)^\circ \quad n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve for $0 < x < 180°$

$$\operatorname{cosec}x - \sin x = \cos x\cot(3x - 50°)$$

(5)

a/ $\frac{1}{\sin\theta} - \sin\theta$

$\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}$

$\frac{(1-\sin^2\theta)}{\sin\theta}$

$\frac{\cos^2\theta}{\sin\theta}$

$\frac{\cos\theta}{\sin\theta}\cos\theta$

$\cot\theta\cos\theta \quad = \cos\theta\cot\theta$

b/

$\cos x \cot x = \cos x \cot(3x-50)$

$0 = \cos x \cot(3x-50) - \cos x \cot x$

$0 = \cos x(\cot(3x-50) - \cot x)$

$\cos x = 0$ or $\cot(3x-50) - \cot x = 0$

$x = 90$

$\cot(3x-50) = \cot x$

$3x - 50 = x$

$2x = 50$

$x = 25$

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Question 12 continued

cot repeats every $180°$

or $3x - 50 = x + 180$

$2x = 230$

$x = \underline{115°}$

$x = \underline{25°, 90°, 115°}$

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Question 12 continued

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Question 12 continued

(Total for Question 12 is 8 marks)

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13. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 2)}{a_n} \qquad n \in \mathbb{N}$$

where k is a constant.

Given that

- the sequence is a periodic sequence of order 3
- $a_1 = 2$

(a) show that

$$k^2 + k - 2 = 0$$

(3)

(b) For this sequence explain why $k \neq 1$

(1)

(c) Find the value of

$$\sum_{r=1}^{80} a_r$$

(3)

a/ $a_{n+1} = \frac{k(a_n + 2)}{a_n}$

$a_2 = \frac{k(2+2)}{2}$

$= 2k$

$a_3 = \frac{k(2k+2)}{2k}$

$= \frac{2k+2}{2}$

$= k+1$

$a_4 = \frac{k(k+1+2)}{k+1}$

$a_4 = a_1$ $\quad \frac{k(k+3)}{k+1} = 2$

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Question 13 continued

$$k(k+3) = 2(k+1)$$
$$k^2+3k = 2k+2$$
$$k^2+k-2 = 0$$

b/ If $K=1$ every term will be 2. (It will not be periodic of order 3)

c/ $k^2+k-2 = 0$

$(k-1)(k+2) = 0$

$k=1$ ✗ $\quad \underline{\underline{k=-2}}$

$a_1 = 2$

$a_2 = 2k = -4$

$a_3 = k+1 = -1$

$a_1 + a_2 + a_3 = 2-4-1 = -3$

$\sum_{r=1}^{78} a_r = 26(-3) = -78 \qquad \left[\frac{78}{3} = 26\right]$

$\sum_{r=1}^{80} a_r = -78+2-4 = \underline{\underline{-80}}$

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Question 13 continued

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Question 13 continued

(Total for Question 13 is 7 marks)

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14. A large spherical balloon is deflating.

At time t seconds the balloon has radius r cm and volume V cm^3

The volume of the balloon is modelled as decreasing at a constant rate.

(a) Using this model, show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 40 cm
- after 5 seconds the radius of the balloon is 20 cm
- the volume of the balloon continues to decrease at a constant rate until the balloon is empty

(b) solve the differential equation to find a complete equation linking r and t.

(5)

(c) Find the limitation on the values of t for which the equation in part (b) is valid.

(2)

a/ Volume of sphere $= \frac{4}{3}\pi r^3$

$V = \frac{4}{3}\pi r^3$ $\qquad \frac{dv}{dt} = -c$

$\frac{dv}{dr} = 4\pi r^2$

$\frac{dr}{dt} = \frac{dr}{dv} \times \frac{dv}{dt}$

$= \frac{1}{4\pi r^2} \times -c$

$= \frac{-c}{4\pi r^2}$

$= \frac{-c}{4\pi} \times \frac{1}{r^2}$ $\qquad$ Let $\frac{c}{4\pi} = k$

$= -\frac{k}{r^2}$

Question 14 continued

b/ $\frac{dr}{dt} = -\frac{k}{r^2}$ $\qquad t=0 \quad r=40$

$t=5 \quad r=20$

$\int r^2 \, dr = \int -k \, dt$

$\frac{1}{3}r^3 = -kt + c$ $\qquad$ when $t=0 \quad r=40$

$\frac{1}{3}(40)^3 = -k(0) + c$

$\frac{64000}{3} = c$

$\frac{1}{3}r^3 = -kt + \frac{64000}{3}$ $\qquad$ when $t=5 \quad r=20$

$\frac{1}{3}(20)^3 = -k(5) + \frac{64000}{3}$

$\frac{8000}{3} = -5k + \frac{64000}{3}$

$5k = \frac{56000}{3}$

$k = \frac{11200}{3}$

$\frac{1}{3}r^3 = -\frac{11200}{3}t + \frac{64000}{3}$

$r^3 = -11200t + 64000$

c/ when $r=0$

$0 = -11200t + 64000$

$11200t = 64000$

$t = \frac{40}{7}$

$0 \leqslant t \leqslant \frac{40}{7}$

Question 14 continued

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Question 14 continued

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15. The curve C has equation

$$x^2 \tan y = 9 \qquad 0 < y < \frac{\pi}{2}$$

$\tan y = \frac{9}{x^2}$

(a) Show that

$$\frac{dy}{dx} = \frac{-18x}{x^4 + 81}$$

(4)

(b) Prove that C has a point of inflection at $x = \sqrt[4]{27}$

(3)

a/ $x^2 \tan y = 9$

$u = x^2 \qquad v = \tan y$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \sec^2 y \frac{dy}{dx}$

$2x \tan y + x^2 \sec^2 y \frac{dy}{dx} = 0$

$x^2 \sec^2 y \frac{dy}{dx} = -2x \tan y$

$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$\frac{dy}{dx} = -\frac{2x \tan y}{x^2 \sec^2 y}$

$1 + \tan^2 x = \sec^2 x$

$= \frac{-2x\left(\frac{9}{x^2}\right)}{x^2(1+\tan^2 x)}$

$\tan x = \frac{9}{x^2}$

$\tan^2 x = \frac{81}{x^4}$

$= \frac{\left(-\frac{18x}{x^2}\right)}{x^2\left(1 + \frac{81}{x^4}\right)}$

$= \frac{\left(\frac{-18x}{x^2}\right)}{\left(x^2 + \frac{81}{x^2}\right)} \qquad \times x^2 \quad \times x^2$

$= \frac{-18x}{x^4 + 81}$

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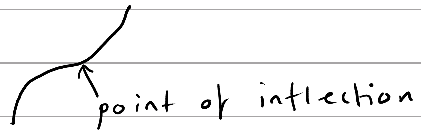
Question 15 continued

b/ A point of inflection is where the concavity changes.

Where the gradient goes from increasing to decreasing or decreasing to increasing



$$\frac{dy}{dx} = \frac{-18x}{x^4+81} \qquad u = -18x \qquad v = x^4 + 81$$

$$\frac{du}{dx} = -18 \qquad \frac{dv}{dx} = 4x^3$$

$$\frac{d^2y}{dx^2} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{-18(x^4+81) - -18x(4x^3)}{(x^4+81)^2}$$

$$= \frac{-18x^4 - 1458 + 72x^4}{(x^4+81)^2}$$

$$= \frac{54x^4 - 1458}{(x^4+81)^2}$$

when $x = \sqrt[4]{27}$ $\quad \frac{d^2y}{dx^2} = 0$ $\qquad \left[\frac{54(27)-1458}{(27+81)^2} = 0\right.$

when $x < \sqrt[4]{27}$ $\quad \frac{d^2y}{dx^2} < 0$ $\qquad \frac{54(26)-1458}{(26+81)^2} < 0$

when $x > \sqrt[4]{27}$ $\quad \frac{d^2y}{dx^2} > 0$ $\qquad \left.\frac{54(28)-1458}{(28+81)^2} > 0\right]$

$\therefore$ when $x = \sqrt[4]{27}$ there is a point of inflection

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Question 15 continued

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Question 15 continued

(Total for Question 15 is 7 marks)

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16. Prove by contradiction that there are no positive integers p and q such that

$$4p^2 - q^2 = 25$$

(4)

$$4p^2 - q^2 = 25$$

$$(2p+q)(2p-q) = 25$$

$25 = 5 \times 5$ or 1×25

Either $2p+q = 5$ and $2p-q = 5$

$$\begin{aligned} 2p+q &= 5 \\ + \quad + \quad & \quad + \\ 2p-q &= 5 \\ 4p &= 10 \end{aligned}$$

$p = \frac{10}{4} = \frac{5}{2}$ (not a +ve integer)

Or $2p+q = 25$ and $2p-q = 1$

$$\begin{aligned} 2p+q &= 25 \\ + \quad + \quad & \quad + \\ 2p-q &= 1 \\ 4p &= 26 \end{aligned}$$

$p = \frac{26}{4} = \frac{13}{2}$ (not a +ve integer)

$\therefore p$ cannot be a positive integer

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Question 16 continued

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Question 16 continued

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TOTAL FOR PAPER IS 100 MARKS

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