Edexcel GCE

Core Mathematics S1

Discrete Random

Variables

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers
without working may gain no credit.

1. The random variable X has probability distribution given in the table below.

x	-1	0	1	2	3
$P(X = x)$	p	q	0.2	0.15	0.15

Given that $E(X) = 0.55$, find

(a) the value of p and the value of q,

(5)

(b) Var (X),

(4)

(c) $E(2X - 4)$.

(2)

a) $p + q + 0.2 + 0.15 + 0.15 = 1$

$$p + q = 0.5$$

$-p + 1(0.2) + 2(0.15) + 3(0.15) = 0.55$

$$-p + 0.95 = 0.55$$
$$p = 0.4$$
$$\therefore q = 0.1$$

b) $Var(x) = E(x^2) - (E(x))^2$

$E(x^2) = 1(0.4) + 1(0.2) + 4(0.15) + 9(0.15)$

$$= 2.55$$

$Var(x) = 2.55 - (0.55)^2$

$$= 2.2475$$

c) $E(2x - 4) = 2(0.55) - 4$

$$= -2.9$$

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2. The random variable X has probability distribution

x	1	2	3	4	5
$P(X=x)$	0.10	p	0.20	q	0.30

(a) Given that $E(X) = 3.5$, write down two equations involving p and q.

(3)

Find

(b) the value of p and the value of q,

(3)

(c) $Var(X)$,

(4)

(d) $Var(3 - 2X)$.

(2)

a/
$$0.1 + p + 0.2 + q + 0.3 = 1$$
$$p + q + 0.6 = 1$$
$$\underline{p + q = 0.4}$$

$$1(0.1) + 2(p) + 3(0.2) + 4q + 5(0.3) = 3.5$$
$$2p + 4q + 2.2 = 3.5$$
$$\underline{2p + 4q = 1.3}$$

b/
$$2p + 2q = 0.8$$
$$2p + 4q = 1.3$$
$$2q = 0.5$$
$$\underline{q = 0.25}$$
$$\underline{\underline{p = 0.15}}$$

c/
$$Var(X) = E(X^2) - (E(x))^2$$
$$E(x^2) = 1(0.1) + 4(0.15) + 9(0.2) + 16(0.25) + 25(0.3)$$
$$= 14$$
$$Var(x) = 14 - (3.5)^2$$
$$\underline{\underline{= 1.75}}$$

d/
$$Var(3 - 2x) = 1.75 \times 4$$
$$\underline{\underline{= 7}}$$

3

3. The random variable X has probability function

$$P(X = x) = \frac{(2x-1)}{36} \qquad x = 1, 2, 3, 4, 5, 6.$$

(a) Construct a table giving the probability distribution of X.

(3)

Find

(b) $P(2 < X \leq 5)$

(2)

(c) the exact value of $E(X)$.

(2)

(d) Show that $Var(X) = 1.97$ to 3 significant figures.

(4)

(e) Find $Var(2 - 3X)$.

(2)

a/

x	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{36}$	$\frac{3}{36}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{9}{36}$	$\frac{11}{36}$

b/ $P(2 < X \leq 5) = P(3) + P(4) + P(5)$

$$= \frac{21}{36}$$

c/ $E(X) = 1\left(\frac{1}{36}\right) + 2\left(\frac{3}{36}\right) + 3\left(\frac{5}{36}\right) + 4\left(\frac{7}{36}\right) + 5\left(\frac{9}{36}\right) + 6\left(\frac{11}{36}\right)$

$$= \frac{161}{36}$$

d/ $E(X^2) = 1\left(\frac{1}{36}\right) + 4\left(\frac{3}{36}\right) + 9\left(\frac{5}{36}\right) + 16\left(\frac{7}{36}\right) + 25\left(\frac{9}{36}\right) + 36\left(\frac{11}{36}\right)$

$$= \frac{791}{36} = \boxed{21.97}$$

$Var(X) = \frac{791}{36} - \left(\frac{161}{36}\right)^2$

$$= 1.97 \quad (3sf)$$

e/ $Var(2 - 3x) = 1.97 \times 9$

$$= \underline{\underline{17.73}}$$

4

4. The random variable X has the discrete uniform distribution

$$P(X=x) = \frac{1}{5}, \quad x = 1, 2, 3, 4, 5.$$

(a) Write down the value of $E(X)$ and show that $Var(X) = 2$.

(3)

Find

(b) $E(3X - 2)$,

(2)

(c) $Var(4 - 3X)$

(2)

a/ $\quad E(x) = 3$

$\quad E(x^2) = 1\left(\frac{1}{5}\right) + 4\left(\frac{1}{5}\right) + 9\left(\frac{1}{5}\right) + 16\left(\frac{1}{5}\right) + 25\left(\frac{1}{5}\right)$

$\quad\quad = 11$

$\quad Var(x) = 11 - 3^2$

$\quad\quad = 2$

b/ $\quad E(3x-2) = 3(3) - 2$

$\quad\quad\quad = 7$

c/ $\quad Var(4-3x) = 9 \times 2$

$\quad\quad\quad = 18$

5. The random variable X has probability distribution

x	1	3	5	7	9
$P(X=x)$	0.2	p	0.2	q	0.15

(a) Given that $E(X) = 4.5$, write down two equations involving p and q.

(3)

Find

(b) the value of p and the value of q,

(3)

(c) $P(4 < X \le 7)$.

(2)

Given that $E(X^2) = 27.4$, find

(d) Var (X),

(2)

(e) $E(19 - 4X)$,

(1)

(f) Var $(19 - 4X)$.

(2)

a/ $\quad 0.2 + p + 0.2 + q + 0.15 = 1$

$\qquad\qquad p + q = 0.45$

$\quad 1(0.2) + 3p + 5(0.2) + 7q + 9(0.15) = 4.5$

$\qquad\qquad 3p + 7q = 1.95$

b/ $\qquad 3p + 7q = 1.95$

$\qquad 3p + 3q = 1.35$

$\qquad\quad 4q = 0.6$

$\qquad\quad q = 0.15$

$\qquad\quad p = 0.3$

c/ $\quad P(5) + P(7) \quad = \quad 0.2 + 0.15 = \quad 0.35$

d/ $\quad Var(x) = E(x^2) - (E(x))^2$

$\qquad\quad = 27.4 - 4.5^2$

$\qquad\quad = 7.15$

6

e/ $\quad 19 - 4(4.5) = 1$

f/ $\quad 16 \times 7.15 = 114.4$

6. The discrete random variable X can take only the values 2, 3 or 4. For these values the cumulative distribution function is defined by

$$F(x) = \frac{(x+k)^2}{25} \text{ for } x = 2, 3, 4,$$

where k is a positive integer.

(a) Find k.

(2)

(b) Find the probability distribution of X.

(3)

a/
$$\frac{(4+k)^2}{25} = 1$$

$$(4+k)^2 = 25$$

$$4 + k = \pm\sqrt{25}$$

$$4 + k = \pm 5$$

$$k = 1$$

(k cannot $= -9$ it is a positive integer)

b/

x	2	3	4
$F(x)$	$\frac{9}{25}$	$\frac{16}{25}$	1

x	2	3	4
$P(X=x)$	$\frac{9}{25}$	$\frac{7}{25}$	$\frac{9}{25}$

7. Tetrahedral dice have four faces. Two fair tetrahedral dice, one red and one blue, have faces numbered 0, 1, 2, and 3 respectively. The dice are rolled and the numbers face down on the two dice are recorded. The random variable R is the score on the red die and the random variable B is the score on the blue die.

(a) Find P($R = 3$ and $B = 0$).

$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

(2)

The random variable T is R multiplied by B.

(b) Complete the diagram below to represent the sample space that shows all the possible values of T.

3	0	3	6	9
2	0	2	4	6
1	0	1	2	3
0	0	0	0	0
B / R	0	1	2	3

Sample space diagram of T

(3)

The table below represents the probability distribution of the random variable T.

t	0	1	2	3	4	6	9
P($T = t$)	a	b	$\frac{1}{8}$	$\frac{1}{8}$	c	$\frac{1}{8}$	d

(c) Find the values of a, b, c and d.

$a = \frac{7}{16}$ $b = \frac{1}{16}$ $c = \frac{1}{16}$ $d = \frac{1}{16}$

(3)

Find the values of

(d) E(T),

$0\left(\frac{7}{16}\right) + 1\left(\frac{1}{16}\right) + 2\left(\frac{1}{8}\right) + 3\left(\frac{1}{8}\right) + 4\left(\frac{1}{16}\right) + 6\left(\frac{1}{8}\right) + 9\left(\frac{1}{16}\right) = \frac{9}{4}$

(2)

(e) Var(T).

(4)

$Var(T) = 1^2\left(\frac{1}{16}\right) + 2^2\left(\frac{1}{8}\right) + 3^2\left(\frac{1}{8}\right) + 4^2\left(\frac{1}{16}\right) + 6^2\left(\frac{1}{8}\right) + 9^2\left(\frac{1}{16}\right) - \left(\frac{9}{4}\right)^2$

$= \frac{115}{16}$

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8. When Rohit plays a game, the number of points he receives is given by the discrete random variable X with the following probability distribution.

x	0	1	2	3
$P(X=x)$	0.4	0.3	0.2	0.1

(a) Find E(X).

(2)

(b) Find F(1.5).

(2)

(c) Show that Var(X) = 1.

(4)

(d) Find Var(5 – 3X).

(2)

Rohit can win a prize if the total number of points he has scored after 5 games is at least 10. After 3 games he has a total of 6 points. You may assume that games are independent.

(e) Find the probability that Rohit wins the prize.

(6)

a) $0(0.4) + 1(0.3) + 2(0.2) + 3(0.1) = \underline{\underline{1}}$

b) $0.4 + 0.3 = \underline{\underline{0.7}}$

c) $E(X^2) = 0 \times 0.4 + 1 \times 0.3 + 2^2 \times 0.2 + 3^2 \times 0.1$

$= 2$

$Var(X) = 2 - 1^2$

$= \underline{\underline{1}}$

d) $1 \times 3^2 = \underline{\underline{9}}$

e)
1 and 3	03×0.1	$= 0.03$
2 and 2	0.2×0.2	$= 0.04$
2 and 3	0.2×0.1	$= 0.02$
3 and 1	0.1×0.3	$= 0.03$
3 and 2	0.1×0.2	$= 0.02$
3 and 3	0.1×0.1	$= 0.01$
	9	$\overline{0.15}$

total $= \underline{\underline{0.15}}$

9. The discrete random variable X has probability function

$$P(X=x) = \begin{cases} a(3-x) & x = 0, 1, 2 \\ b & x = 3 \end{cases}$$

(a) Find $P(X=2)$ and copy and complete the table below. $\quad (3-2)a = a$

x	0	1	2	3
$P(X=x)$	$3a$	$2a$	a	b

$\qquad\quad$ 0.3 $\quad$ 0.2 $\quad$ 0.1 $\quad$ 0.4

(1)

Given that $E(X) = 1.6$,

(b) find the value of a and the value of b.

$3a$

(5)

Find

(c) $P(0.5 < X < 3)$,

(2)

(d) $E(3X - 2)$.

(2)

(e) Show that the $Var(X) = 1.64$

(3)

(f) Calculate $Var(3X - 2)$.

(2)

b/ $\quad 3a + 2a + a + b = 1$

$\qquad\qquad 6a + b = 1$

$\qquad 2a + 2a + 3b = 1.6$

$\qquad\qquad 4a + 3b = 1.6$

$\qquad\qquad 18a + 3b = 3$

$\qquad\qquad\qquad 14a = 1.4$

$\qquad\qquad\qquad\quad a = 0.1$

$\qquad\qquad\qquad\quad b = 0.4$

c/ $\quad P(1) + P(2) = 0.2 + 0.1 = \underline{\underline{0.3}}$

d/ $\quad 3(1.6) - 2 = \underline{\underline{2.8}}$

e/ $\quad E(x^2) = 1^2 \times 0.2 + 2^2 \times 0.1 + 3^2 \times 0.4 = 4.2$

$\quad 4.2 - \&1.6^2 = \underline{\underline{1.64}}$

f/ $\quad 9 \times 1.64 = \underline{\underline{14.76}}$

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