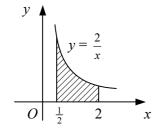
Worksheet I

> INTEGRATION

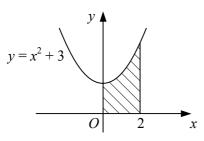


The shaded region in the diagram is bounded by the curve $y = \frac{2}{x}$, the x-axis and the lines $x = \frac{1}{2}$ and x = 2. Show that when the shaded region is rotated through 360° about the x-axis, the volume of the solid formed is 6π .

2

C4

1

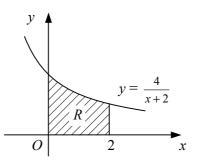


The shaded region in the diagram, bounded by the curve $y = x^2 + 3$, the coordinate axes and the line x = 2, is rotated through 2π radians about the *x*-axis.

Show that the volume of the solid formed is approximately 127.

- 3 The region enclosed by the given curve, the x-axis and the given ordinates is rotated through 360° about the x-axis. Find the exact volume of the solid formed in each case.
 - **a** $y = 2e^{\frac{x}{2}}$, x = 0, x = 1 **b** $y = \frac{3}{x^2}$, x = -2, x = -1 **c** $y = 1 + \frac{1}{x}$, x = 3, x = 9 **d** $y = \frac{3x^2 + 1}{x}$, x = 1, x = 2 **e** $y = \frac{1}{\sqrt{x+2}}$, x = 2, x = 6**f** $y = e^{1-x}$, x = -1, x = 1

4

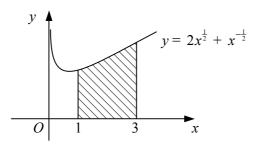


The diagram shows part of the curve with equation $y = \frac{4}{x+2}$.

The shaded region, R, is bounded by the curve, the coordinate axes and the line x = 2.

- **a** Find the area of *R*, giving your answer in the form $k \ln 2$.
- The region *R* is rotated through 2π radians about the *x*-axis.
- **b** Show that the volume of the solid formed is 4π .





The diagram shows the curve with equation $y = 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$.

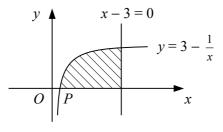
The shaded region bounded by the curve, the x-axis and the lines x = 1 and x = 3 is rotated through 2π radians about the x-axis. Find the volume of the solid generated, giving your answer in the form $\pi(a + \ln b)$ where a and b are integers.

6 a Sketch the curve $y = 3x - x^2$, showing the coordinates of any points where the curve intersects the coordinate axes.

The region bounded by the curve and the x-axis is rotated through 360° about the x-axis.

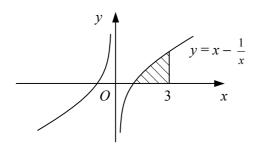
b Show that the volume of the solid generated is $\frac{81}{10}\pi$.

7



The diagram shows the curve with equation $y = 3 - \frac{1}{x}$, x > 0.

- **a** Find the coordinates of the point *P* where the curve crosses the *x*-axis.
- The shaded region is bounded by the curve, the straight line x 3 = 0 and the x-axis.
- **b** Find the area of the shaded region.
- **c** Find the volume of the solid formed when the shaded region is rotated completely about the *x*-axis, giving your answer in the form $\pi(a + b \ln 3)$ where *a* and *b* are rational.
- 8



The diagram shows the curve $y = x - \frac{1}{x}$, $x \neq 0$.

a Find the coordinates of the points where the curve crosses the *x*-axis.

The shaded region is bounded by the curve, the *x*-axis and the line x = 3.

b Show that the area of the shaded region is $4 - \ln 3$.

The shaded region is rotated through 360° about the *x*-axis.

c Find the volume of the solid generated as an exact multiple of π .