1


The shaded region in the diagram is bounded by the curve $y=\frac{2}{x}$, the $x$-axis and the lines $x=\frac{1}{2}$ and $x=2$. Show that when the shaded region is rotated through $360^{\circ}$ about the $x$-axis, the volume of the solid formed is $6 \pi$.

2


The shaded region in the diagram, bounded by the curve $y=x^{2}+3$, the coordinate axes and the line $x=2$, is rotated through $2 \pi$ radians about the $x$-axis.
Show that the volume of the solid formed is approximately 127.
3 The region enclosed by the given curve, the $x$-axis and the given ordinates is rotated through $360^{\circ}$ about the $x$-axis. Find the exact volume of the solid formed in each case.
a $y=2 \mathrm{e}^{\frac{x}{2}}$,
$x=0, \quad x=1$
b $y=\frac{3}{x^{2}}$,
$x=-2, \quad x=-1$
c $y=1+\frac{1}{x}$,
$x=3, \quad x=9$
d $y=\frac{3 x^{2}+1}{x}$,
$x=1, \quad x=2$
e $y=\frac{1}{\sqrt{x+2}}$,
$x=2, \quad x=6$
f $y=\mathrm{e}^{1-x}$,
$x=-1, \quad x=1$


The diagram shows part of the curve with equation $y=\frac{4}{x+2}$.
The shaded region, $R$, is bounded by the curve, the coordinate axes and the line $x=2$.
a Find the area of $R$, giving your answer in the form $k \ln 2$.
The region $R$ is rotated through $2 \pi$ radians about the $x$-axis.
b Show that the volume of the solid formed is $4 \pi$.

5


The diagram shows the curve with equation $y=2 x^{\frac{1}{2}}+x^{-\frac{1}{2}}$.
The shaded region bounded by the curve, the $x$-axis and the lines $x=1$ and $x=3$ is rotated through $2 \pi$ radians about the $x$-axis. Find the volume of the solid generated, giving your answer in the form $\pi(a+\ln b)$ where $a$ and $b$ are integers.

6 a Sketch the curve $y=3 x-x^{2}$, showing the coordinates of any points where the curve intersects the coordinate axes.
The region bounded by the curve and the $x$-axis is rotated through $360^{\circ}$ about the $x$-axis.
b Show that the volume of the solid generated is $\frac{81}{10} \pi$.
7


The diagram shows the curve with equation $y=3-\frac{1}{x}, x>0$.
a Find the coordinates of the point $P$ where the curve crosses the $x$-axis.
The shaded region is bounded by the curve, the straight line $x-3=0$ and the $x$-axis.
b Find the area of the shaded region.
c Find the volume of the solid formed when the shaded region is rotated completely about the $x$-axis, giving your answer in the form $\pi(a+b \ln 3)$ where $a$ and $b$ are rational.

8


The diagram shows the curve $y=x-\frac{1}{x}, x \neq 0$.
a Find the coordinates of the points where the curve crosses the $x$-axis.
The shaded region is bounded by the curve, the $x$-axis and the line $x=3$.
b Show that the area of the shaded region is $4-\ln 3$.
The shaded region is rotated through $360^{\circ}$ about the $x$-axis.
c Find the volume of the solid generated as an exact multiple of $\pi$.

