# Edexcel GCE

## Core Mathematics C3

# Numerical Methods

| **Materials required for examination** | **Items included with question papers** |
|---|---|
| Mathematical Formulae (Green) | Nil |

**Advice to Candidates**

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

**1.**

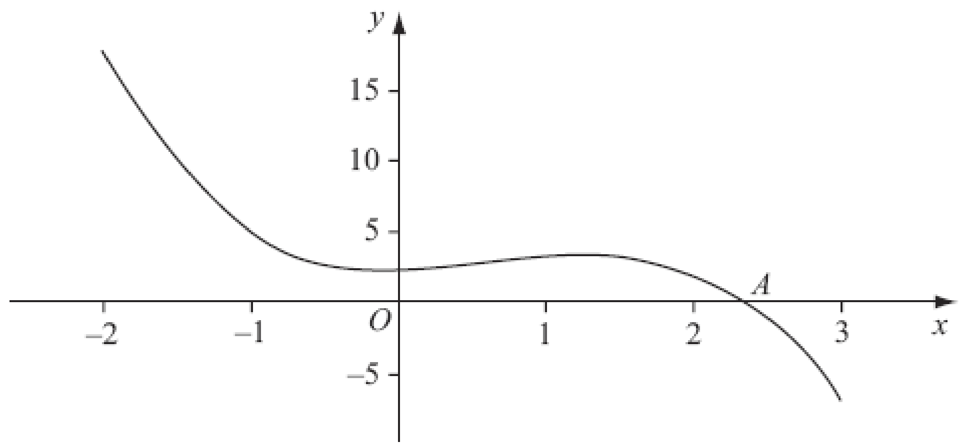


**Figure 1**

Figure 1 shows part of the curve with equation $y = -x^3 + 2x^2 + 2$, which intersects the $x$-axis at the point $A$ where $x = \alpha$.

To find an approximation to $\alpha$, the iterative formula

$$x_{n+1} = \frac{2}{(x_n)^2} + 2$$

is used.

($a$) Taking $x_0 = 2.5$, find the values of $x_1$, $x_2$, $x_3$ and $x_4$.
Give your answers to 3 decimal places where appropriate.

**(3)**

($b$) Show that $\alpha = 2.359$ correct to 3 decimal places.

**(3)**

**2.**

$$f(x) = \ln(x + 2) - x + 1, \qquad x > -2, x \in \mathbb{R}.$$

($a$) Show that there is a root of $f(x) = 0$ in the interval $2 < x < 3$.

**(2)**

($b$) Use the iterative formula

$$x_{n+1} = \ln(x_n + 2) + 1, \quad x_0 = 2.5,$$

to calculate the values of $x_1$, $x_2$ and $x_3$, giving your answers to 5 decimal places.

**(3)**

($c$) Show that $x = 2.505$ is a root of $f(x) = 0$ correct to 3 decimal places.

**(2)**

**3.** $$\mathrm{f}(x) = 3xe^x - 1.$$

The curve with equation $y = \mathrm{f}(x)$ has a turning point $P$.

(*a*) Find the exact coordinates of $P$.

**[This question involves C3 differentiation, you may wish to skip to part B]**

**(5)**

The equation $\mathrm{f}(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$.

(*b*) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}.$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of $x_1$, $x_2$ and $x_3$.

**(3)**

(*c*) By choosing a suitable interval, show that a root of $\mathrm{f}(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

**(3)**

**4.** $$\mathrm{f}(x) = 3x^3 - 2x - 6.$$

(*a*) Show that $\mathrm{f}(x) = 0$ has a root, $\alpha$, between $x = 1.4$ and $x = 1.45$.

**(2)**

(*b*) Show that the equation $\mathrm{f}(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}, \quad x \neq 0.$$

**(3)**

(*c*) Starting with $x_0 = 1.43$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{2}{3}\right)}$$

to calculate the values of $x_1$, $x_2$ and $x_3$, giving your answers to 4 decimal places.

**(3)**

(*d*) By choosing a suitable interval, show that $\alpha = 1.435$ is correct to 3 decimal places.

**(3)**

**5.** $$f(x) = -x^3 + 3x^2 - 1.$$

($a$) Show that the equation $f(x) = 0$ can be rewritten as

$$x = \sqrt{\left(\frac{1}{3-x}\right)}.$$

**(2)**

($b$) Starting with $x_1 = 0.6$, use the iteration

$$x_{n+1} = \sqrt{\left(\frac{1}{3-x_n}\right)}$$

to calculate the values of $x_2$, $x_3$ and $x_4$, giving all your answers to 4 decimal places.

**(2)**

($c$) Show that $x = 0.653$ is a root of $f(x) = 0$ correct to 3 decimal places.

**(3)**

**6.** $$f(x) = 2x^3 - x - 4.$$

($a$) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{2}{x} + \frac{1}{2}\right)}.$$

**(3)**

The equation $2x^3 - x - 4 = 0$ has a root between 1.35 and 1.4.

($b$) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{2}{x_n} + \frac{1}{2}\right)},$$

with $x_0 = 1.35$, to find, to 2 decimal places, the value of $x_1$, $x_2$ and $x_3$.

**(3)**

The only real root of $f(x) = 0$ is $\alpha$.

($c$) By choosing a suitable interval, prove that $\alpha = 1.392$, to 3 decimal places.

**(3)**

**7.** The function f is defined by

$$f : x \mapsto \ln (4 - 2x), \quad x < 2 \text{ and } x \in \mathbb{R}.$$

(*a*) Show that the inverse function of f is defined by

$$f^{-1} : x \mapsto 2 - \frac{1}{2} e^{x}$$

and write down the domain of $f^{-1}$.

**(4)**

(*b*) Write down the range of $f^{-1}$.

**(1)**

(*c*) Sketch the graph of $y = f^{-1}(x)$. State the coordinates of the points of intersection with the $x$ and $y$ axes.

**(4)**

The graph of $y = x + 2$ crosses the graph of $y = f^{-1}(x)$ at $x = k$.

The iterative formula

$$x_{n+1} = -\frac{1}{2} e^{x_n}, \quad x_0 = -0.3,$$

is used to find an approximate value for $k$.

(*d*) Calculate the values of $x_1$ and $x_2$, giving your answer to 4 decimal places.

**(2)**

(*e*) Find the values of $k$ to 3 decimal places.

**(2)**

**8.** **Figure 2**

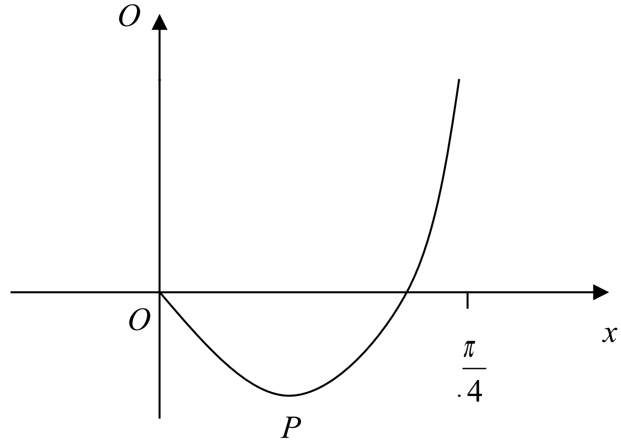


Figure 2 shows part of the curve with equation

$$y = (2x - 1)\tan 2x, \quad 0 \le x < \frac{\pi}{4}.$$

The curve has a minimum at the point $P$. The $x$-coordinate of $P$ is $k$.

(*a*) Show that $k$ satisfies the equation

$$4k + \sin 4k - 2 = 0.$$

**[This question involves C3 differentiation and trigonometry, you may wish to skip to part b]**

**(6)**

The iterative formula

$$x_{n+1} = \frac{1}{4}(2 - \sin 4x_n), \quad x_0 = 0.3,$$

is used to find an approximate value for $k$.

(*b*) Calculate the values of $x_1$, $x_2$, $x_3$ and $x_4$, giving your answers to 4 decimals places.

**(3)**

(*c*) Show that $k = 0.277$, correct to 3 significant figures.

**(2)**

**9.** $$\mathrm{f}'(x) = 3\mathrm{e}^x - \frac{1}{2x}, \quad x > 0.$$

The curve with equation $y = \mathrm{f}(x)$ has a turning point at $P$. The $x$-coordinate of $P$ is $\alpha$.

(*a*) Show that $\alpha = \frac{1}{6}\mathrm{e}^{-\alpha}$.

**(2)**

The iterative formula

$$x_{n+1} = \frac{1}{6}\mathrm{e}^{-x_n}, \quad x_0 = 1,$$

is used to find an approximate value for $\alpha$.

(*b*) Calculate the values of $x_1$, $x_2$, $x_3$ and $x_4$, giving your answers to 4 decimal places.

**(2)**

(*c*) By considering the change of sign of $\mathrm{f}'(x)$ in a suitable interval, prove that $\alpha = 0.1443$ correct to 4 decimal places.

**(2)**