Name: ______________________________

GCSE (1 – 9)

Proof

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 Prove algebraically that the sum of any two consecutive integers is always an odd number.

$n + n+1$

$2n + 1$

even

even + 1 is odd

(Total for question 1 is 2 marks)

2 Prove algebraically that the sum of any three consecutive even integers is always a multiple of 6.

$2n + 2n+2 + 2n+4$

$6n + 6$

$\underline{\underline{6(n+1)}}$

(Total for question 2 is 2 marks)

3 Prove that $(3n+1)^2 - (3n-1)^2$ is always a multiple of 12, for all positive integer values of n.

$$\left((3n+1)(3n+1)\right) - \left((3n-1)(3n-1)\right)$$

$$(9n^2+3n+3n+1) - (9n^2-3n-3n+1)$$

$$(9n^2+6n+1) - (9n^2-6n+1)$$

$$9n^2+6n+1-9n^2+6n-1$$

$$\underline{\underline{12n}}$$

(Total for question 3 is 2 marks)

4 n is an integer.
Prove algebraically that the sum of $n(n+1)$ and $n+1$ is always a square number.

$$n(n+1)+n+1$$

$$n^2+n+n+1$$

$$n^2+2n+1$$

$$(n+1)(n+1)$$

$$\underline{\underline{(n+1)^2}}$$

(Total for question 4 is 2 marks)

5 Prove that $(2n + 3)^2 - (2n - 3)^2$ is always a multiple of 12, for all positive integer values of n.

$$((2n+3)(2n+3)) - ((2n-3)(2n-3))$$

$$(4n^2+6n+6n+9) - (4n^2-6n-6n+9)$$

$$(4n^2+12n+9) - (4n^2-12n+9)$$

$$4n^2+12n+9-4n^2+12n-9$$

$$24n$$

$$\underline{\underline{12(2n)}}$$

(Total for question 5 is 2 marks)

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6 n is an integer.
Prove algebraically that the sum of $(n + 2)(n + 1)$ and $n + 2$ is always a square number.

$$(n+2)(n+1) + n+2$$

$$n^2+n+2n+2+n+2$$

$$n^2+4n+4$$

$$(n+2)(n+2)$$

$$\underline{\underline{(n+2)^2}}$$

(Total for question 6 is 2 marks)

7 Prove that the sum of 3 consecutive odd numbers is always a multiple of 3.

$2n + 1 + 2n + 3 + 2n + 5$

$6n + 9$

$\underline{\underline{3(2n + 3)}}$

(Total for question 7 is 2 marks)

8 Prove that the sum of 3 consecutive even numbers is always a multiple of 6.

$2n + 2n + 2 + 2n + 4$

$6n + 6$

$\cancel{2(3n + 3)}$

$\underline{\underline{6(n + 1)}}$

(Total for question 8 is 2 marks)

9 Prove algebraically that the sum of the squares of any 2 even positive integers is always a multiple of 4.

$$(2n)^2 + (2m)^2$$

$$4n^2 + 4m^2$$

$$4(n^2 + m^2)$$

(Total for question 9 is 2 marks)

10 Prove algebraically that the sum of the squares of any 2 odd positive integers is always even.

$$(2n+1)^2 + (2m+1)^2$$

$$(2n+1)(2n+1) + (2m+1)(2m+1)$$

$$4n^2 + 2n + 2n + 1 + 4m^2 + 2m + 2m + 1$$

$$4n^2 + 4n + 4m^2 + 4m + 2$$

$$2(2n^2 + 2n + 2m^2 + 2m + 1)$$

(Total for question 10 is 2 marks)

11 Prove that the sum of the squares of any two consecutive integers is always an odd number.

$$n^2 + (n+1)^2$$

$$n^2 + (n+1)(n+1)$$

$$n^2 + n^2 + n + n + 1$$

$$n^2 + n^2 + 2n + 1$$

$$2n^2 + 2n + 1$$

$$\underbrace{2(n^2 + n)}_{\text{even}} + 1$$

even + 1 is odd.

(Total for question 11 is 3 marks)

12 Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8

$$(2n+1)^2 + (2n+3)^2$$

$$(2n+1)(2n+1) + (2n+3)(2n+3)$$

$$4n^2 + 2n + 2n + 1 + 4n^2 + 6n + 6n + 9$$

$$8n^2 + 16n + 10$$

$$8n^2 + 16n + 8 + 2$$

$$\underbrace{8(n^2 + 2n + 1)}_{\text{multiple of } 8} \underbrace{+ 2}_{+2}$$

(Total for question 12 is 2 marks)

13 Prove that the difference between the squares of any 2 consecutive integers is equal to the sum of these integers.

n and $n+1$

$n+n+1 = \underline{\underline{2n+1}}$

$(n+1)^2 - n^2$

$(n+1)(n+1) - n^2$

$n^2 + n + n + 1 - n^2$

$\underline{\underline{2n+1}}$

(Total for question 13 is 3 marks)

14 Prove algebraically that the sums of the squares of any 2 consecutive even number is always 4 more than a multiple of 8.

$(2n)^2 + (2n+2)^2$

$4n^2 + (2n+2)(2n+2)$

$4n^2 + 4n^2 + 4n + 4n + 4$

$\cancel{4}\ 8n^2 + 8n + 4$

$\underbrace{8(n^2+n)}_{\text{multiple of 8}} \underbrace{+4}_{+4}$

(Total for question 14 is 3 marks)