Edexcel GCE

Core Mathematics C2

Advanced Subsidiary

Sequences and Series

Materials required for examination
Mathematical Formulae (Pink or Green)

Items included with question papers
Nil

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. The fourth term of a geometric series is 10 and the seventh term of the series is 80.

For this series, find

(*a*) the common ratio, **(2)**

(*b*) the first term, **(2)**

(*c*) the sum of the first 20 terms, giving your answer to the nearest whole number. **(2)**

1a) $u_4 = 10$ $\quad u_7 = 80$

$ar^3 = 10$ $\quad ar^6 = 80$

$$\frac{ar^6}{ar^3} = \frac{80}{10}$$

$$r^3 = 8$$

$$\underline{\underline{r = 2}}$$

b/ $a(2)^3 = 10$

$$a = \frac{10}{8}$$

$$\underline{\underline{a = 5/4}}$$

c/ $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_{20} = \frac{5/4\,(1-2^{20})}{1-2}$$

$= 1310718.75$

$= 1310719$ (nearest whole number)

2. The first term of a geometric series is 120. The sum to infinity of the series is 480.

(*a*) Show that the common ratio, r, is $\frac{3}{4}$. **(3)**

(*b*) Find, to 2 decimal places, the difference between the 5th and 6th terms. **(2)**

(*c*) Calculate the sum of the first 7 terms. **(2)**

The sum of the first n terms of the series is greater than 300.

(*d*) Calculate the smallest possible value of n. **(4)**

a/ $a = 120$ $\quad S_\infty = \frac{a}{1-r}$

$480 = \frac{120}{1-r}$

$1-r = \frac{1}{4}$

$\underline{r = 3/4}$

b/ $u_5 = 120\left(\frac{3}{4}\right)^4 = 37.96875$

$u_6 = 120\left(\frac{3}{4}\right)^5 = 28.4765625$

$u_5 - u_6 = 9.49$ (2dp)

c/ $S_n = \frac{a(1-r^n)}{1-r}$

$S_7 = \frac{120\left(1-\left(\frac{3}{4}\right)^7\right)}{1-3/4}$

$= 415.93$ (2dp)

d/ $300 < \frac{120\left(1-\left(\frac{3}{4}\right)^n\right)}{1-3/4}$

$75 < 120\left(1-\left(\frac{3}{4}\right)^n\right)$

$\frac{5}{8} < \left(1-\left(\frac{3}{4}\right)^n\right)$

$\left(\frac{3}{4}\right)^n < \frac{3}{8}$

$\log_{\left(\frac{3}{4}\right)}\left(\frac{3}{8}\right) = 3.409\ldots$

$\underline{n = 4}$

3. The third term of a geometric sequence is 324 and the sixth term is 96.

(*a*) Show that the common ratio of the sequence is $\frac{2}{3}$. **(2)**

(*b*) Find the first term of the sequence. **(2)**

(*c*) Find the sum of the first 15 terms of the sequence. **(3)**

(*d*) Find the sum to infinity of the sequence. **(2)**

a) $U_3 = 324 \quad U_6 = 96$

$ar^2 = 324$

$ar^5 = 96$

$r^3 = \frac{96}{324} \quad r = \sqrt[3]{\frac{96}{324}} = \sqrt[3]{\frac{8}{27}}$

$\underline{\underline{r = \frac{2}{3}}}$

b/ $a\left(\frac{2}{3}\right)^2 = 324$

$\underline{a = 729}$

c/ $S_n = \frac{a(1-r^n)}{1-r}$

$S_{15} = \frac{729\left(1-\left(\frac{2}{3}\right)^{15}\right)}{1-\frac{2}{3}}$

$= 2182.005639$

$= 2182.01$ (2DP)

d/ $S_\infty = \frac{729}{1-\frac{2}{3}}$

$= 2187$

4. A geometric series has first term 5 and common ratio $\frac{4}{5}$.

Calculate

(*a*) the 20th term of the series, to 3 decimal places,

(2)

(*b*) the sum to infinity of the series.

(2)

Given that the sum to *k* terms of the series is greater than 24.95,

(*c*) show that $k > \frac{\log 0.002}{\log 0.8}$,

(4)

(*d*) find the smallest possible value of *k*.

(1)

a/ $U_{20} = 5 \times \left(\frac{4}{5}\right)^{19}$

$= 0.072$ (3dp)

b/ $S_\infty = \frac{5}{1 - \frac{4}{5}}$

$= 25$

c/ $24.95 < \frac{5\left(1 - \left(\frac{4}{5}\right)^k\right)}{1 - 4/5}$

$4.99 < 5\left(1 - \left(\frac{4}{5}\right)^k\right)$

$0.998 < 1 - \left(\frac{4}{5}\right)^k$

$\left(\frac{4}{5}\right)^k < 0.002$

$\log 0.8^k < \log 0.002$

$k \log 0.8 < \log 0.002$

$k > \frac{\log 0.002}{\log 0.8}$

$\boxed{\log 0.8 < 0 \therefore \text{ inequality sign changes}}$

d/ $k > 27.85\ldots$

$k = 28$

5. A trading company made a profit of £50 000 in 2006 (Year 1).

A model for future trading predicts that profits will increase year by year in a geometric sequence with common ratio r, $r > 1$.

The model therefore predicts that in 2007 (Year 2) a profit of £50 000r will be made.

(*a*) Write down an expression for the predicted profit in Year n.

(1)

The model predicts that in Year n, the profit made will exceed £200 000.

(*b*) Show that $n > \dfrac{\log 4}{\log r} + 1$.

(3)

Using the model with $r = 1.09$,

(*c*) find the year in which the profit made will first exceed £200 000,

(2)

(*d*) find the total of the profits that will be made by the company over the 10 years from 2006 to 2015 inclusive, giving your answer to the nearest £10 000.

(3)

a) $u_n = 50000r^{n-1}$

b) $200000 < 50000r^{n-1}$

$4 < r^{n-1}$

$\log 4 < \log r^{n-1}$

$\log 4 < (n-1)\log r$

$\dfrac{\log 4}{\log r} < n-1$

$n > \dfrac{\log 4}{\log r} + 1$

c) $\dfrac{\log 4}{\log 1.09} + 1 = 17.086\ldots$

$n = 18$

d) $S_{10} = \dfrac{50000(1-1.09^{10})}{1-1.09}$

$= £759646.47$

$= £760000$

6. (*a*) A geometric series has first term a and common ratio r. Prove that the sum of the first n terms of the series is

$$\frac{a(1-r^n)}{1-r}.$$

(4)

Mr King will be paid a salary of £35 000 in the year 2005. Mr King's contract promises a 4% increase in salary every year, the first increase being given in 2006, so that his annual salaries form a geometric sequence.

(*b*) Find, to the nearest £100, Mr King's salary in the year 2008.

(2)

Mr King will receive a salary each year from 2005 until he retires at the end of 2024.

(*c*) Find, to the nearest £1000, the total amount of salary he will receive in the period from 2005 until he retires at the end of 2024.

(4)

a) $S_n = a + ar + ar^2 + \dots + ar^{n-3} + ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \dots + ar^{n-2} + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a(1-r^n)$

$$S_n = \frac{a(1-r^n)}{1-r}$$

b) $a = £35000 \quad r = 1.04$

$u_4 = 35000(1.04)^3$

$= £39400$ (nearest £100)

c) $$S_{20} = \frac{35000(1-1.04^{20})}{1-1.04}$$

$= £1042000$

7. A geometric series has first term a and common ratio r. The second term of the series is 4 and the sum to infinity of the series is 25.

(*a*) Show that $25r^2 - 25r + 4 = 0$. **(4)**

(*b*) Find the two possible values of r. **(2)**

(*c*) Find the corresponding two possible values of a. **(2)**

(*d*) Show that the sum, S_n, of the first n terms of the series is given by

$$S_n = 25(1 - r^n).$$

(1)

Given that r takes the larger of its two possible values,

(*e*) find the smallest value of n for which S_n exceeds 24. **(2)**

$u_2 = 4$ $\quad S_\infty = 25$

$4 = ar$ $\quad 25 = \frac{a}{1-r}$

$\frac{4}{r} = a$ $\quad 25(1-r) = a$

$25(1-r) = \frac{4}{r}$

$25r(1-r) = 4$

$25r - 25r^2 = 4$

$\underline{0 = 25r^2 - 25r + 4}$

b) $(5r-4)(5r-1) = 0$

$r = 4/5 \quad r = 1/5$

c) $a = \frac{4}{4/5} = 5 \quad a = \frac{4}{1/5} = 20$

d) $S_n = \frac{\cancel{2}5(1 - \frac{4}{5}^n)}{1 - 4/5}$ $\quad S_n = \frac{20(1 - \frac{1}{5}^n)}{1 - 1/5}$

$= 25(1 - \frac{4}{5}^n)$ $\quad = 25(1 - \frac{1}{5}^n)$

$\underline{= 25(1 - r^n)}$

e) $24 < 25(1 - \frac{4}{5}^n)$

$\frac{24}{25} < 1 - \frac{4}{5}^n$

$\frac{4}{5}^n < \frac{1}{25}$

$n \log 0.8 < \log 0.04$

$n > \frac{\log 0.04}{\log 0.8}$

$n > 14.425...$

$\underline{n = 15}$

8. The first three terms of a geometric series are $(k + 4)$, k and $(2k - 15)$ respectively, where k is a positive constant.

(*a*) Show that $k^2 - 7k - 60 = 0$. **(4)**

(*b*) Hence show that $k = 12$. **(2)**

(*c*) Find the common ratio of this series. **(2)**

(*d*) Find the sum to infinity of this series. **(2)**

a) $\frac{k}{k+4} = \frac{2k-15}{k}$

$k^2 = (2k-15)(k+4)$

$k^2 = 2k^2 + 8k - 15k - 60$

$0 = k^2 - 7k - 60$

b) $(k-12)(k+5) = 0$

$k = 12 \quad k = -5$

k is +ve $\therefore k = 12$

c) $r = \frac{12}{12+4} = \frac{12}{16} = \frac{3}{4}$

d) $a = 16 \quad r = 3/4$

$S_\infty = \frac{16}{1 - 3/4}$

$= 64$

9. The second and fourth terms of a geometric series are 7.2 and 5.832 respectively.

The common ratio of the series is positive.

For this series, find

(*a*) the common ratio, **(2)**

(*b*) the first term, **(2)**

(*c*) the sum of the first 50 terms, giving your answer to 3 decimal places, **(2)**

(*d*) the difference between the sum to infinity and the sum of the first 50 terms, giving your answer to 3 decimal places. **(2)**

a) $ar = 7.2 \qquad ar^3 = 5.832$

$$r^2 = \frac{5.832}{7.2} = \frac{81}{100}$$

$$r = \frac{9}{10}$$

b) $$a = \frac{7.2}{0.9}$$

$$= 8$$

c) $$S_{50} = \frac{8(1-0.9^{50})}{1-0.9}$$

$$= 79.588 \text{ (3dp)}$$

d) $$S_\infty = \frac{8}{1-0.9}$$

$$= 80$$

$$\cancel{8} \quad S_\infty - S_{50} = 0.412 \quad \text{3dp}$$

10. A geometric series is $a + ar + ar^2 + \ldots$

(*a*) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

(4)

(*b*) Find

$$\sum_{k=1}^{10} 100(2^k).$$

(3)

(*c*) Find the sum to infinity of the geometric series

$$\frac{5}{6} + \frac{5}{18} + \frac{5}{54} + \ldots$$

(3)

(*d*) State the condition for an infinite geometric series with common ratio r to be convergent.

(1)

a/ $S_n = a + ar + ar^2 + \ldots + ar^{n-3} + ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots + ar^{n-2} + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$

$S_n(1-r) = a(1-r^n)$

$S_n = \dfrac{a(1-r^n)}{1-r}$

b/ $a = 200 \quad r = 2$

$S_{10} = \dfrac{200(1-2^{10})}{1-2}$

$= 204600$

c/ $a = \frac{5}{6} \quad r = \frac{1}{3}$

$S_\infty = \dfrac{5/6}{1 - 1/3} = \dfrac{5}{4}$

d/ $|r| < 1$