

$$\begin{aligned}
 1) \quad & x^2 + y^2 - 2x + 6y = 26 \\
 & x^2 - 2x + y^2 + 6y = 26 \\
 & (x-1)^2 - 1 + (y+3)^2 - 9 = 26 \\
 & (x-1)^2 + (y+3)^2 = 36
 \end{aligned}$$

i centre:  $(1, -3)$

ii radius: 6

$$2) \quad (x-a)^2 + (y-b)^2 = r^2$$

$$(x-2)^2 + (y-5)^2 = r^2 \quad (4, 9)$$

$$(4-2)^2 + (9-5)^2 = r^2$$

$$20 = r^2$$

$$r = \sqrt{20}$$

$$(x-2)^2 + (y-5)^2 = 20$$

$$\begin{aligned}
 3) \quad (x - a)^2 + (y - b)^2 &= r^2 \\
 (x + 2)^2 + (y - 3)^2 &= r^2 && (1, 8) \\
 (1 + 2)^2 + (8 - 3)^2 &= r^2 \\
 &34 = r^2 \\
 (x + 2)^2 + (y - 3)^2 &= 34
 \end{aligned}$$

$$\begin{aligned}
 b) \quad (3 + 2)^2 + (6 - 3)^2 &= 34 \\
 &34 = 34
 \end{aligned}$$

$$\begin{aligned}
 c) \quad \text{centre } (-2, 3) & \quad \text{point } (3, 6) \\
 & \quad \quad \quad x_1 \quad y_1 \quad \quad \quad x_2 \quad y_2 \\
 \text{gradient of radius} &= \frac{6 - 3}{3 - (-2)} \\
 &= \frac{3}{5}
 \end{aligned}$$

$$\text{gradient of tangent} = -\frac{5}{3}$$

$$y = -\frac{5}{3}x + c \quad (3, 6)$$

$$6 = -\frac{5}{3}(3) + c$$

$$6 = -5 + c$$

$$c = 11$$

$$y = -\frac{5}{3}x + 11$$

$$3y = -5x + 33$$

$$\underline{5x + 3y - 33 = 0}$$

$$40) \quad (x-2)^2 + (y-5)^2 = 49$$

$$b) \quad y = (3x - 1)$$

$$(x-2)^2 + (3x-1-5)^2 = 49$$

$$(x-2)^2 + (3x-6)^2 = 49$$

$$(x-2)(x-2) + (3x-6)(3x-6) = 49$$

$$x^2 - 2x - 2x + 4 + 9x^2 - 18x - 18x + 36 = 49$$

$$10x^2 - 40x + 40 = 49$$

$$10x^2 - 40x - 9 = 0$$

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(10)(-9)}}{2(10)}$$

$$x = \frac{20 + 7\sqrt{10}}{10}$$

$$x = \frac{20 - 7\sqrt{10}}{10}$$

$$y = 3\left(\frac{20 + 7\sqrt{10}}{10}\right) - 1$$

$$y = 3\left(\frac{20 - 7\sqrt{10}}{10}\right) - 1$$

$$= \frac{50 + 21\sqrt{10}}{10}$$

$$= \frac{50 - 21\sqrt{10}}{10}$$

$$\left(\frac{20 + 7\sqrt{10}}{10}, \frac{50 + 21\sqrt{10}}{10}\right) \text{ and } \left(\frac{20 - 7\sqrt{10}}{10}, \frac{50 - 21\sqrt{10}}{10}\right)$$



$$5) \quad x^2 + y^2 + 8x - 4y + k = 0 \quad (1, 5)$$

$$(1)^2 + (5)^2 + 8(1) - 4(5) + k = 0$$

$$14 + k = 0$$

$$\underline{\underline{k = -14}}$$

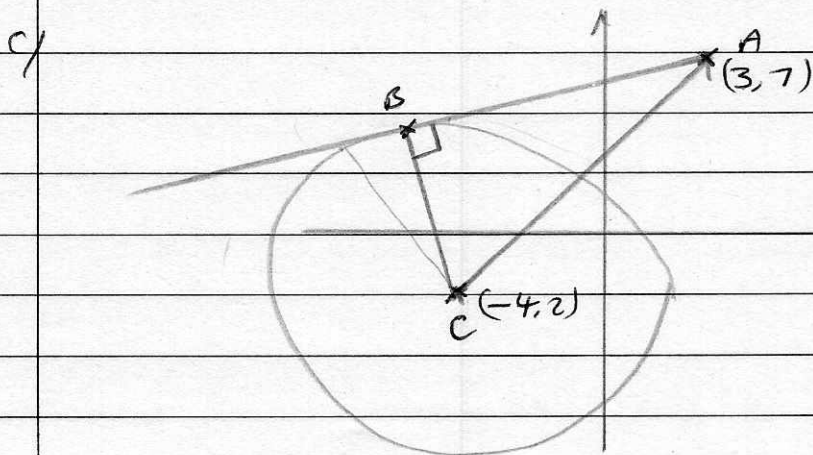
$$b) \quad x^2 + y^2 + 8x - 4y - 14 = 0$$

$$(x + 4)^2 - 16 + (y - 2)^2 - 4 - 14 = 0$$

$$(x + 4)^2 + (y - 2)^2 = 34$$

centre:  $(-4, 2)$

radius:  $\sqrt{34}$



BC is a radius:  $\sqrt{34}$

$$AC = \sqrt{7^2 + 5^2} = \sqrt{74}$$

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 34 = 74$$

$$AB^2 = 40$$

$$AB = \sqrt{40}$$

$$\underline{\underline{= 2\sqrt{10}}}$$

6a)  $D(-3, 2)$   $E(4, -1)$   $F(1, -8)$

$$DE^2 = 7^2 + 3^2 \quad DF^2 = 4^2 + 10^2 \quad EF^2 = 3^2 + 7^2$$
$$DE = \sqrt{58} \quad DF = \sqrt{116} \quad EF = \sqrt{58}$$

DEF is a right angled triangle as  $DE^2 + EF^2 = DF^2$   
( $58 + 58 = 116$ )

DF is the hypotenuse  $\therefore \underline{\underline{DEF = 90^\circ}}$

6b) DEF is  $90^\circ$   $\therefore$  DF is a diameter.

$$\left( \frac{-3+1}{2}, \frac{2-8}{2} \right)$$

$$\underline{\underline{(-1, -3)}}$$

6c)  $(x+1)^2 + (y+3)^2 = r^2$

$$r = \frac{\sqrt{116}}{2} = \sqrt{29}$$

$$r^2 = 29$$

$$\underline{\underline{(x+1)^2 + (y+3)^2 = 29}}$$



7a)

$$x^2 + y^2 - 6x + 2y = 6$$

$$(x-3)^2 - 9 + (y+1)^2 - 1 = 6$$

$$(x-3)^2 + (y+1)^2 = 16$$

Centre:  $(3, -1)$

radius: 4

b) crosses y when  $x=0$

$$(0-3)^2 + (y+1)^2 = 16$$

$$9 + (y+1)^2 = 16$$

$$(y+1)^2 = 7$$

$$y+1 = \pm\sqrt{7}$$

$$y = -1 \pm\sqrt{7}$$

$$(0, -1+\sqrt{7}) \quad (0, -1-\sqrt{7})$$

$$8/ \quad (-3, 5) \quad (13, -4)$$

centre = midpoint of diameter

$$\left( \frac{-3+13}{2}, \frac{5-4}{2} \right)$$
$$\left( 5, \frac{1}{2} \right)$$

$$(x-5)^2 + \left(y - \frac{1}{2}\right)^2 = r^2$$

$$\text{diameter length} = \sqrt{16^2 + 9^2}$$
$$= \sqrt{337}$$

$$\text{radius} = \frac{\sqrt{337}}{2}$$

$$r^2 = \frac{337}{4}$$

$$(x-5)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{337}{4}$$

$$9a) (x-1)^2 + (y-5)^2 = r^2 \quad (-4, 3)$$

$$(-5)^2 + (-2)^2 = r^2$$

$$29 = r^2$$

$$\underline{(x-1)^2 + (y-5)^2 = 29}$$

$$b) \text{ gradient of radius} = \frac{3-5}{-4-1}$$

$$= \frac{-2}{-5}$$

$$= \frac{2}{5}$$

$$\text{gradient of tangent} = -\frac{5}{2}$$

$$y = -\frac{5}{2}x + c \quad (-4, 3)$$

$$3 = -\frac{5}{2}(-4) + c$$

$$3 = 10 + c$$

$$\underline{c = -7}$$

$$y = -\frac{5}{2}x - 7$$

$$2y = -5x - 14$$

$$\underline{5x + 2y + 14 = 0}$$



10)

gradient of tangent = 2

gradient of radius =  $-\frac{1}{2}$ 

$$-\frac{1}{2} = \frac{7-k}{3-5}$$

$$-\frac{1}{2} = \frac{7-k}{-2}$$

$$-1(-2) = 2(7-k)$$

$$2 = 14 - 2k$$

$$2k = 12$$

$$\underline{k = 6}$$

$$(x-5)^2 + (y-6)^2 = r^2 \quad (3, 7)$$

$$(3-5)^2 + (7-6)^2 = r^2$$

$$5 = r^2$$

$$\underline{(x-5)^2 + (y-6)^2 = 5}$$