C1 - SEQUENCES AND SERIES

1a) $a_{n+1} = 2a_n - 7$

$a_2 = 2(a_1) - 7$ $\qquad a_1 = k$

$= 2k - 7$

b) $a_3 = 2(a_2) - 7$

$= 2(2k-7) - 7$

$= 4k - 14 - 7$

$= 4k - 21$

c) $a_4 = 2(a_3) - 7$

$= 2(4k-21) - 7$

$= 8k - 42 - 7$

$= 8k - 49$

$k + 2k - 7 + 4k - 21 + 8k - 49 = 43$

$15k - 77 = 43$

$15k = 120$

$\underline{\underline{k = 8}}$

2) 1951 = year 1 $\qquad$ 1990 = year 40.

$a_{10} = 2400$

$a_{40} = 600$

$a + 9d = 2400$

$a + 39d = 600$

$30d = -1800$

$\underline{\underline{d = -60}}$

b) $a + 9(-60) = 2400$

$a - 540 = 2400$ $\qquad \underline{\underline{a = 2940}}$

c)

$$\cancel{S_n = \frac{n}{2}(2a + (n-1)d)}$$
$$\cancel{S_{40} = \frac{40}{2}(2(2940) + 39(-60))}$$

$$S_n = \frac{n}{2}(a + l)$$
$$S_{40} = \frac{40}{2}(2940 + 600)$$
$$= 20(3540)$$
$$= \underline{\underline{70800}}$$

3a)

$$a = 30$$
$$d = -1.5$$
$$U_n = a + (n-1)d$$
$$U_{25} = 30 + 24(-1.5)$$
$$= 30 - 36$$
$$= \underline{\underline{-6}}$$

b)

$$U_r = a + (r-1)d$$
$$0 = 30 + (r-1)(-1.5)$$
$$-30 = (r-1)(-1.5)$$
$$20 = r - 1$$
$$\underline{\underline{r = 21}}$$

c)

$$S_n = \frac{n}{2}(a + l)$$
$$S_{21} = \frac{21}{2}(30 + 0)$$
$$= \frac{21}{2}(30)$$
$$= \underline{\underline{315}}$$

4a)

$$U_{18} = 25 \qquad U_{21} = 32.5$$

$$a + 17d = 25$$
$$a + 20d = 32.5$$

b)

$$3d = 7.5$$
$$\underline{\underline{d = 2.5}}$$

$$a + 20(2.5) = 32.5$$
$$a + 50 = 32.5$$
$$\underline{\underline{a = -17.5}}$$

c/ $S_n = \frac{n}{2}(2a + (n-1)d)$

$2750 = \frac{n}{2}(2a + (n-1)d)$ $\quad a = -17.5 \quad d = 2.5$

$$2750 = \frac{n}{2}(2(-17.5) + (n-1)(2.5))$$
$$2750 = n/2(-35 + 2.5n - 2.5)$$
$$2750 = n/2(-37.5 + 2.5n)$$
$$5500 = n(-37.5 + 2.5n)$$
$$5500 = -37.5n + 2.5n^2$$
$$5500\cancel{0} = 2.5n^2 - 37.5n \cancel{-5500}$$
$$11000 = 5n^2 - 75n$$
$$2200 = n^2 - 15n$$
$$55 \times 40 = n^2 - 15n$$
$$0 = n^2 - 15n - 55 \times 40$$
$$0 = (n - 55)(n + 40)$$
$$\underline{\underline{n = 55}} \quad n = -40$$

n cannot be negative $\therefore$ $\underline{\underline{n = 55}}$

5a/ $x_1 = 1$
$$x_2 = a(x_1) - 3$$
$$= a - 3$$

b/ $x_3 = a(x_2) - 3$
$$= a(a-3) - 3$$
$$= a^2 - 3a - 3$$

c/ $a^2 - 3a - 3 = 7$
$$a^2 - 3a - 10 = 0$$
$$(a+2)(a-5) = 0$$
$$\underline{\underline{a = -2}} \quad \underline{\underline{a = 5}}$$

6a) $U_4 = a + 3d$ $a = 5$ $d = 2$

$= 5 + 3(2)$

$= 11$ (km)

b) $U_n = a + (n-1)d$

$= 5 + (n-1)(2)$

$= 5 + 2n - 2$

$= 2n + 3$ (km)

c) $S_n = \frac{n}{2}(2a + (n-1)d)$

$= \frac{n}{2}(2(5) + (n-1)(2))$

$= \frac{n}{2}(10 + 2n - 2)$

$= \frac{n}{2}(8 + 2n)$

$= n(4 + n)$ (km)

d) $2n + 3 = 43$

$2n = 40$

$n = 20$ [km]

e) $S_n = n(4+n)$

$S_{20} = 20(4+20)$

$= 480$ km

7a) $x_1 = 1$

$x_2 = x_1(p + x_1)$

$= 1(p+1)$

$= p + 1$

b) $x_3 = x_2(p + x_2)$

$= (p+1)(p+p+1)$

$= (p+1)(2p+1)$

$= 2p^2 + p + 2p + 1$

$= 1 + 3p + 2p^2$

c/ $1 + 3p + 2p^2 = 1$

$3p + 2p^2 = 0$

$p(3+2p) = 0$

$p = 0 \quad p = -3/2$

$p \neq 0 \quad \therefore \quad \underline{\underline{p = -3/2}}$

d/ $x_1 = 1$

$x_2 = -1/2$

$x_3 = 1$

$x_4 = -1/2$

$\therefore \quad \underline{\underline{x_{2008} = -1/2}}$

8a/ $a = 5 \quad d = 2$

$U_n = a + (n-1)d$

$U_{200} = 5 + (199)(2)$

$= 5 + 398$

$= 403p \qquad$ or $\quad \underline{\underline{£4.03}}$

b/ $S_n = \frac{n}{2}(a + l)$

$= \frac{200}{2}(5 + 403)$

$= 100(408$

$= 40800p \qquad$ or $\quad \underline{\underline{£408}}$

9a/ $a_1 = 3$

$a_2 = 3(a_1) - 5$

$= 3(3) - 5$

$= \underline{\underline{4}}$

$$a_3 = 3(a_2) - 5$$
$$= 3(4) - 5$$
$$= \underline{\underline{7}}$$

b/
$$a_4 = 3(a_3) - 5$$
$$= 3(7) - 5$$
$$= \underline{\underline{16}}$$
$$a_5 = 3(16) - 5$$
$$= \underline{\underline{43}}$$

$$\sum_{r=1}^{5} a_r = 3 + 4 + 7 + 16 + 43$$
$$= \underline{\underline{73}}$$

10a/
$$a_1 = k$$
$$a_2 = 3(a_1) + 5$$
$$= 3k + 5$$

b/
$$a_3 = 3(a_2) + 5$$
$$= 3(3k+5) + 5$$
$$= 9k + 15 + 5$$
$$= \underline{\underline{9k + 20}}$$

c/
$$a_4 = 3(a_3) + 5$$
$$= 3(9k + 20) + 5$$
$$= 27k + 60 + 5$$
$$= 27k + 65$$

$$\sum_{r=1}^{4} a_r = k + 3k + 5 + 9k + 20 + 27k + 65$$
$$= \underline{\underline{40k + 90}}$$

ii/ $\underline{\underline{10(4k+9)}}$

11/ $a=4 \quad d=3$

a/ $U_n = a+(n-1)d$

$= 4+(n-1)3$

$= 4+3n-3$

$= \underline{\underline{3n+1}}$

b/ $S_n = \frac{n}{2}(2a+(n-1)d)$

$S_{10} = \frac{10}{2}(2(4)+9(3))$

$= 5(8+27)$

$= 5(35)$

$= \underline{\underline{175}}$

c/ $S_n < 1750$

$\frac{k}{2}(2(4)+(k-1)(3)) < 1750$

$\frac{k}{2}(8+3k-3) < 1750$

$\frac{k}{2}(5+3k) < 1750$

$k(5+3k) < 3500$

$5k+3k^2 < 3500$

$3k^2+5k-3500 < 0$

$(3k-100)(k+35) < 0$

d/ $k=\frac{100}{3} \quad k=-35$

$-35 < k < \frac{100}{3}$

$\therefore \underline{\underline{k=33}}$

12/ $U_{11}=9 \quad S_{11}=77$

$\cancel{a+(n-1)d=9} \qquad a+10d=9$

$\cancel{\frac{n}{2}(2a+(n-1)d)=77} \qquad \frac{11}{2}(2a+10d)=77$

$$a + 10d = 9$$
$$11a + 55d = 77$$
$$a + 5d = 7$$
$$5d = 2$$
$$d = 2/5 = 0.4 \text{ km.}$$

$$a + 10(0.4) = 9$$
$$a + 4 = 9$$
$$\underline{\underline{a = 5}} \text{ km.}$$

13a/
$$u_1 = 1$$
$$u_2 = (u_1 - 3)^2$$
$$= (1-3)^2$$
$$= \underline{4}$$
$$u_3 = (u_2 - 3)^2$$
$$= (4-3)^2$$
$$= \underline{1}$$
$$u_4 = (u_3 - 3)^2$$
$$= (1-3)^2$$
$$= \underline{4}$$

b/ $\underline{\underline{u_{20} = 4}}$

14a/
$$u_1 = 2(1) - 5$$
$$= -3$$
$$u_2 = 2(2) - 5$$
$$= -1$$
$$u_3 = 2(3) - 5$$
$$= 1$$

b/ $\underline{\underline{d = 2}}$

c/ $a = -3 \quad d = 2$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$= \frac{n}{2}(2(-3) + (n-1)(2))$$
$$= \frac{n}{2}(-6 + 2n - 2)$$
$$= \frac{n}{2}(2n - 8)$$
$$= n(n-4)$$

15a) $a = 500 \qquad d = 200$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$
$$S_2 = \frac{2}{2}(2(500) + 200)$$
$$= (1000 + 200)$$
$$= 1200$$

b/ $U_n = a + (n-1)d$
$$U_8 = 500 + 7(200)$$
$$= 500 + 1400$$
$$= £1900$$

c/ $S_n = \frac{n}{2}(2a + (n-1)d)$
$$S_8 = \frac{8}{2}(2(500) + 7(200))$$
$$= 4(1000 + 1400)$$
$$= 4(2400)$$
$$= £9600$$

d/ $32000 = \frac{n}{2}(2(500) + (n-1)(200))$
$$32000 = \frac{n}{2}(1000 + 200n - 200)$$
$$32000 = \frac{n}{2}(800 + 200n)$$
$$32000 = 400n + 100n^2$$
$$320 = 4n + n^2$$
$$0 = n^2 + 4n - 320$$

$$(n+20)(n-16)=0$$

$$n=-20 \quad n=16$$

$$n \neq -20 \quad \therefore \quad n=16$$

$\therefore$ Alice ~~is~~ was 26

16a)

$$S_n = a + (a+d) + \cancel{(a+2d)} + \dots + (a+(n-2)d) + (a+(n-1)d)$$

$$S_n = a+(n-1)d + a+(n-2)d + \dots + a+d + a$$

$$2S_n = (2a+(n-1)d) + (2a+(n-1)d) + \dots + (2a+(n-1)d) + (2a+(n-1)d)$$

$$2S_n = n(2a+(n-1)d)$$

$$S_n = \frac{n}{2}(2a+(n-1)d)$$

b)

$$a = 149$$

$$d = -2$$

$$U_n = a + (n-1)d$$

$$U_{21} = 149 + 20(-2)$$

$$= 109$$

c)

$$S_n = \frac{n}{2}(2a+(n-1)d)$$

$$S_n = \frac{n}{2}(2(149)+(n-1)(-2))$$

$$5000 = \frac{n}{2}(298-2n+2)$$

$$5000 = \frac{n}{2}(300-2n)$$

$$5000 = 150n - n^2$$

$$n^2 - 150n + 5000 = 0$$

d)

$$(n-100)(n-50)=0$$

$$n=100 \quad n=50$$

e) $n=100$ is not sensible. He would be repaying a negative amount of money.