1

a $= -\sin x$

b $= 5\cos x$

c $= -3\sin 3x$

d $= \frac{1}{4}\cos \frac{1}{4}x$

e $= \cos(x + 1)$

f $= -3\sin(3x - 2)$

g $= -4\cos(\frac{\pi}{3} - x)$

h $= -\frac{1}{2}\sin(\frac{1}{2}x + \frac{\pi}{6})$

i $= 2\sin x\cos x$

j $= 6\cos^2 x \times (-\sin x)$
$= -6\cos^2 x \sin x$

k $= 2\cos(x - 1) \times [-\sin(x - 1)]$
$= -2\cos(x - 1)\sin(x - 1)$

l $= 4\sin^3 2x \times 2\cos 2x$
$= 8\sin^3 2x\cos 2x$

2

a $= \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$
$= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$
$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$
$= \frac{1}{\cos^2 x}$
$= \sec^2 x$

b $= \frac{d}{dx}[(\cos x)^{-1}]$
$= -(\cos x)^{-2} \times (-\sin x)$
$= \frac{\sin x}{\cos^2 x}$
$= \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$
$= \sec x \tan x$

c $= \frac{d}{dx}[(\sin x)^{-1}]$
$= -(\sin x)^{-2} \times \cos x$
$= -\frac{\cos x}{\sin^2 x}$
$= -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$
$= -\operatorname{cosec} x \cot x$

d $= \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right)$
$= \frac{-\sin x \times \sin x - \cos x \times \cos x}{\sin^2 x}$
$= -\frac{\sin^2 x + \cos^2 x}{\sin^2 x}$
$= -\frac{1}{\sin^2 x}$
$= -\operatorname{cosec}^2 x$

3

a $= -2\operatorname{cosec}^2 2t$

b $= \sec(t + 2)\tan(t + 2)$

c $= 4\sec^2(4t - 3)$

d $= -3\operatorname{cosec} 3t \cot 3t$

e $= 2\tan t \times \sec^2 t$
$= 2\tan t \sec^2 t$

f $= -3\operatorname{cosec}(t + \frac{\pi}{6})\cot(t + \frac{\pi}{6})$

g $= 3\cot^2 t \times (-\operatorname{cosec}^2 t)$
$= -3\cot^2 t \operatorname{cosec}^2 t$

h $= 2\sec \frac{1}{2}t \tan \frac{1}{2}t$

i $= -2\operatorname{cosec}^2(2t - 3)$

j $= 2\sec 2t \times 2\sec 2t \tan 2t$
$= 4\sec^2 2t \tan 2t$

k $= -2\sec^2(\pi - 4t)$

l $= 2\operatorname{cosec}(3t + 1) \times -3\operatorname{cosec}(3t + 1)\cot(3t + 1)$
$= -6\operatorname{cosec}^2(3t + 1)\cot(3t + 1)$

4 **a** $= \frac{1}{\sin x} \times \cos x$
$= \cot x$

b $= 6e^{\tan x} \times \sec^2 x$
$= 6e^{\tan x} \sec^2 x$

c $= \frac{1}{2}(\cos 2x)^{-\frac{1}{2}} \times (-2 \sin 2x)$
$= -\frac{\sin 2x}{\sqrt{\cos 2x}}$

d $= e^{\sin 3x} \times 3 \cos 3x$
$= 3e^{\sin 3x} \cos 3x$

e $= -2 \operatorname{cosec}^2 x^2 \times 2x$
$= -4x \operatorname{cosec}^2 x^2$

f $= \frac{1}{2}(\sec x)^{-\frac{1}{2}} \times \sec x \tan x$
$= \frac{1}{2}\sqrt{\sec x} \tan x$

g $= 3e^{-\operatorname{cosec} 2x} \times 2 \operatorname{cosec} 2x \cot 2x$
$= 6 \operatorname{cosec} 2x \cot 2x \, e^{-\operatorname{cosec} 2x}$

h $= \frac{1}{\tan 4x} \times 4 \sec^2 4x$
$= \frac{\cos 4x}{\sin 4x} \times \frac{4}{\cos^2 4x}$
$= 4 \sec 4x \operatorname{cosec} 4x$

5 **a** $\frac{dy}{dx} = 1 + 2 \cos x$
SP: $1 + 2 \cos x = 0$
$\cos x = -\frac{1}{2}$
$x = \pi - \frac{\pi}{3}, \pi + \frac{\pi}{3}$
$x = \frac{2\pi}{3}, \frac{4\pi}{3}$
$\therefore (\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3}),$
$(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3})$

b $\frac{dy}{dx} = 2 \sec x \tan x - \sec^2 x$
$= \sec x(2 \tan x - \sec x)$
SP: $\sec x(2 \tan x - \sec x) = 0$
$2 \tan x - \sec x = 0$
$\frac{2 \sin x}{\cos x} = \frac{1}{\cos x}$
$\sin x = \frac{1}{2}$
$x = \frac{\pi}{6}, \pi - \frac{\pi}{6}$
$x = \frac{\pi}{6}, \frac{5\pi}{6}$
$\therefore (\frac{\pi}{6}, \sqrt{3}), (\frac{5\pi}{6}, -\sqrt{3})$

c $\frac{dy}{dx} = \cos x - 2 \sin 2x$
SP: $\cos x - 2 \sin 2x = 0$
$\cos x - 4 \sin x \cos x = 0$
$\cos x(1 - 4 \sin x) = 0$
$\cos x = 0$ or $\sin x = \frac{1}{4}$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $0.253, \pi - 0.25268$
$x = 0.253$ (3sf), $\frac{\pi}{2}$, 2.89 (3sf), $\frac{3\pi}{2}$
$\therefore (0.253, \frac{9}{8}), (\frac{\pi}{2}, 0),$
$(2.89, \frac{9}{8}), (\frac{3\pi}{2}, -2)$

6 **a** $x = 0 \therefore y = 1$
$\frac{dy}{dx} = 2 \cos 2x$
grad = 2
$\therefore y - 1 = 2(x - 0)$
[$y = 2x + 1$]

b $x = \frac{\pi}{3} \therefore y = \frac{1}{2}$
$\frac{dy}{dx} = -\sin x$
grad $= -\frac{\sqrt{3}}{2}$
$\therefore y - \frac{1}{2} = -\frac{\sqrt{3}}{2}(x - \frac{\pi}{3})$
[$3\sqrt{3}x + 6y - 3 - \sqrt{3}\pi = 0$]

c $x = \frac{\pi}{4} \therefore y = -1$
$\frac{dy}{dx} = 3 \sec^2 3x$
grad = 6
$\therefore y + 1 = 6(x - \frac{\pi}{4})$
[$12x - 2y - 2 - 3\pi = 0$]

d $x = \frac{\pi}{6} \therefore y = 1$
$\frac{dy}{dx} = -\operatorname{cosec} x \cot x - 2 \cos x$
grad $= -3\sqrt{3}$
$\therefore y - 1 = -3\sqrt{3}(x - \frac{\pi}{6})$
[$6\sqrt{3}x + 2y - 2 - \sqrt{3}\pi = 0$]

7 **a** $= 1 \times \sin x + x \times \cos x$

$= \sin x + x\cos x$

b $= \dfrac{-2\sin 2x \times x - \cos 2x \times 1}{x^2}$

$= -\dfrac{2x\sin 2x + \cos 2x}{x^2}$

c $= e^x \times \cos x + e^x \times (-\sin x)$
$= e^x(\cos x - \sin x)$

d $= \cos x \times \cos x + \sin x \times (-\sin x)$
$= \cos^2 x - \sin^2 x = \cos 2x$

e $= 2x \times \operatorname{cosec} x + x^2 \times (-\operatorname{cosec} x \cot x)$
$= x \operatorname{cosec} x(2 - x\cot x)$

f $= \sec x \tan x \times \tan x + \sec x \times \sec^2 x$
$= \sec x(\tan^2 x + \sec^2 x)$

g $= \dfrac{1 \times \tan x - x \times \sec^2 x}{\tan^2 x}$

$= \cot x - x\operatorname{cosec}^2 x$

h $= \dfrac{2\cos 2x \times e^{3x} - \sin 2x \times 3e^{3x}}{(e^{3x})^2}$

$= \dfrac{2\cos 2x - 3\sin 2x}{e^{3x}}$

i $= 2\cos x(-\sin x)\times\cot x + \cos^2 x\times(-\operatorname{cosec}^2 x)$

$= -2\cos^2 x - \cot^2 x$

j $= \dfrac{2\sec 2x \tan 2x \times x^2 - \sec 2x \times 2x}{x^4}$

$= \dfrac{2\sec 2x(x\tan 2x - 1)}{x^3}$

k $= 1 \times \tan^2 4x + x \times 2\tan 4x \times 4\sec^2 4x$

$= \tan 4x(\tan 4x + 8x\sec^2 4x)$

l $= \dfrac{\cos x \times \cos 2x - \sin x \times (-2\sin 2x)}{\cos^2 2x}$

$= \dfrac{\cos x(1 - 2\sin^2 x) + 4\sin^2 x\cos x}{\cos^2 2x} = \dfrac{\cos x(1 + 2\sin^2 x)}{\cos^2 2x}$

8 **a** $f'(x) = 3\cos 3x \times \cos 5x + \sin 3x \times (-5\sin 5x)$
$= 3\cos 3x\cos 5x - 5\sin 3x\sin 5x$

$f'(\tfrac{\pi}{4}) = 3(-\tfrac{1}{\sqrt{2}})(-\tfrac{1}{\sqrt{2}}) - 5\times\tfrac{1}{\sqrt{2}}(-\tfrac{1}{\sqrt{2}})$

$= \tfrac{3}{2} + \tfrac{5}{2} = 4$

b $f'(x) = 2\sec^2 2x \times \sin x + \tan 2x \times \cos x$
$= 2\sec^2 2x\sin x + \tan 2x\cos x$

$f'(\tfrac{\pi}{3}) = 2\times 4\times\tfrac{\sqrt{3}}{2} + (-\sqrt{3})\times\tfrac{1}{2}$

$= 4\sqrt{3} - \tfrac{1}{2}\sqrt{3} = \tfrac{7}{2}\sqrt{3}$

c $f'(x) = \dfrac{\frac{1}{2\cos x}\times(-2\sin x)\times\sin x - \ln(2\cos x)\times\cos x}{\sin^2 x}$

$= -\sec x - \dfrac{\cos x\ln(2\cos x)}{\sin^2 x}$

$f'(\tfrac{\pi}{3}) = -2 - 0 = -2$

d $f'(x) =$

$2\sin x\cos x \times \cos^3 x + \sin^2 x \times 3\cos^2 x \times (-\sin x)$

$= \sin x\cos^2 x(2\cos^2 x - 3\sin^2 x)$

$f'(\tfrac{\pi}{6}) = \tfrac{1}{2}\times\tfrac{3}{4}(2\times\tfrac{3}{4} - 3\times\tfrac{1}{4}) = \tfrac{9}{32}$

9 $x = 0 \;\therefore\; y = 3$

$\dfrac{dy}{dx} = 1 \times \cos 2x + x \times (-2\sin 2x)$

$= \cos 2x - 2x\sin 2x$

grad = 1

$\therefore$ grad of normal = –1

$\therefore\; y - 3 = -(x - 0)$

$[\, y = 3 - x \,]$

10 **a** $= \dfrac{\cos x \times (1 - \sin x) - (2 + \sin x)\times(-\cos x)}{(1 - \sin x)^2}$

$= \dfrac{3\cos x}{(1 - \sin x)^2}$

b SP: $\dfrac{3\cos x}{(1 - \sin x)^2} = 0$

$\cos x = 0$

$x \neq \tfrac{\pi}{2} \;\therefore\; x = \tfrac{3\pi}{2}$

$\therefore\; (\tfrac{3\pi}{2}, \tfrac{1}{2})$

c $x = \tfrac{\pi}{6} \;\therefore\; y = 5$

grad $= 6\sqrt{3}$

$y - 5 = 6\sqrt{3}(x - \tfrac{\pi}{6})$

$y = 6\sqrt{3}x + 5 - \sqrt{3}\pi$

11 **a** $\frac{dy}{dx} = -e^{-x} \times \sin x + e^{-x} \times \cos x$

$= e^{-x}(\cos x - \sin x)$

$\frac{d^2y}{dx^2} = -e^{-x} \times (\cos x - \sin x)$

$+ e^{-x} \times (-\sin x - \cos x)$

$= -2e^{-x} \cos x$

b SP: $e^{-x}(\cos x - \sin x) = 0$

$\cos x - \sin x = 0$

$\tan x = 1$

$x = \frac{\pi}{4},\ \frac{\pi}{4} - \pi = -\frac{3\pi}{4},\ \frac{\pi}{4}$

$x = -\frac{3\pi}{4}$: $\frac{d^2y}{dx^2} = \sqrt{2}\,e^{\frac{3\pi}{4}}$ (> 0)

$x = \frac{\pi}{4}$: $\frac{d^2y}{dx^2} = -\sqrt{2}\,e^{-\frac{\pi}{4}}$ (< 0)

$\therefore$ $(-\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}})$, minimum

$(\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}})$, maximum

12 **a** $\frac{dy}{dx} = 1 \times \sec x + x \times \sec x \tan x$

$= \sec x(1 + x \tan x)$

SP: $\sec x(1 + x \tan x) = 0$

no real values of x for which $\sec x = 0$

$\therefore$ $1 + x \tan x = 0$

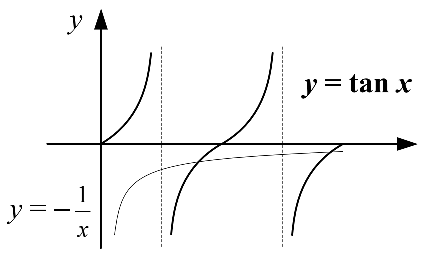
b $1 + x \tan x = 0$

$\Rightarrow$ $x \tan x = -1$

$\tan x = -\frac{1}{x}$

$\therefore$ x-coord of SP where curves

$y = \tan x$ and $y = -\frac{1}{x}$ intersect



intersect at 2 points $\therefore$ 2 SP in interval

13 **a** $f'(x) = -\sin x \times \sin 2x + \cos x \times 2\cos 2x$

$= 2\cos x(1 - 2\sin^2 x) - 2\sin^2 x \cos x$

$= 2\cos x(1 - 3\sin^2 x)$

b SP: $2\cos x(1 - 3\sin^2 x) = 0$

$\cos x = 0$ or $\sin x = \pm\frac{1}{\sqrt{3}}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ or $0.615, \pi - 0.61548,$

$\pi + 0.61548, 2\pi - 0.61548$

$x = 0.615, \frac{\pi}{2}, 2.53, 3.76, \frac{3\pi}{2}, 5.67$

c using graph, max. f(x) when $\sin x = \frac{1}{\sqrt{3}}$

$\cos^2 x = 1 - \sin^2 x = 1 - \frac{1}{3} = \frac{2}{3}$

$f(x) = \cos x \times 2\sin x \cos x = 2\sin x \cos^2 x$

$\therefore$ max. $f(x) = 2 \times \frac{1}{\sqrt{3}} \times \frac{2}{3}$

$= \frac{4}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{4}{9}\sqrt{3}$

d period of $\cos x = 2\pi$, period of $\sin 2x = \pi$

$\therefore$ period of $f(x) = 2\pi$

$\therefore$ values of $f(x)$ in this interval are repeated

14 **a** $x = 0$ $\therefore$ $y = -2$

$\frac{dy}{dx} = -\text{cosec}\,(x - \frac{\pi}{6})\cot(x - \frac{\pi}{6})$

grad $= -2\sqrt{3}$

$\therefore$ grad of normal $= \frac{1}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{6}\sqrt{3}$

$\therefore$ $y + 2 = \frac{1}{6}\sqrt{3}\,(x - 0)$

[$y = \frac{1}{6}\sqrt{3}\,x - 2$]

b $x = \frac{\pi}{3}$ $\therefore$ $y = 2$

grad $= -2\sqrt{3}$

$\therefore$ $y - 2 = -2\sqrt{3}\,(x - \frac{\pi}{3})$

[$6\sqrt{3}\,x + 3y - 6 - 2\sqrt{3}\,\pi = 0$]

c $\frac{1}{6}\sqrt{3}\,x - 2 = 2 - 2\sqrt{3}\,(x - \frac{\pi}{3})$

$x - 4\sqrt{3} = 4\sqrt{3} - 12x + 4\pi$

$13x = 8\sqrt{3} + 4\pi$

$x = \frac{8\sqrt{3} + 4\pi}{13}$