Name: _____

GCSE (1 – 9)

Vectors Proof Questions

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

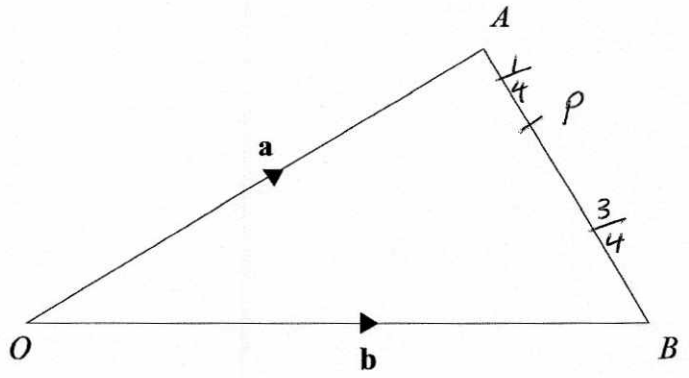
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1



$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = \boldsymbol{b}$

P is the point on AB such that AP:PB = 1:3

$\overrightarrow{OP} = k(3\boldsymbol{a} + \boldsymbol{b})$

Find the value of k

$$\overrightarrow{AB} = -a + b$$

$$\overrightarrow{AP} = \frac{1}{4}(-a + b)$$

$$\overrightarrow{OP} = a + \frac{1}{4}(-a + b)$$

$$= a - \frac{1}{4}a + \frac{1}{4}b$$

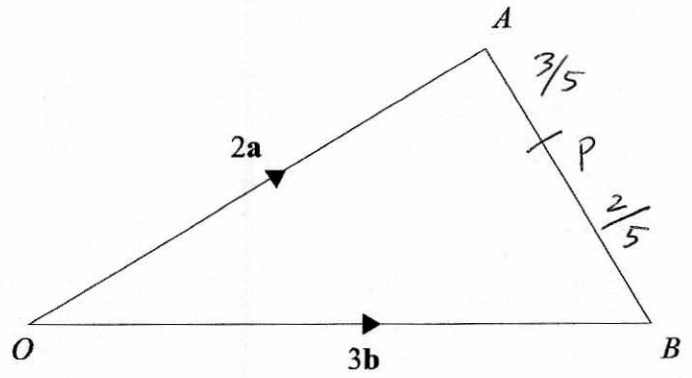
$$= \frac{3}{4}a + \frac{1}{4}b$$

$$= \frac{1}{4}(3a + b)$$

$k = 1/4$

(Total for question 1 is 4 marks)

2



$\overrightarrow{OA} = 2a$

$\overrightarrow{OB} = 3b$

P is the point on AB such that AP:PB = 3:2

$\overrightarrow{OP} = k(4\boldsymbol{a} + 9\boldsymbol{b})$

Find the value of k

$$\overrightarrow{AB} = -2a + 3b$$

$$\overrightarrow{AP} = \frac{3}{5}(-2a + 3b)$$

$$\overrightarrow{OP} = 2a + \frac{3}{5}(-2a + 3b)$$

$$= 2a - \frac{6}{5}a + \frac{9}{5}b$$

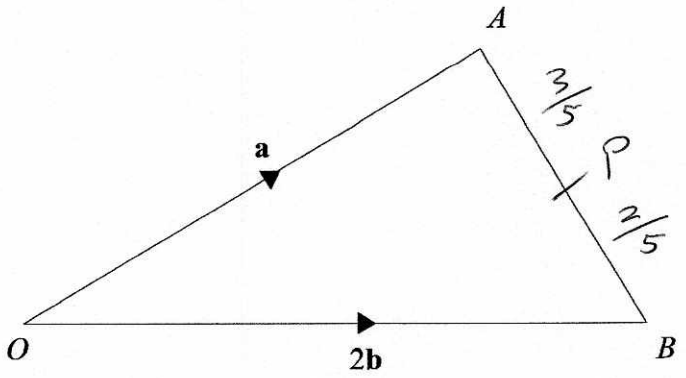
$$= \frac{4}{5}a + \frac{9}{5}b$$

$$= \frac{1}{5}(4a + 9b)$$

$k = 1/5$

(Total for question 2 is 4 marks)

3



$\overrightarrow{OA} = \boldsymbol{a}$

$\overrightarrow{OB} = 2\boldsymbol{b}$

P is the point on AB such that AP:PB = 3:2

$\overrightarrow{OP} = k(\boldsymbol{a} + 3\boldsymbol{b})$

Find the value of k

$$\overrightarrow{AB} = -a + 2b$$

$$\overrightarrow{AP} = \frac{3}{5}(-a + 2b)$$

$$\overrightarrow{OP} = a + \frac{3}{5}(-a + 2b)$$

$$= a - \frac{3}{5}a + \frac{6}{5}b$$

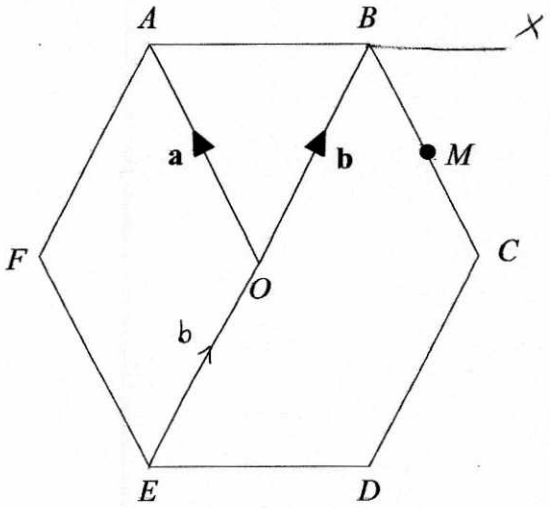
$$= \frac{2}{5}a + \frac{6}{5}b$$

$$= \frac{2}{5}(a + 3b)$$

$k = 2/5$

(Total for question 3 is 4 marks)

4 *ABCDEF* is a regular hexagon with centre *O*.



$\overrightarrow{OA} = a$

$\overrightarrow{OB} = b$

M is the midpoint of *BC*.
X is the point on *AB* extended, such that *AB*:*BX* = 3:2

Prove that *E*, *M* and *X* are on the same straight line.

$\overrightarrow{BX} = \frac{2}{3}\overrightarrow{AB}$

$\overrightarrow{AB} = -a + b$

$\overrightarrow{BX} = \frac{2}{3}(-a + b)$

$\overrightarrow{BM} = -\frac{1}{2}a$

$\overrightarrow{EM} = 2b - \frac{1}{2}a$

$= \frac{1}{2}(4b - a)$

$\overrightarrow{EX} = 2b + \frac{2}{3}(-a + b)$

$= 2b - \frac{2}{3}a + \frac{2}{3}b$

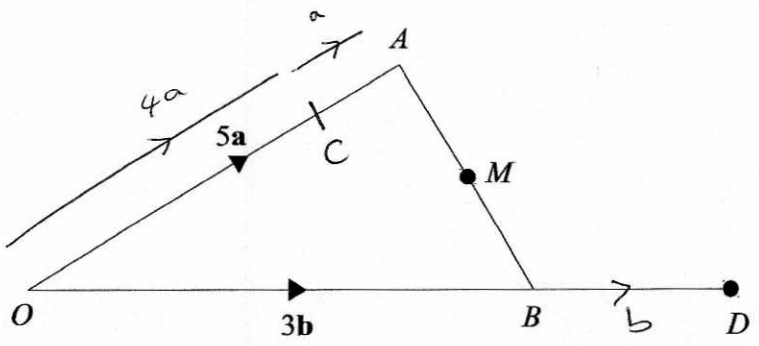
$= \frac{8}{3}b - \frac{2}{3}a$

$= \frac{2}{3}(4b - a)$

$\overrightarrow{EM}$ and $\overrightarrow{EX}$ are both multiples of $4b - a$ and both pass through E.

(Total for question 4 is 5 marks)

5



$\overrightarrow{OA}=5a$

$\overrightarrow{OB}=3b$

C is the point such that OC:CA = 4:1
M is the midpoint of AB
D is the point such that OB:OD = 3:4

$\overrightarrow{OD} = \frac{4}{3}\overrightarrow{OB}$

Show that C, M and D are on the same straight line. $\therefore \overrightarrow{OD} = 4b$

$\overrightarrow{AB} = -5a + 3b$

$\overrightarrow{AM} = \frac{1}{2}(-5a + 3b)$

$\overrightarrow{CM} = a + \frac{1}{2}(-5a + 3b)$

$= a - \frac{5}{2}a + \frac{3}{2}b$

$= -\frac{3}{2}a + \frac{3}{2}b$

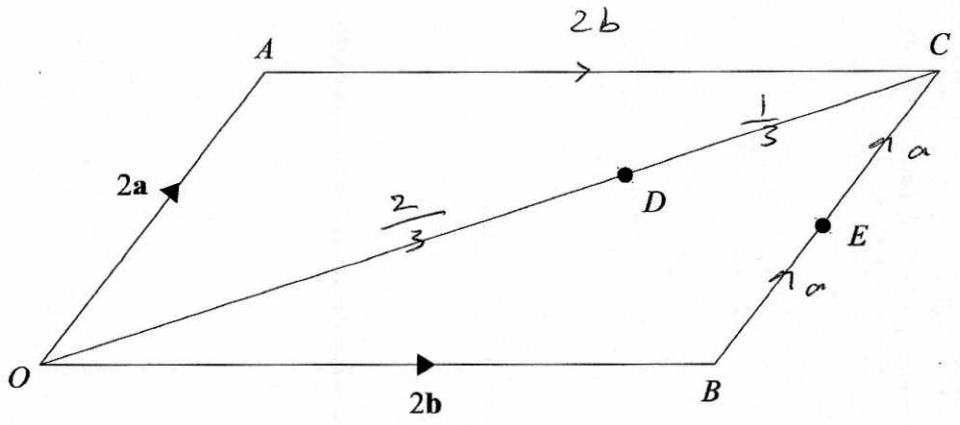
$\overrightarrow{CD} = -4a + 4b$

$\overrightarrow{CM} = \frac{3}{2}(-a + b)$ $\quad \overrightarrow{CD} = 4(-a + b)$

$\overrightarrow{CM}$ and $\overrightarrow{CD}$ are both multiples of $-a+b$ and they both pass through C.

(Total for question 5 is 5 marks)

6 The diagram shows a parallelogram.



$\overrightarrow{OA} = 2a$

$\overrightarrow{OB} = 2b$

D is the point on OC such that OD:DC = 2:1

E is the midpoint of BC

Show that A, D and E are on the same straight line.

$\overrightarrow{OC} = 2a + 2b$

$\overrightarrow{OD} = \frac{2}{3}(2a + 2b)$

$\overrightarrow{AD} = -2a + \frac{2}{3}(2a + 2b)$

$= -2a + \frac{4}{3}a + \frac{4}{3}b$

$= -\frac{2}{3}a + \frac{4}{3}b$

$= \frac{2}{3}(-a + 2b)$

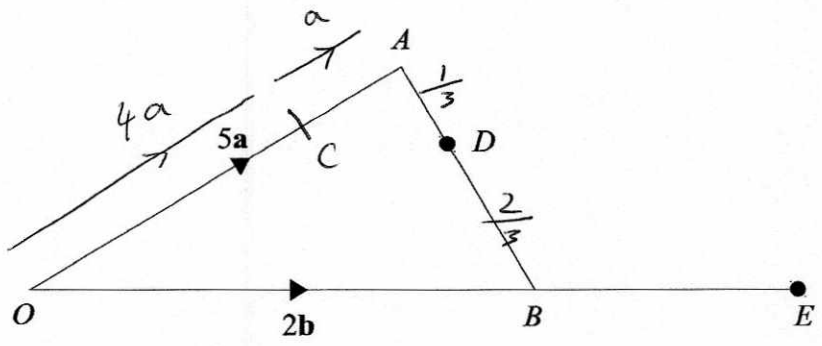
$\overrightarrow{AE} = 2b - a$

$= -a + 2b$

$\overrightarrow{AD}$ and $\overrightarrow{AE}$ are both multiples of $-a + 2b$ and both pass through A.

(Total for question 6 is 4 marks)

7



$\overrightarrow{OA}$=5a

$\overrightarrow{OB}$=2b

C is the point on OA such that OC:CA = 4:1
D is the point such that AD:DB = 1:2
The line OB is extended to point E

Given that C, D and E are on the same straight line find $\overrightarrow{BE}$

$\overrightarrow{AB} = -5a + 2b$

$\overrightarrow{AD} = \frac{1}{3}(-5a + 2b)$

$\overrightarrow{CD} = a + \frac{1}{3}(-5a + 2b)$

$= a - \frac{5}{3}a + \frac{2}{3}b$

$= -\frac{2}{3}a + \frac{2}{3}b$

$\overrightarrow{CE}$ must be a multiple of $-\frac{2}{3}a + \frac{2}{3}b$

$= \frac{2}{3}(-a + b)$

$\overrightarrow{CE} = -4a + xb$

$= -4a + 4b$ x must be 4

$\therefore \overrightarrow{BE} = 2b$

2b

(Total for question 7 is 5 marks)