Edexcel GCE

Core Mathematics S1

Probability

Materials required for examination
Mathematical Formulae (Green)

Items included with question papers
Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A disease is known to be present in 2% of a population. A test is developed to help determine whether or not someone has the disease.

Given that a person has the disease, the test is positive with probability 0.95.

Given that a person does not have the disease, the test is positive with probability 0.03.

(*a*) Draw a tree diagram to represent this information.

(3)

A person is selected at random from the population and tested for this disease.

(*b*) Find the probability that the test is positive.

(3)

A doctor randomly selects a person from the population and tests him for the disease. Given that the test is positive,

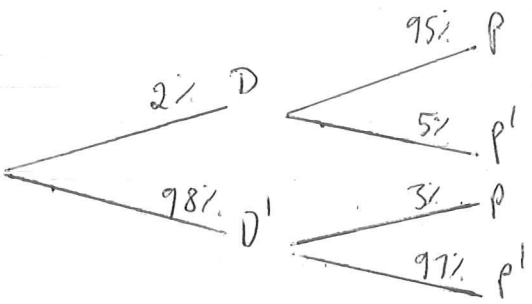
(*c*) find the probability that he does not have the disease.

(2)

(*d*) Comment on the usefulness of this test.

(1)

a)



b/ $0.02 \times 0.95 = 0.019$

$0.98 \times 0.03 = 0.0294$

0.0484

4.84%

c/ $\frac{0.0294}{0.0484} = 60.7\%$ (3sf)

d/ The test is not very useful as more than half of those who test positive do not have the disease.

2. In a factory, machines A, B and C are all producing metal rods of the same length. Machine A produces 35% of the rods, machine B produces 25% and the rest are produced by machine C. Of their production of rods, machines A, B and C produce 3%, 6% and 5% defective rods respectively.

(a) Draw a tree diagram to represent this information.

(3)

(b) Find the probability that a randomly selected rod is

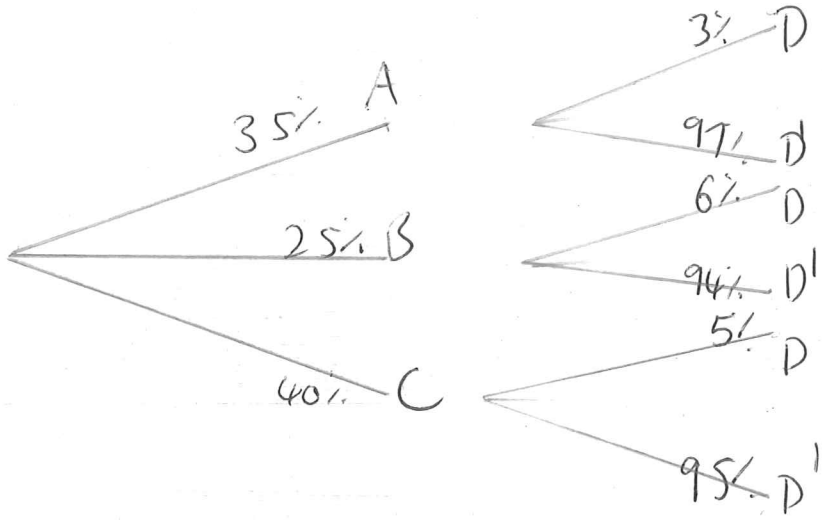
(i) produced by machine A and is defective,

(ii) is defective.

(5)

(c) Given that a randomly selected rod is defective, find the probability that it was produced by machine C.

(3)



b i) $0.35 \times 0.03 = 0.0105$

ii) $(0.35 \times 0.03) + (0.25 \times 0.06) + (0.4 \times 0.05)$

$= 0.0455$

c) $\frac{0.4 \times 0.05}{0.0455} = \frac{40}{91}$

3. A group of office workers were questioned for a health magazine and $\frac{2}{5}$ were found to take regular exercise. When questioned about their eating habits $\frac{2}{3}$ said they always eat breakfast and, of those who always eat breakfast $\frac{9}{25}$ also took regular exercise.

Find the probability that a randomly selected member of the group

(a) always eats breakfast and takes regular exercise,

(2)

(b) does not always eat breakfast and does not take regular exercise.

(4)

(c) Determine, giving your reason, whether or not always eating breakfast and taking regular exercise are statistically independent.

(2)

a) $P(E) = \frac{2}{5}$

$P(B) = \frac{2}{3}$

$P(E|B) = \frac{9}{25}$

$P(E \cap B) = \frac{2}{5} \times \frac{9}{25}$

$= \frac{6}{25}$

b) $P(E \cup B) = P(E) + P(B) - P(E \cap B)$

$= \frac{2}{5} + \frac{2}{3} - \frac{6}{25}$

$= \frac{62}{75}$

$P(E \cup B)' = \frac{13}{75}$

c) if ~~indende~~ independent $P(A) \times P(B) = P(A \cap B)$

$\frac{2}{5} \times \frac{2}{3} = \frac{4}{15}$

$P(E \cap B) \neq \frac{4}{15}$ 4

They are not statistically independent.

4. A survey of the reading habits of some students revealed that, on a regular basis, 25% read quality newspapers, 45% read tabloid newspapers and 40% do not read newspapers at all.

(*a*) Find the proportion of students who read both quality and tabloid newspapers.

(3)

(*b*) Draw a Venn diagram to represent this information.

(3)

A student is selected at random. Given that this student reads newspapers on a regular basis,

(*c*) find the probability that this student only reads quality newspapers.

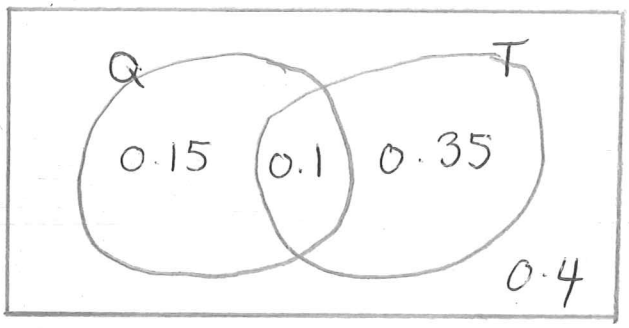
(3)

a) $P(Q \cap T) = P(Q) + P(T) - P(Q \cup T)$

$= 0.25 + 0.45 - 0.6$

$= 0.1$

b)



c) $\frac{0.15}{0.6} = 0.25$

5. The following shows the results of a wine tasting survey of 100 people.

96 like wine *A*,
93 like wine *B*,
96 like wine *C*,
92 like *A* and *B*,
91 like *B* and *C*,
93 like *A* and *C*,
90 like all three wines.

(*a*) Draw a Venn Diagram to represent these data.

(6)

Find the probability that a randomly selected person from the survey likes

(*b*) none of the three wines,

(1)

(*c*) wine *A* but not wine *B*,

(2)

(*d*) any wine in the survey except wine *C*,

(2)

(*e*) exactly two of the three kinds of wine.

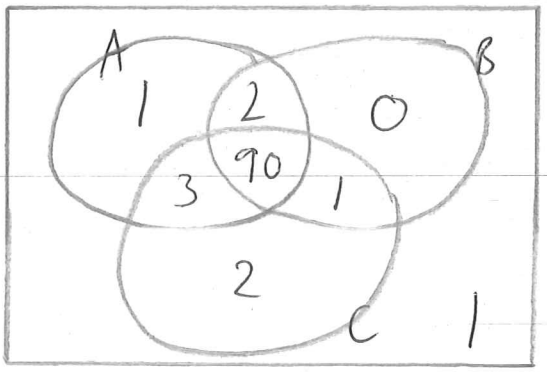
(2)

Given that a person from the survey likes wine *A*,

(*f*) find the probability that the person likes wine *C*.

(3)

a/



b/ $\frac{1}{100}$

c/ $\frac{4}{100}$

d/ $\frac{3}{100}$

e/ $\frac{6}{100}$

f/ $\frac{94}{96}$

6. A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left-handed and having dark hair.

Glasses were worn by 36 people, 28 were left-handed and 36 had dark hair. There were 17 who wore glasses and were left-handed, 19 who wore glasses and had dark hair and 15 who were left-handed and had dark hair. Only 10 people wore glasses, were left-handed and had dark hair.

(*a*) Represent these data on a Venn diagram.

(6)

A person was selected at random from this group.

Find the probability that this person

(*b*) wore glasses but was not left-handed and did not have dark hair,

(1)

(*c*) did not wear glasses, was not left-handed and did not have dark hair,

(1)

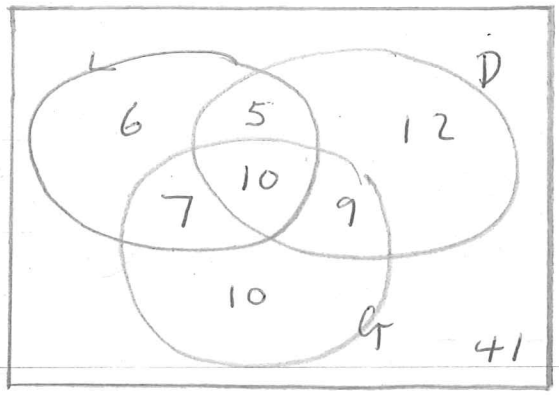
(*d*) had only two of the attributes,

(2)

(*e*) wore glasses, given they were left-handed and had dark hair.

(3)

a/



b/ $\frac{10}{100}$

c/ $\frac{41}{100}$

d/ $\frac{21}{100}$

e/ $\frac{10}{15}$

7. A person's blood group is determined by whether or not it contains any of 3 substances *A*, *B* and *C*.

A doctor surveyed 300 patients' blood and produced the table below.

Blood contains	No. of Patients
only *C*	100
A and *C* but not *B*	100
only *A*	30
B and *C* but not *A*	25
only *B*	12
A, *B* and *C*	10
A and *B* but not *C*	3

(*a*) Draw a Venn diagram to represent this information.

(4)

(*b*) Find the probability that a randomly chosen patient's blood contains substance *C*.

(2)

Harry is one of the patients. Given that his blood contains substance *A*,

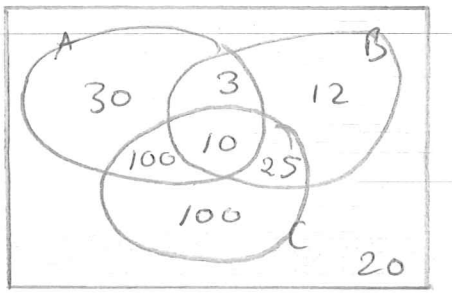
(*c*) find the probability that his blood contains all 3 substances.

(2)

Patients whose blood contains none of these substances are called universal blood donors.

(*d*) Find the probability that a randomly chosen patient is a universal blood donor.

(2)



b/ $\frac{235}{300}$

c/ $\frac{10}{143}$

d/ $\frac{20}{300}$

8. On a randomly chosen day the probability that Bill travels to school by car, by bicycle or on foot is $\frac{1}{2}$, $\frac{1}{6}$ and $\frac{1}{3}$ respectively. The probability of being late when using these methods of travel is $\frac{1}{5}$, $\frac{2}{5}$ and $\frac{1}{10}$ respectively.

(a) Draw a tree diagram to represent this information.

(3)

(*b*) Find the probability that on a randomly chosen day

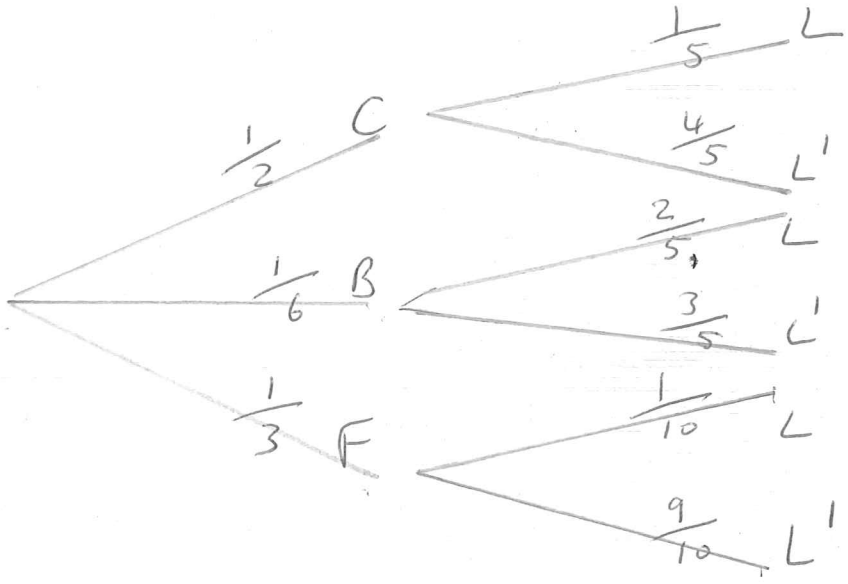
(i) Bill travels by foot and is late,

(ii) Bill is not late.

(4)

(*c*) Given that Bill is late, find the probability that he did not travel on foot.

(4)



b/ $\frac{1}{3} \times \frac{1}{10} = \frac{1}{30}$

c(ii) $\frac{1}{2} \times \frac{4}{5} + \frac{1}{6} \times \frac{3}{5} + \frac{1}{3} \times \frac{9}{10} = \frac{4}{5}$

c/ $\frac{\frac{1}{2} \times \frac{1}{5} + \frac{1}{6} \times \frac{2}{5}}{1/5} = \frac{5}{6}$

9. (*a*) Given that P(A) = a and P(B) = b express P($A \cup B$) in terms of a and b when

(i) A and B are mutually exclusive,

(ii) A and B are independent.

(2)

Two events R and Q are such that

$$P(R \cap Q') = 0.15, \quad P(Q) = 0.35 \quad \text{and} \quad P(R \mid Q) = 0.1$$

Find the value of

(*b*) P($R \cup Q$),

(1)

(*c*) P($R \cap Q$),

(2)

(*d*) P(R).

(2)

9 a i) $a + b$

ii) $a + b - ab$

b) $0.15 + 0.35 = 0.5$

c) $P(R \cap Q) = P(R|Q) \times P(Q)$

$= 0.1 \times 0.35$

$= 0.035$

d) ~~$P(R \cap Q) = P(R) + P(Q) - P(R \cup Q)$~~

~~$0.035 =$~~

$0.035 + 0.15$

$= 0.185$

10. A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded.

(*a*) Draw a tree diagram to represent the information.

(3)

Find the probability that

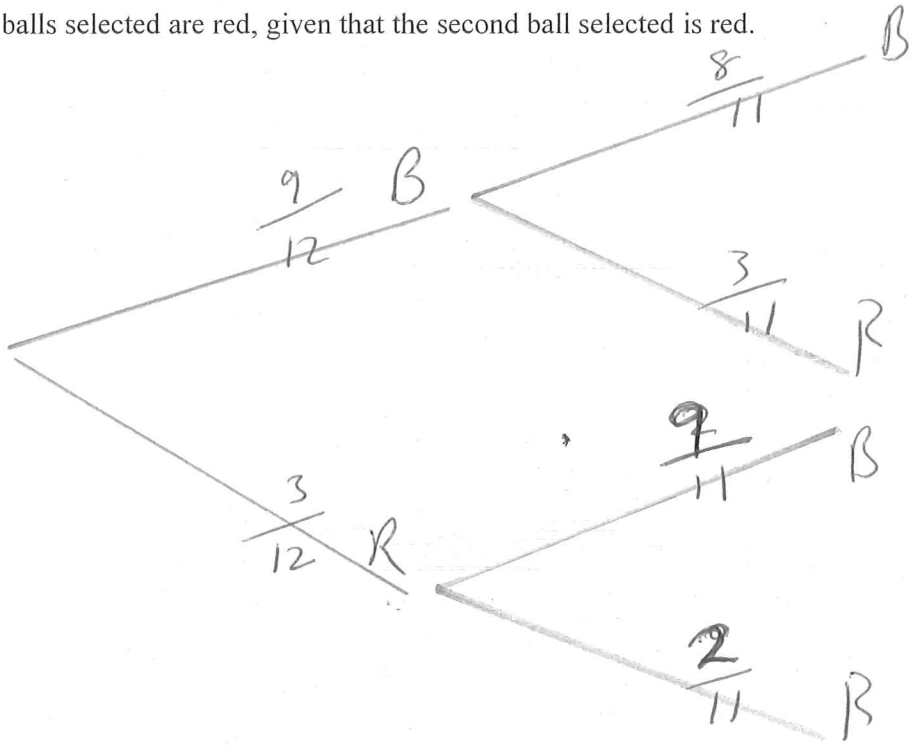
(*a*) the second ball selected is red,

(2)

(*b*) both balls selected are red, given that the second ball selected is red.

(2)

a/



b/ $\frac{9}{12} \times \frac{3}{11} + \frac{3}{12} \times \frac{2}{11} = \frac{1}{4}$

c/ $\frac{\frac{3}{12} \times \frac{2}{11}}{\frac{1}{4}} = \frac{2}{11}$

11. For the events A and B,

$$P(A \cap B') = 0.32,\ P(A' \cap B) = 0.11 \text{ and } P(A \cup B) = 0.65.$$

(*a*) Draw a Venn diagram to illustrate the complete sample space for the events A and B.

(3)

(*b*) Write down the value of $P(A)$ and the value of $P(B)$.

(3)

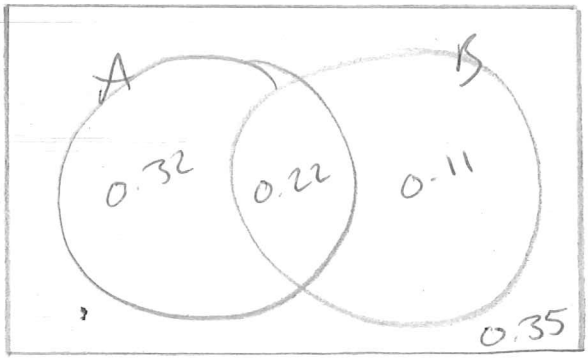
(*c*) Find $P(A \mid B')$.

(2)

(*d*) Determine whether or not A and B are independent.

(3)

a/



b/ $P(A) = 0.54$

$P(B) = 0.33$

c/ $\frac{0.32}{0.67} = \frac{32}{67}$

d/ $0.54 \times 0.33 = 0.1782$

$P(A) \times P(B) \neq P(A \cap B)$

A and B are not independent.