1a) ~~$\frac{dP}{dt} = 0.02P$~~

$$\frac{dy}{dt} = 0.02y$$

b) $$\int \frac{1}{y}\, dy = \int 0.02\, dt$$

$$\ln y = 0.02t + C$$

$$y = e^{0.02t + C}$$

$$y = e^{0.02t} \cdot e^{C}$$

$$\underline{y = Ae^{0.02t}}$$

c) $A = 2.5$

$$y = 2.5e^{0.02t}$$

when $t = 10$

$$y = 2.5e^{0.02(10)}$$

$$y = \underline{3.05} \quad (2dp) \quad \text{million}$$

2a/ $\frac{dy}{dt} = k(y-20)$

$$\int \frac{1}{y-20}\,dy = \int k\,dt$$

b/ $\ln(y-20) = kt + c$

$$y - 20 = e^{kt+c}$$
$$y - 20 = e^{kt} \cdot e^{c}$$
$$y - 20 = Ae^{kt}$$
$$y = 20 + Ae^{kt}$$

c/ $A = 80$

d/ $y = 20 + 80e^{kt}$

$t = 8 \quad y = 60$

$$60 = 20 + 80e^{8k}$$
$$40 = 80e^{8k}$$
$$\frac{1}{2} = e^{8k}$$
$$\ln\frac{1}{2} = 8k$$
$$\frac{1}{8}\ln\frac{1}{2} = k$$
$$k = \frac{1}{8}\ln 2^{-1}$$
$$= -\frac{1}{8}\ln 2$$

3a/ $$\frac{3x-3}{(x+1)(2x-1)} = \frac{A}{x+1} + \frac{B}{2x-1}$$

$$3x - 3 = A(2x-1) + B(x+1)$$

Let $x = -1$ $\quad -6 = -3A$

$$A = 2$$

Let $x = \frac{1}{2}$ $\quad -\frac{3}{2} = \frac{3}{2}B$

$$B = -1$$

$$\frac{2}{x+1} - \frac{1}{2x-1}$$

b/ $$\int \frac{1}{y}\,dy = \int \frac{3x-3}{(x+1)(2x-1)}\,dx$$

$$\int \frac{1}{y}\,dy = \int \frac{2}{x+1} - \frac{1}{2x-1}\,dx$$

$$\ln y = 2\ln|x+1| - \tfrac{1}{2}\ln|2x-1| + c$$

c/ $$\ln 6 = 2\ln 6 - \tfrac{1}{2}\ln 9 + c$$

$$\ln 6 = 2\ln 6 - \ln 3 + c$$

$$\ln 3 - \ln 6 = c$$

$$\ln y = 2\ln|x+1| - \tfrac{1}{2}\ln|2x-1| + \ln 3 - \ln 6$$

$$\ln y = \ln(x+1)^2 - \tfrac{1}{2}\ln(2x-1)^{\frac{1}{2}} + \ln 3 - \ln 6$$

$$\ln y = \ln\left(\frac{3(x+1)^2}{6(2x-1)^{\frac{1}{2}}}\right)$$

$$y = \frac{3(x+1)^2}{6(2x-1)^{\frac{1}{2}}}$$

$$y = \frac{(x+1)^2}{2(2x-1)^{\frac{1}{2}}}$$

4/ $\frac{dy}{dx} = xy \sin x$

$\int \frac{1}{y}\, dy = \int x \sin x \, dx$

~~$\ln y = 1$~~ $u = x \quad \frac{dv}{dx} = \sin x$

$\frac{du}{dx} = 1 \quad v = -\cos x$

$\ln y = -x \cos x - \int -\cos x \, dx$

$\ln y = -x \cos x + \sin x + c$

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4/

$$\frac{dy}{dx} = y^2 \ln x$$

$$\int \frac{1}{y^2}\, dy = \int \ln x \, dx$$

$$\int y^{-2}\, dy = \int \ln x \, dx$$

$$u = \ln x \qquad \frac{dv}{dx} = 1$$

$$\frac{du}{dx} = \frac{1}{x} \qquad v = x$$

~~$-2y^{-3}$~~

$$-y^{-1} = x \ln x - \int 1 \, dx$$

$$-y^{-1} = x \ln x - x + c$$

6 $\frac{dy}{dx} = (y+1)^2$

$\int \frac{1}{(y+1)^2}\, dy = \int 1 \, dx$

$\int (y+1)^{-2}\, dy = \int 1 \, dx$

$-(y+1)^{-1} = x + c$

$y = 0 \quad x = 2$

$-1 = 2 + c$

$c = -3$

$-(y+1)^{-1} = x - 3$

$-\frac{1}{y+1} = x - 3$

$-\frac{1}{x-3} = y + 1$

$-\frac{1}{x-3} - 1 = y$

$y = -\frac{1}{x-3} - 1$

$$\left[y = \frac{x-2}{3-x} \right]$$