

θ is small

$$\sin \theta \approx \theta$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2}$$

$$\tan \theta \approx \theta$$

1a)

$$\frac{\cos \theta}{\sin \theta}$$

$$\frac{1 - \frac{\theta^2}{2} \quad \times 2}{\theta \quad \times 2}$$

$$\frac{2 - \theta^2}{2\theta}$$

b/

$$\frac{2 - \left(\frac{\pi}{24}\right)^2}{2\left(\frac{\pi}{24}\right)} = 7.573987421$$

c/

$$\frac{\cos \frac{\pi}{24}}{\sin \frac{\pi}{24}} = 7.595754113$$

$$\frac{7.595754113 - 7.573987421 \times 100}{7.595754113}$$

$$= \underline{\underline{0.29\%}} \quad (\text{2 dp})$$

$$\begin{aligned}
 2a) \quad \tan(3x) \cos(2x) &\approx 3x \left(1 - \frac{(2x)^2}{2}\right) \\
 &\approx 3x(1 - 2x^2) \\
 &\approx 3x - 6x^3
 \end{aligned}$$

$$b) \quad x = 0.1$$

$$3(0.1) - 6(0.1)^3 = 0.294$$

$$c) \quad \tan(0.3) \cos(0.2) = 0.3031701196$$

$$\begin{aligned}
 \% \text{ Error} &= \frac{0.3031701196 - 0.294}{0.3031701196} \times 100 \\
 &= \underline{\underline{3.02\%}} \quad 2dp
 \end{aligned}$$

$$\begin{aligned}
 3a) \quad 4 \cos \theta + \cos^2(2\theta) &\approx 4 \left(1 - \frac{\theta^2}{2}\right) + \left(1 - \frac{(2\theta)^2}{2}\right)^2 \\
 &\approx 4 - 2\theta^2 + (1 - 2\theta^2)^2 \\
 &\approx 4 - 2\theta^2 + 1 - 4\theta^2 + 4\theta^4 \\
 &\approx 5 - 6\theta^2 + 4\theta^4
 \end{aligned}$$

$$b) \quad 3^\circ = \frac{3}{180} \times \pi^\circ = \frac{1}{60} \pi^\circ$$

$$\begin{aligned}
 &5 - 6 \left(\frac{1}{60} \pi\right)^2 + 4 \left(\frac{1}{60} \pi\right)^4 \\
 &= 4.983580724
 \end{aligned}$$

$$c) \quad 4 \cos\left(\frac{1}{60} \pi\right) + \cos^2\left(\frac{1}{30} \pi\right) = 4.983591939$$

$$\begin{aligned}
 \% \text{ Error} &= \frac{4.983591939 - 4.983580724}{4.983591939} \times 100 \\
 &= \underline{\underline{2.25 \times 10^{-4} \%}} \quad 3st
 \end{aligned}$$