Name: ______________________________

GCSE (1 – 9)

Probability Equation Questions

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– there may be more space than you need.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The marks for each question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 There are some red counters and some blue counters in a bag.

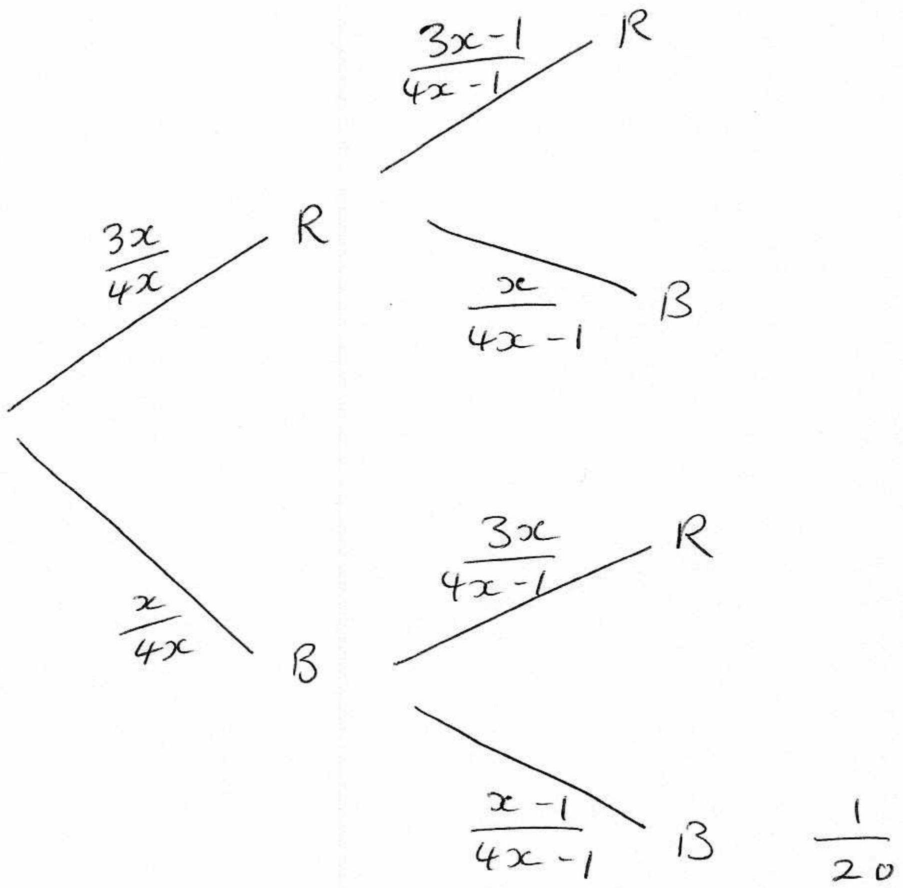
The ratio of red counters to blue counters is 3:1.

Two counters are removed at random.

The probability that both the counters taken are blue is $\frac{1}{20}$

Work how many counters were in the bag before any counters were removed.

$3x$ Red $\quad x$ Blue. $\quad 4x$ Total.



$$\frac{x}{4x} \times \frac{x-1}{4x-1} = \frac{1}{20}$$

$$\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{1}{20}$$

$$\frac{x-1}{4(4x-1)} = \frac{1}{20}$$

$$20(x-1) = 4(4x-1)$$

$$20x - 20 = 16x - 4$$

$$4x - 20 = -4$$

$$4x = 16$$

$$x = 4$$

16

(Total for question 1 is 5 marks)

2 There are some red counters and some blue counters in a bag.

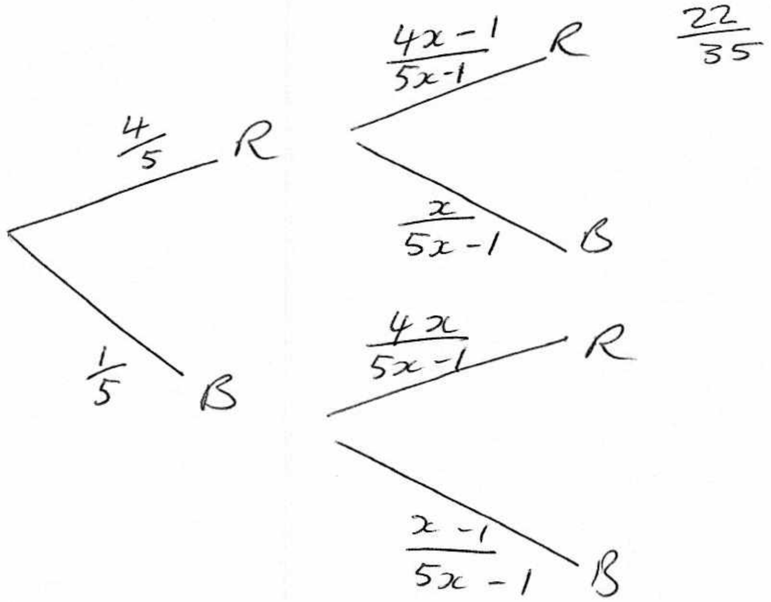
The ratio of red counters to blue counters is 4:1.

Two counters are removed at random.

The probability that both the counters taken are red is $\frac{22}{35}$

Work how many blue counters are in the bag.

$4x$ Red $\quad x$ Blue $\quad 5x$ Total.



$$\frac{4}{5} \times \frac{4x-1}{5x-1} = \frac{22}{35}$$

$$\frac{4(4x-1)}{5(5x-1)} = \frac{22}{35}$$

$$\frac{16x-4}{25x-5} = \frac{22}{35}$$

$$35(16x-4) = 22(25x-5)$$

$$560x - 140 = 550x - 110$$

$$10x - 140 = -110$$

$$10x = 30$$

$$\underline{\underline{x = 3}}$$

......3......

(Total for question 2 is 5 marks)

3 There are 5 red counters and y blue counters in a bag.

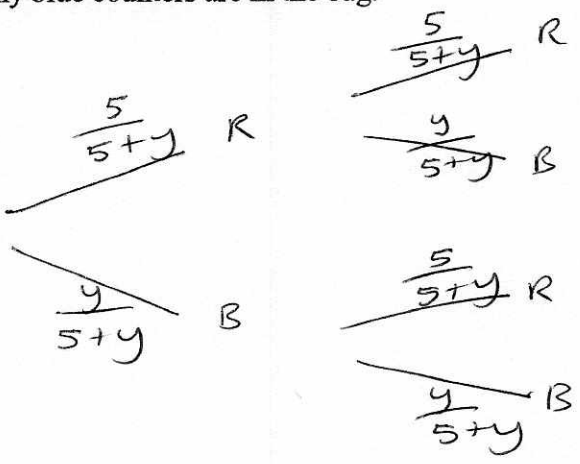
5 Red
y Blue

Imogen takes a counter from the bag at random.
She puts the counter back into the bag.
Imogen then takes another counter at random from the bag. 5+y Total

The probability that the first counter Imogen takes is red and the second counter Imogen takes is red is $\frac{1}{9}$

Work how many blue counters are in the bag.

$\frac{1}{9}$



$$\frac{5}{5+y} \times \frac{5}{5+y} = \frac{1}{9}$$

$$\frac{25}{(5+y)(5+y)} = \frac{1}{9}$$

$$225 = (5+y)(5+y)$$

$$225 = 25+5y+5y+y^2$$

$$225 = 25+10y+y^2$$

$$0 = y^2+10y-200$$

$$0 = (y+20)(y-10)$$

$$y=-20 \quad y=10$$

(y cannot be negative)

.......10.......

(Total for question 3 is 5 marks)

4 There are 4 red counters and x blue counters in a bag.

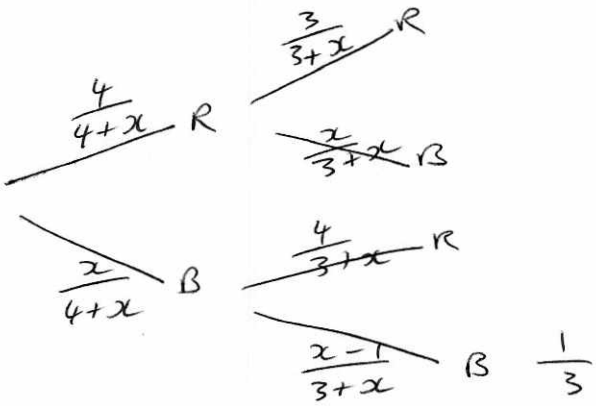
4 Red x Blue

$4 + x$ Total

2 counters are removed from the bag at random.

The probability that both the counters taken are blue is $\frac{1}{3}$

Work out the value of x.



$$\frac{x}{4+x} \times \frac{x-1}{3+x} = \frac{1}{3}$$

$$\frac{x(x-1)}{(4+x)(3+x)} = \frac{1}{3}$$

$$3x(x-1) = (4+x)(3+x)$$

$$3x^2 - 3x = 12 + 4x + 3x + x^2$$

$$3x^2 - 3x = 12 + 7x + x^2$$

$$2x^2 - 10x - 12 = 0$$

$$x^2 - 5x - 6 = 0$$

$$(x-6)(x+1) = 0$$

$$x = 6 \quad x = -1$$

x cannot be negative

$x = 6$

(Total for question 4 is 6 marks)

5 There are 5 red counters and x blue counters in a bag.

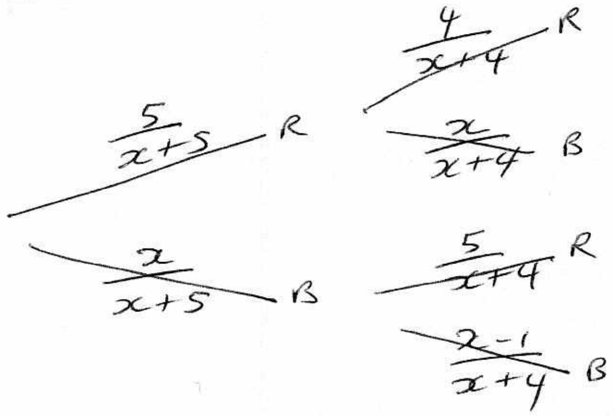
5 Red
x Blue
$x+5$ Total

2 counters are removed from the bag at random.

The probability that both the counters taken are red is $\frac{5}{33}$

Work out the value of x.

$\frac{5}{33}$



$$\frac{5}{x+5} \times \frac{4}{x+4} = \frac{5}{33}$$

$$\frac{20}{(x+5)(x+4)} = \frac{5}{33}$$

$$660 = 5(x+5)(x+4)$$

$$132 = (x+5)(x+4)$$

$$132 = x^2+4x+5x+20$$

$$132 = x^2+9x+20$$

$$0 = x^2+9x-112$$

$$0 = (x+16)(x-7)$$

$$x = -16 \quad x = 7$$

$$\therefore x = 7$$

7

(Total for question 5 is 7 marks)

6 There are n counters in a bag.

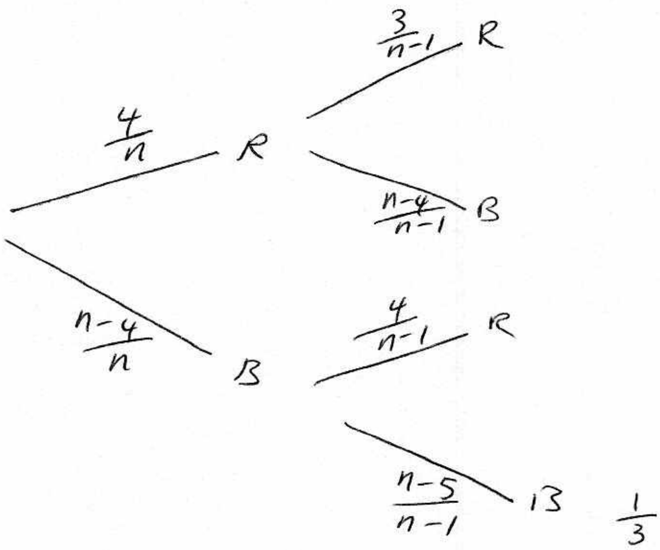
Total n
Red 4
Blue $n-4$

4 of the counters are red and the rest are blue.

Ross takes a counter from the bag at random and does not replace it.
He then takes another counter at random from the bag.

The probability that Ross takes two blue counters is $\frac{1}{3}$

(a) Show that $n^2 - 13n + 30 = 0$



$$\frac{n-4}{n} \times \frac{n-5}{n-1} = \frac{1}{3}$$

$$\frac{(n-4)(n-5)}{n(n-1)} = \frac{1}{3}$$

$$\frac{n^2 - 5n - 4n + 20}{n^2 - n} = \frac{1}{3}$$

$$\frac{n^2 - 9n + 20}{n^2 - n} = \frac{1}{3}$$

$$3(n^2 - 9n + 20) = n^2 - n$$

$$3n^2 - 27n + 60 = n^2 - n$$

$$2n^2 - 26n + 60 = 0$$

$$\underline{n^2 - 13n + 30 = 0}$$

(5)

(b) Find the value of n.

$$n^2 - 13n + 30 = 0$$

$$(n-3)(n-10) = 0$$

$n = 3$ $n = 10$

n cannot be less than 4 so $n = 10$

10

(2)

(Total for question 6 is 7 marks)

7 There are n counters in a bag.

n Total
8 Red
$n-8$ Blue

8 of the counters are red and the rest are blue.

Adam takes a counter from the bag at random and does not replace it.
He then takes another counter at random from the bag.

The probability that Adam takes two blue counters is $\frac{1}{5}$

(a) Show that $n^2 - 21n + 90 = 0$

$$\frac{n-8}{n} \times \frac{n-9}{n-1} = \frac{1}{5}$$

$$\frac{(n-8)(n-9)}{n(n-1)} = \frac{1}{5}$$

$$\frac{n^2 - 9n - 8n + 72}{n^2 - n} = \frac{1}{5}$$

$$\frac{n^2 - 17n + 72}{n^2 - n} = \frac{1}{5}$$

$$5(n^2 - 17n + 72) = n^2 - n$$

$$5n^2 - 85n + 360 = n^2 - n$$

$$4n^2 - 84n + 360 = 0$$

$$\underline{\underline{n^2 - 21n + 90 = 0}}$$

(5)

(b) Find the value of n.

$$(n-6)(n-15) = 0$$

$$n = 6 \qquad n = 15$$

n cannot be less than 8 $\therefore$ $n = 15$

15

(2)

(Total for question 1 is 7 marks)