

1. A random sample of 35 homeowners was taken from each of the villages Greenslax and Penville and their ages were recorded. The results are summarised in the back-to-back stem and leaf diagram below.

Totals	Greenslax						Penville					Totals							
(2)				8	7	2	5	5	6	7	8	8	9	(7)					
(3)				9	8	7	3	1	1	1	2	3	4	4	5	6	7	9	(11)
(4)			4	4	4	0	4	0	1	2	4	7		(5)					
(5)			6	6	5	2	2	5	0	0	5	5	5		(5)				
(7)	8	6	5	4	2	1	1	6	2	5	6	6		(4)					
(8)	8	6	6	6	4	3	1	1	7	0	5		(2)						
(5)			9	8	4	3	2	8						(0)					
(1)						4	9	9					(1)						

Key: 7 | 3 | 1 means 37 years for Greenslax and 31 years for Penville

Some of the quartiles for these two distributions are given in the table below.

	Greenslax	Penville
^{9th} Lower quartile, Q_1	a	31
Median, Q_2	64	39
^{27th} Upper quartile, Q_3	b	55

- (a) Find the value of a and the value of b . (2)

An outlier is a value that falls either

more than $1.5 \times (Q_3 - Q_1)$ above Q_3

or more than $1.5 \times (Q_3 - Q_1)$ below Q_1

- (b) On the graph paper opposite draw a box plot to represent the data from Penville. Show clearly any outliers. (4)

- (c) State the skewness of each distribution. Justify your answers. (3)

a/ a = 44
b = 76



Question 1 continued

$$b) \text{ Penville: } IQR = 55 - 31 = 24$$

$$1.5 \times 24 = 36$$

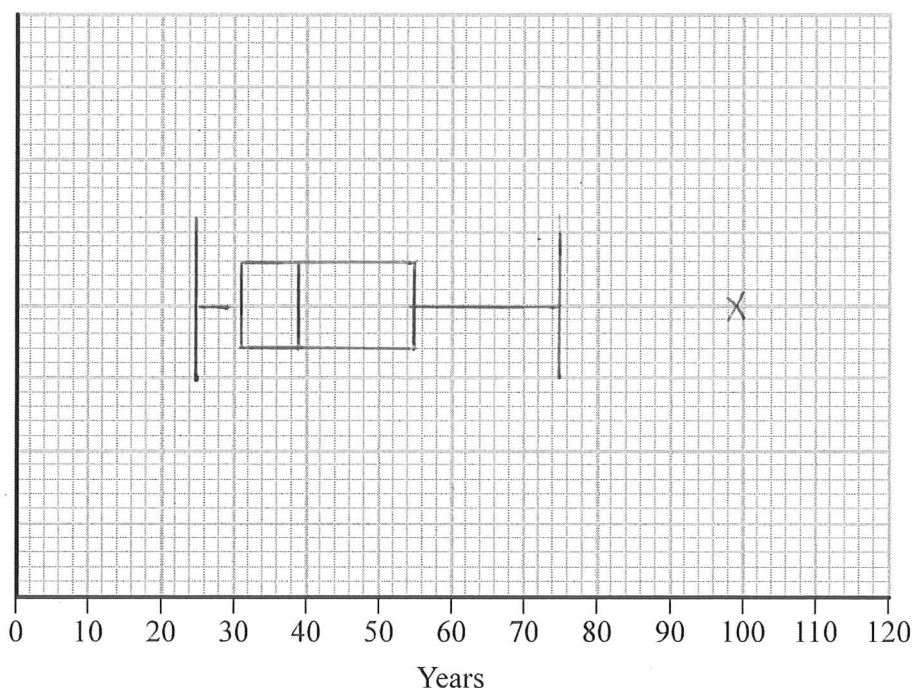
$$LQ - 1.5(IQR) = -5 \quad UQ + 1.5(IQR) = 91$$

99 is an outlier.

Penville:

c) The median is closer to the lower quartile.
There is a positive skew

Greenstax: The median is closer to the upper quartile.
There is a negative skew.



2. The mark, x , scored by each student who sat a statistics examination is coded using

$$y = 1.4x - 20$$

The coded marks have mean 60.8 and standard deviation 6.60

Find the mean and the standard deviation of x .

(4)

standard deviation:

$$6.60 = 1.4\sigma$$

$$\sigma = \frac{6.60}{1.4} = \underline{\underline{4.71}}$$

mean

$$60.8 = 1.4\mu - 20$$

$$\mu = \frac{60.8 + 20}{1.4} = \underline{\underline{57.7}}$$

Q2

(Total 4 marks)



3. The table shows data on the number of visitors to the UK in a month, v (1000s), and the amount of money they spent, m (£ millions), for each of 8 months.

Number of visitors v (1000s)	2450	2480	2540	2420	2350	2290	2400	2460
Amount of money spent m (£ millions)	1370	1350	1400	1330	1270	1210	1330	1350

You may use

$$S_{vv} = 42587.5 \quad S_{vm} = 31512.5 \quad S_{mm} = 25187.5 \quad \sum v = 19390 \quad \sum m = 10610$$

- (a) Find the product moment correlation coefficient between m and v . (2)
- (b) Give a reason to support fitting a regression model of the form $m = a + bv$ to these data. (1)
- (c) Find the value of b correct to 3 decimal places. (2)
- (d) Find the equation of the regression line of m on v . (2)
- (e) Interpret your value of b . (2)
- (f) Use your answer to part (d) to estimate the amount of money spent when the number of visitors to the UK in a month is 2 500 000 (2)
- (g) Comment on the reliability of your estimate in part (f). Give a reason for your answer. (2)

$$a) \quad r = \frac{S_{vm}}{\sqrt{S_{vv} S_{mm}}}$$

$$r = \frac{31512.5}{\sqrt{(42587.5)(25187.5)}}$$

$$= 0.962 \text{ (3sf)}$$

b) There is a positive correlation (close to 1)



Question 3 continued

$$c/ \quad b = \frac{S_{vm}}{S_{vv}} = \frac{31512.5}{42587.5}$$
$$= 0.740 \quad (3dp)$$

$$d/ \quad a = \bar{m} - b\bar{v}$$
$$= \frac{10616}{8} - 0.740 \left(\frac{19390}{8} \right)$$
$$= -467 \quad (3sf)$$

$$m = -467 + 0.740v$$

e/ For each additional 1000 visitors an extra £0.740 million is spent.

$$f/ \quad v = 2500$$
$$m = -467 + 0.740(2500)$$
$$= 1383$$
$$= \underline{\underline{1380}} \text{ million pounds} \quad (3sf)$$

g/ it is within the range so the estimate would be fairly reliable



4. In a factory, three machines, J , K and L , are used to make biscuits.

Machine J makes 25% of the biscuits.

Machine K makes 45% of the biscuits.

The rest of the biscuits are made by machine L .

It is known that 2% of the biscuits made by machine J are broken, 3% of the biscuits made by machine K are broken and 5% of the biscuits made by machine L are broken.

(a) Draw a tree diagram to illustrate all the possible outcomes and associated probabilities. (2)

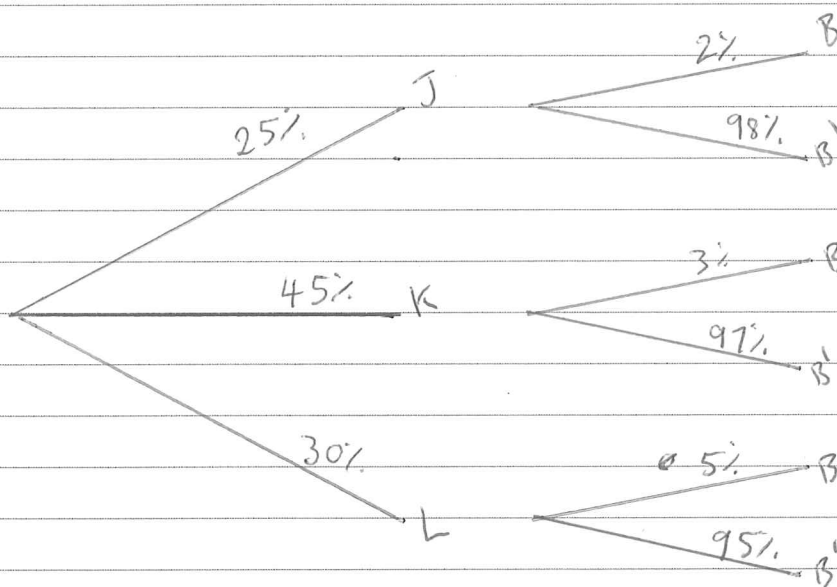
A biscuit is selected at random.

(b) Calculate the probability that the biscuit is made by machine J and is not broken. (2)

(c) Calculate the probability that the biscuit is broken. (2)

(d) Given that the biscuit is broken, find the probability that it was not made by machine K . (3)

4a)



b) $0.25 \times 0.98 = 0.245$

c) $(0.25 \times 0.02) + (0.45 \times 0.03) + 0.3(0.05) = \frac{67}{2000}$

d) $\frac{(0.25 \times 0.02) + (0.3 \times 0.05)}{\frac{67}{2000}} = \frac{40}{67}$



5. The discrete random variable X has the probability function

$$P(X = x) = \begin{cases} kx & x = 2, 4, 6 \\ k(x - 2) & x = 8 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Show that $k = \frac{1}{18}$ (2)

(b) Find the exact value of $F(5)$. (1)

(c) Find the exact value of $E(X)$. (2)

(d) Find the exact value of $E(X^2)$. (2)

(e) Calculate $\text{Var}(3 - 4X)$ giving your answer to 3 significant figures. (3)

a/

x	2	4	6	8
$P(X=x)$	$2k$	$4k$	$6k$	$6k$
	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{1}{3}$	$\frac{1}{3}$

$$2k + 4k + 6k + 6k = 1$$

$$18k = 1$$

$$k = \frac{1}{18}$$

b/ $F(5) = 2k + 4k = 6k$
 $= 6\left(\frac{1}{18}\right)$
 $= \frac{1}{3}$

c/ $E(X) = 2\left(\frac{1}{9}\right) + 4\left(\frac{2}{9}\right) + 6\left(\frac{1}{3}\right) + 8\left(\frac{1}{3}\right)$
 $= \frac{52}{9}$

d/ $E(X^2) = 2^2\left(\frac{1}{9}\right) + 4^2\left(\frac{2}{9}\right) + 6^2\left(\frac{1}{3}\right) + 8^2\left(\frac{1}{3}\right)$
 $= \frac{112}{3}$



Question 5 continued

$$\begin{aligned} \text{e/ } \text{Var}(x) &= E(x^2) - (E(x))^2 \\ &= \frac{112}{3} - \left(\frac{52}{9}\right)^2 \\ &= \frac{320}{81} \end{aligned}$$

$$\begin{aligned} \text{Var}(3-4x) &= 16 \times \frac{320}{81} \\ &= \underline{\underline{63.2}} \quad (3 \text{ sf}) \end{aligned}$$



6. The times, in seconds, spent in a queue at a supermarket by 85 randomly selected customers, are summarised in the table below.

Time (seconds)	Number of customers, f
0 – 30	2
30 – 60	10
60 – 70	17
70 – 80	25
80 – 100	25
100 – 150	6

A histogram was drawn to represent these data. The 30 – 60 group was represented by a bar of width 1.5 cm and height 1 cm.

- (a) Find the width and the height of the 70 – 80 group. (3)
- (b) Use linear interpolation to estimate the median of this distribution. (2)

Given that x denotes the midpoint of each group in the table and

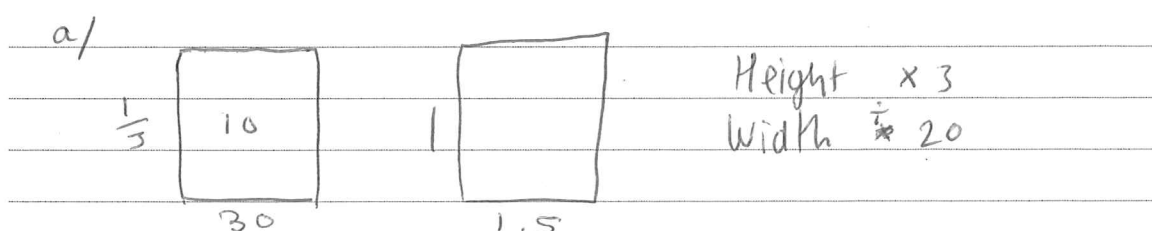
$$\sum fx = 6460 \quad \sum fx^2 = 529\,400$$

- (c) calculate an estimate for
- (i) the mean,
- (ii) the standard deviation,
- for the above data. (3)

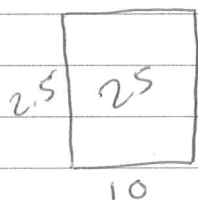
One measure of skewness is given by

$$\text{coefficient of skewness} = \frac{3(\text{mean} - \text{median})}{\text{standard deviation}}$$

- (d) Evaluate this coefficient and comment on the skewness of these data. (3)



Question 6 continued



$$\text{Height} = 2.5 \times 3 = 7.5 \text{ cm}$$

$$\text{Width} = 10 \div 20 = 0.5 \text{ cm}$$

$$b/ \quad \frac{85}{2} = 42.5$$

~~$$60 + 13.5$$~~

$$70 + \frac{13.5(10)}{25}$$

$$= 75.4 \text{ seconds}$$

$$c/ \quad \text{mean} = \frac{\sum fx}{n}$$

$$= \frac{6460}{85}$$

$$= 76 \text{ seconds}$$

$$\sigma = \sqrt{\frac{\sum fx^2}{n} - \left(\frac{\sum fx}{n}\right)^2}$$

$$= \sqrt{\frac{529400}{85} - (76)^2}$$

$$= 21.3 \text{ (3sf)}$$

$$d/ \quad \frac{3(\text{Mean} - \text{median})}{\sigma}$$

$$\frac{3(76 - 75.4)}{21.3} = 0.0846 \text{ (3sf)}$$

There is a very small positive skew.



7. The heights of adult females are normally distributed with mean 160 cm and standard deviation 8 cm.

- (a) Find the probability that a randomly selected adult female has a height greater than 170 cm. (3)

Any adult female whose height is greater than 170 cm is defined as tall.

An adult female is chosen at random. Given that she is tall,

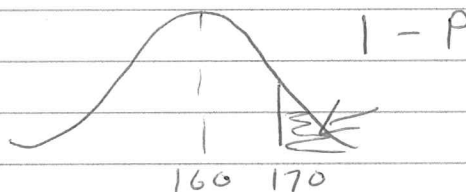
- (b) find the probability that she has a height greater than 180 cm. (4)

Half of tall adult females have a height greater than h cm.

- (c) Find the value of h . (5)

$$\mu = 160 \quad \sigma = 8$$

$$\begin{aligned} a/ \quad z &= \frac{x - \mu}{\sigma} \\ &= \frac{170 - 160}{8} \\ &= 1.25 \end{aligned}$$

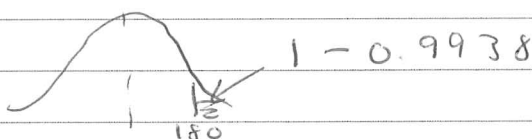


$$\begin{aligned} P(x < 170) &= 0.8944 \\ P(x > 170) &= \underline{\underline{0.1056}} \end{aligned}$$

$$b/ \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A/B) = \frac{P(x > 180)}{0.1056}$$

$$\begin{aligned} z &= \frac{180 - 160}{8} \\ &= 2.5 \end{aligned}$$



$$P(A/B) = \frac{1 - 0.9938}{0.1056} = \frac{0.0062}{0.1056} = \frac{31}{528}$$

0.0588
351



Question 7 continued

$$c) \text{ half of tall females: } \frac{0.1056}{2}$$

$$1 - \left(\frac{0.1056}{2}\right) = 0.9472$$

$$z = 1.62$$

$$1.62 = \frac{X - 160}{8}$$

$$X = 172.96$$
$$= \underline{\underline{173 \text{ cm}}} \text{ (3st)}$$



8. For the events A and B ,

$$P(A' \cap B) = 0.22 \text{ and } P(A' \cap B') = 0.18$$

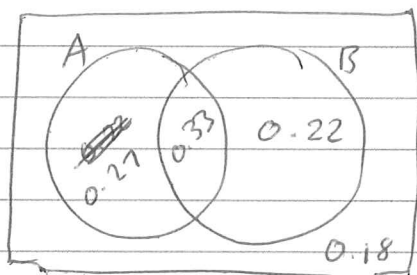
(a) Find $P(A)$. (1)

(b) Find $P(A \cup B)$. (1)

Given that $P(A|B) = 0.6$

(c) find $P(A \cap B)$. (3)

(d) Determine whether or not A and B are independent. (2)



$$\begin{aligned} \text{a/ } P(A) &= 1 - 0.22 - 0.18 \\ &= \underline{\underline{0.6}} \end{aligned}$$

$$\begin{aligned} \text{b/ } P(A \cup B) &= 1 - 0.18 \\ &= \underline{\underline{0.82}} \end{aligned}$$

$$\begin{aligned} \text{c/ } P(A|B) &= \frac{P(A \cap B)}{P(B)} \\ 0.6 &= \frac{P(A \cap B)}{P(B)} \end{aligned}$$

$$\begin{aligned} P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.6 + P(B) - 0.82 \end{aligned}$$

$$P(A \cap B) = P(B) - 0.22$$

$$0.6P(B) = P(B) - 0.22$$

$$0.22 = 0.4(P(B))$$

$$P(B) = \frac{11}{20}$$

$$P(A \cap B) = 0.6 \left(\frac{11}{20} \right) = 0.33$$



Question 8 continued

if independent

$$P(A) \times P(B) = P(A \cap B)$$

$$0.6 \times 0.55 = 0.33$$

$$0.33 = P(A \cap B)$$

\therefore They are independent.

