Name: ______________________________

IGCSE

Circle Theorems

Instructions

• Use **black** ink or ball-point pen.
• Answer all questions.
• Answer the questions in the spaces provided
– there may be more space than you need.
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working out.**

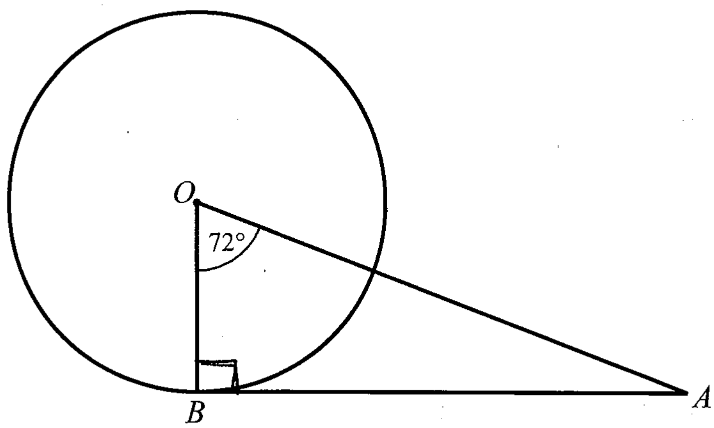
Information

• The marks for each question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end

1



B is a point on the circumference of a circle, centre O.
AB is a tangent to the circle.

Angle $BOA = 72°$

Work out the size of angle BAO.
You must show all your working.

OBA = 90° Tangent meets radius at 90°

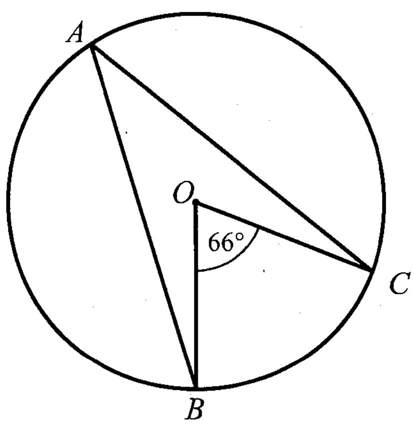
BAO = 180 − 90 − 72 Angles in a triangle add to 180°

= 18°

18 °

(Total for Question 1 is 2 marks)

2



A, B, C and D are points on the circumference of a circle.

Angle $BOC = 66°$

(i) Find the size of angle BAC.

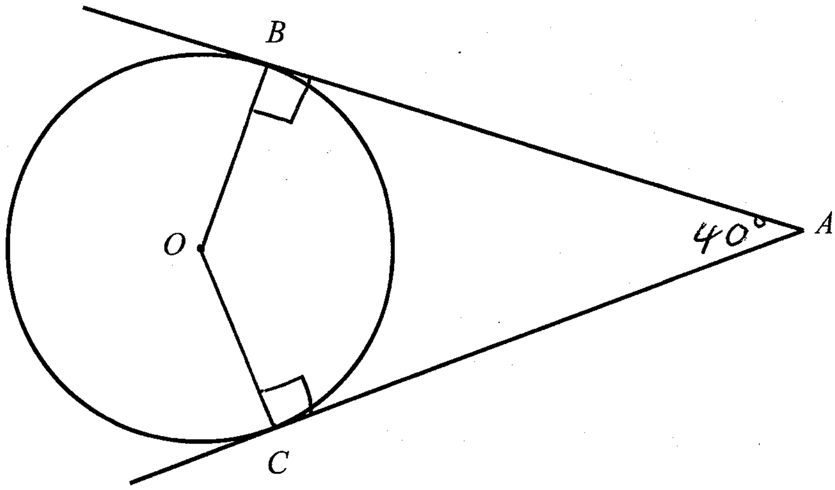
33 °

(ii) Give a reason for your answer.

The angle at the circumference is half the angle at the centre

(Total for Question 2 is 2 marks)

3



B and *C* are points on a circle, centre *O*.
AB and *AC* are tangents to the circle.

Angle $BAC = 40°$

Work out the size of angle *BOC*.
You must show all your working.

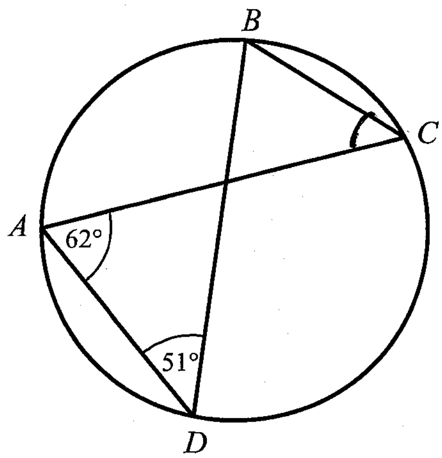
OBA and OCA = 90°
Tangent meets radius at 90°

360 − 90 − 90 − 40 = 140°
Angles in a quadrilateral add to 360°

140 °

(Total for Question 3 is 3 marks)

4



A, B, C and *D* are points on the circumference of a circle.

Angle $CAD = 62°$
Angle $ADB = 51°$

(i) Find the size of angle *ACB*.

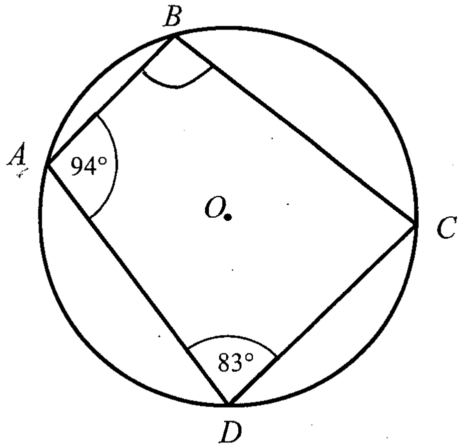
51 °

(ii) Give a reason for your answer.

Angles from the same points (to the circumference) are equal

(Total for Question 4 is 2 marks)

5



A, B, C and D are points on the circumference of a circle.

Angle $BAD = 94°$
Angle $ADC = 83°$

180 − 83

(i) Find the size of angle ABC.

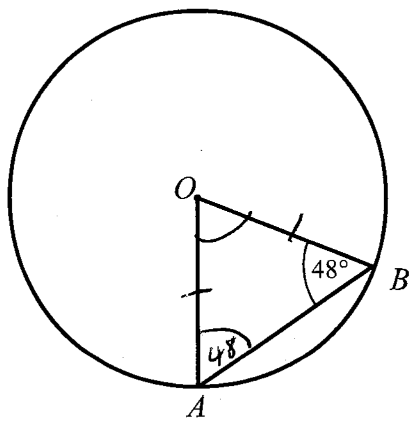
97 °

(ii) Give a reason for your answer.

opposite angles in a cyclic quadrilateral add to 180°

(Total for Question 5 is 2 marks)

6



OAB = 48°
Angles at base of isosceles are equal

A and B are points on the circumference of a circle, centre O.

Angle $ABO = 48°$

(i) Find the size of angle AOB.

180 − 48 − 48

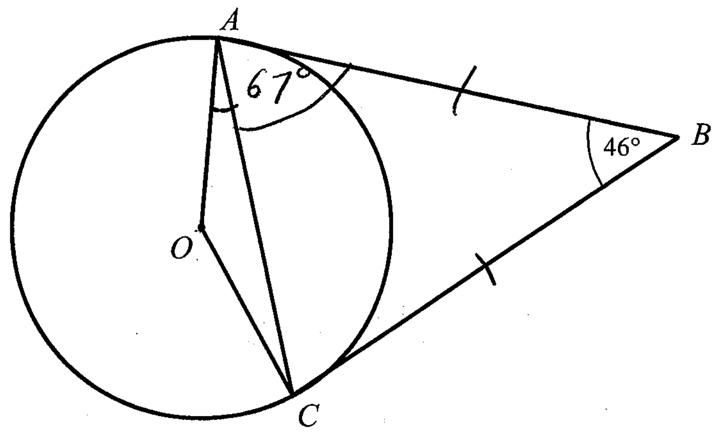
84 °

(ii) Give a reason for your answer.

Angles at the base of an isosceles triangle are equal + angles in a triangle add to 180°

(Total for Question 6 is 2 marks)

7



A and *C* are points on the circumference of a circle, centre *O*.
AB and *BC* are tangents to the circle.

Angle *ABC* = 46°

Find the size of angle *OAC*.
Give reasons for each stage of your working.

ABC is an isosceles triangle, 2 tangents from the same point are equal

$CAB = \frac{180-46}{2} = \frac{134}{2} = 67°$

Angles at the base of an isosceles triangle are equal

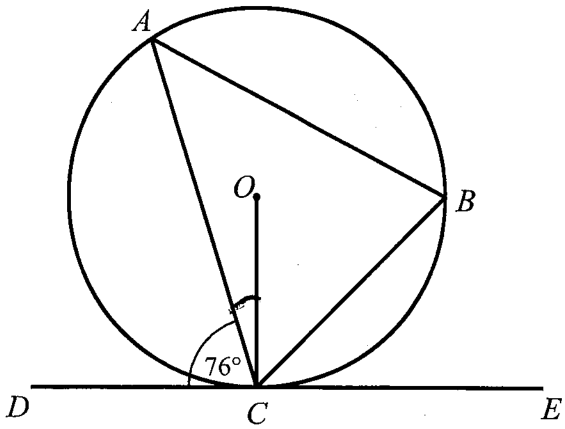
$OAB = 90°$ Tangent meets radius at 90°

$OAC = 90 - 67 = \underline{\underline{23°}}$

23 °

(Total for Question 7 is 4 marks)

8



A and B are points on the circumference of a circle, centre O.
DCE is a tangent to the circle.

Angle $ACD = 76°$

(a) Find the size of angle ACO.
You must ~~show~~ all your working.
Give reasons for

OCD = 90° Tangent meets radius at 90°

90 − 76 = 14°

14 °

(2)

(b) Find the size of angle ABC.
You must ~~show~~ all your working.
give reasons for

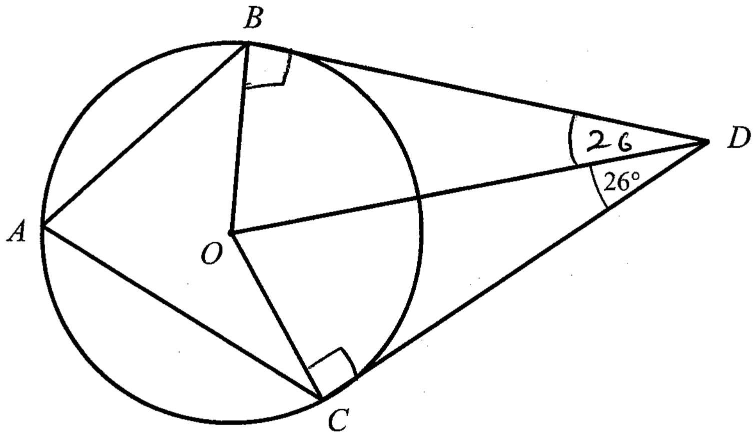
ABC = 76° Alternate segment theorem

76 °

(2)

(Total for Question 8 is 4 marks)

9



A, B and *C* are points on the circumference of a circle, centre *O*.
BD and *CD* are tangents to the circle.

Angle *ODC* = 26°

Find the size of angle *BAC*.
Give reasons for each stage of your working.

Triangle BOD = Triangle DOC all sides are equal

OBD and OCD = 90° Tangent meets radius at 90°

BOD and COD = 180 − 90 − 26
= 64°

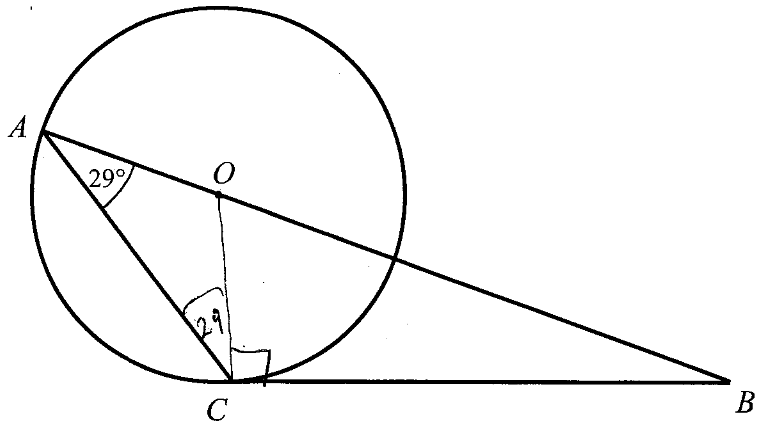
BOC = 2 × 64 = 128°

BAC = $\frac{128}{2}$ = 64° Angle at circumference is half the angle at the centre

64 °

(Total for Question 9 is 4 marks)

10



A and *C* are points on the circumference of a circle, centre *O*.
BC is a tangent to the circle.

Angle *CAB* = 29°

Find the size of angle *ABC*.
You must show all your working.

OCA = 29° Angles at the base of an isosceles triangle are equal

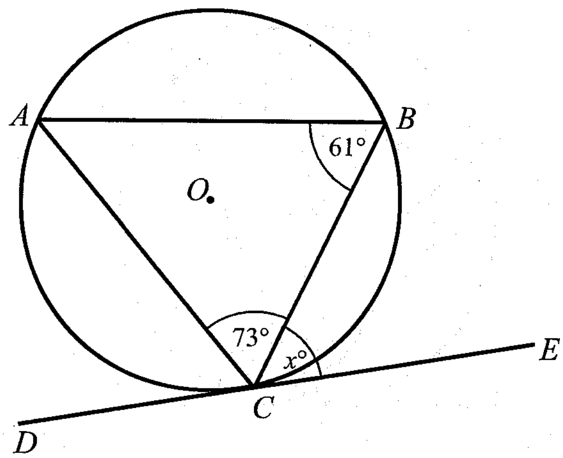
OCB = 90° Tangent meets radius at 90°

ABC = 180 − 90 − 29 − 29
= 32° Angles in a triangle add to 180°

32 °

(Total for Question 10 is 4 marks)

11



A, B and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Angle $ABC = 61°$
Angle $ACB = 73°$
Angle $BCE = x°$

Find the value of x.
You must show all your working.

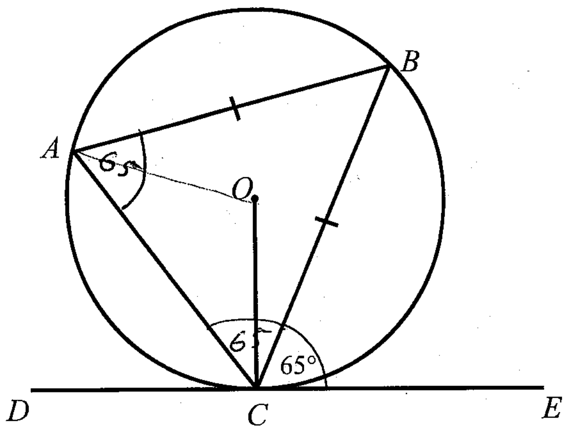
BAC = 180 − 73 − 61
= 46° Angles in a triangle add to 180°

x = 46° Alternate segment theorem

46°

(Total for Question 11 is 3 marks)

12



A, B and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

AB = *BC*
Angle *BCE* = 65°

Find the size of angle *AOC*.
You must show all your working.

BAC = 65° Alternate segment theorem

ACB = 65° Angles at the base of an isosceles triangle are equal

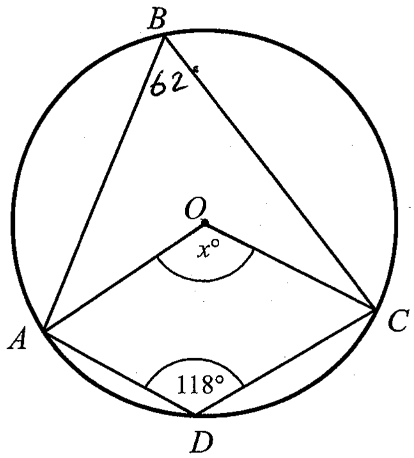
ABC = 180 − 65 − 65
= 50° Angles in a triangle add to 180°

AOC = 2 × 50
= 100° Angle at centre is twice angle at circumference

100 °

(Total for Question 12 is 4 marks)

13



A, B, C and *D* are points on the circumference of a circle, centre *O*.

Angle $ADC = 118°$
Angle $AOC = x°$

Work out the value of *x*.
You must show all your working.

ABC = 180 − 118
= 62°

opposite angles in a cyclic quadrilateral add to 180°

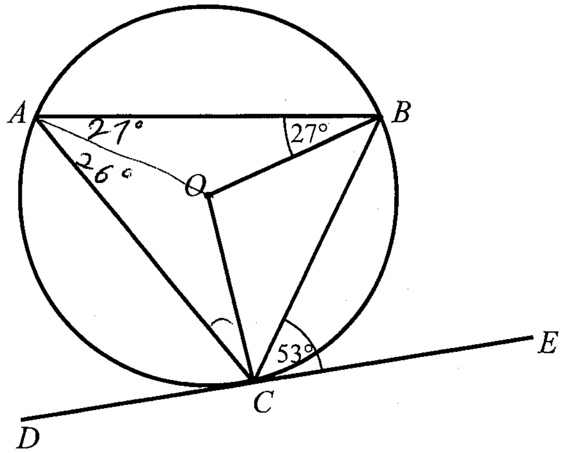
AOC = 2 × 62
= 124°

Angle at centre is twice angle at circumference

124 °

(Total for Question 13 is 3 marks)

14



A, B and *C* are points on the circumference of a circle, centre *O*.
DCE is a tangent to the circle.

Angle *ABO* = 27°
Angle *BCE* = 53°

Find the size of angle *ACO*.
~~You must show all your working~~.
Give reasons for each stage of your working

BAO = 27° Angles at base of isosceles triangle are equal

BAC = 53° Alternate segment theorem

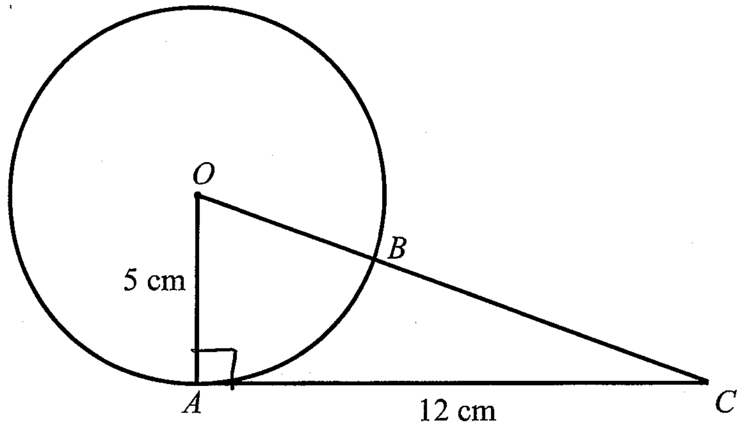
OAC = 53 − 27 = 26°

ACO = 26° Angles at base of isosceles triangle are equal

26 °

(Total for Question 14 is ~~3~~ 4 marks)

15



A and B is a point on the circumference of a circle, centre O.
AC is a tangent to the circle.
OBC is a straight line.

$OA = 5$ cm
$AC = 12$ cm

Find the length of BC.
You must show all your working.

OAC = 90° Tangent meets radius at 90°

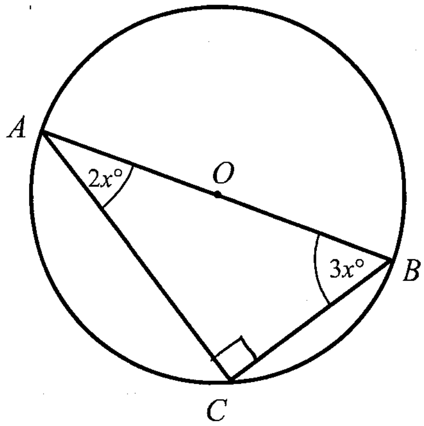
$5^2 + 12^2 = OC^2$
$169 = OC^2$
$OC = \sqrt{169}$
$= 13$ cm

BC = 13 − 5 = 8cm (radius = 5cm)

8 cm

(Total for Question 15 is 4 marks)

16



A, B and *C* are points on the circumference of a circle, centre *O*.

Angle $CAB = 2x°$
Angle $ABC = 3x°$

Find the value of x.
You must show all your working.

ACB = 90° Angle in a semi circle is 90°

$2x + 3x + 90 = 180$

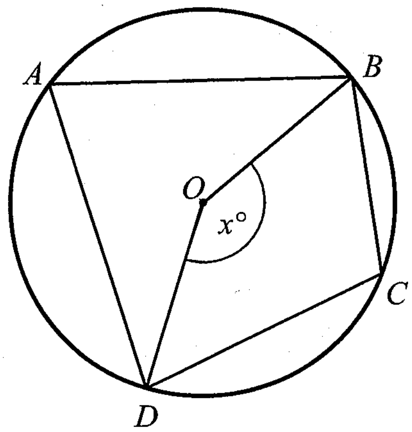
$5x = 90$

$x = 18°$

$x =$ 18

(Total for Question 16 is 3 marks)

17



A, B, C and *D* are points on the circumference of a circle, centre *O*.

Angle $BOD = x°$

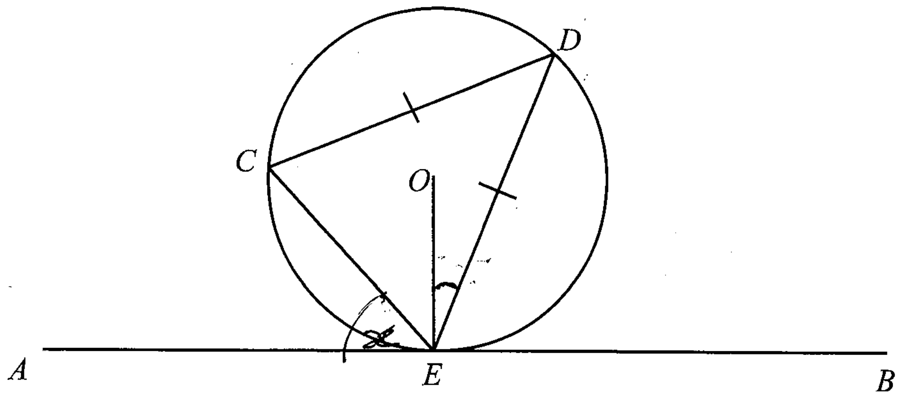
Find the size of angle *BCD*, in terms of *x*.
Give reasons for each stage of your working.

$BAD = \frac{1}{2}x$ Angle at circumference is half angle at centre

$BCD = \underline{180 - \frac{1}{2}x}$ opposite angles in a cyclic quadrilateral add to 180°

(Total for Question 17 is 3 marks)

18



C, D and E are points on a circle, centre O.
AEB is a tangent to the circle at E.

$CD = DE$
Angle $AEC = x°$

Find the size of angle OED, in terms of x.
Give reasons for each stage of your working.

$CDE = x$ Alternate segment theorem

$CED = \frac{180 - x}{2}$ Angles at the base of an isosceles triangle are equal

$OEC = 90 - x$ $OEA = 90°$ Tangent meets radius at $90°$

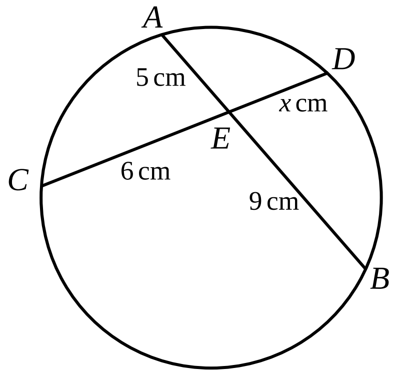
$OED = \frac{180 - x}{2} - (90 - x)$

$= 90 - \frac{1}{2}x - 90 + x$

$= \frac{1}{2}x$

(Total for Question 18 is 5 marks)

19



AB and *CD* are chords of a circle that intersect at E.

AE = 5 cm
B*E* = 9 cm
CE = 9 cm
DE = *x* cm

Find the value of *x*.

$6x = 5 \times 9$

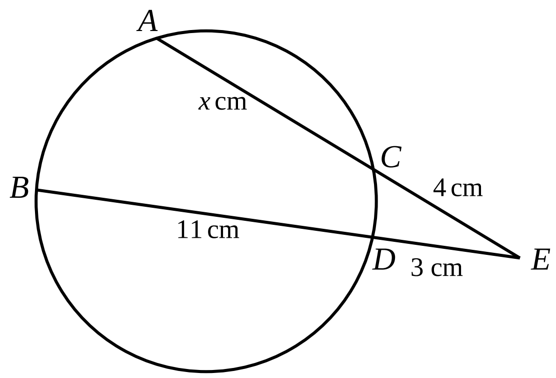
$6x = 45$

$x = \frac{45}{6} = \frac{15}{2}$

$x =$ 7.5

(Total for Question 19 is 2 marks)

20



A, *B*, C and *D* are points on a circle.
ACE and *BDE* are straight lines.

AC = *x* cm, *BD* = 10 cm, *CE* = 4 cm and *DE* = 3 cm

Find the value of *x*.

$4(x+4) = 3(3+11)$

$4x + 16 = 42$

$4x = 26$

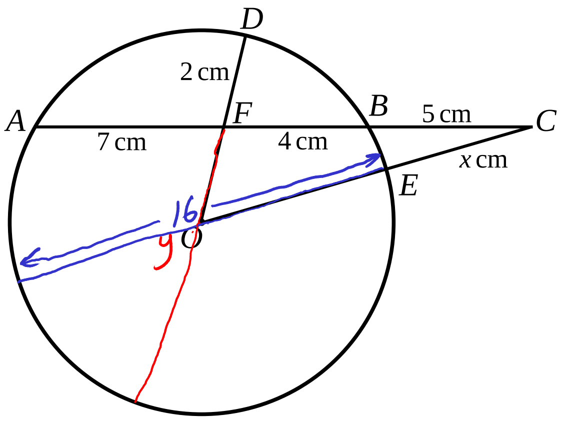
$x = \frac{13}{2}$

$x =$ 6.5

(Total for Question 20 is 3 marks)

June 2018 Paper 2H Question 22

21



A, *D*, *B* and *E* are points on a circle, centre *O*.
AFBC, *OEC* and *OFD* are straight lines.

AF = 7 cm, *FB* = 4 cm, *BC* = 5 cm, *FD* = 2 cm and *CE* = *x* cm.

Work out the value of *x*.
Show your working clearly.

$7(4) = 2(y)$
$28 = 2y$
$y = 14$

diameter = $14 + 2 = 16$ cm

$x(16 + x) = 5(7 + 4 + 5)$

$16x + x^2 = 5(16)$

$16x + x^2 = 80$

$x^2 + 16x - 80 = 0$

$(x - 4)(x + 20) = 0$

$x = 4$ $\quad x = -20$

x cannot be negative

x = ...4........

(Total for Question 21 is 5 marks)