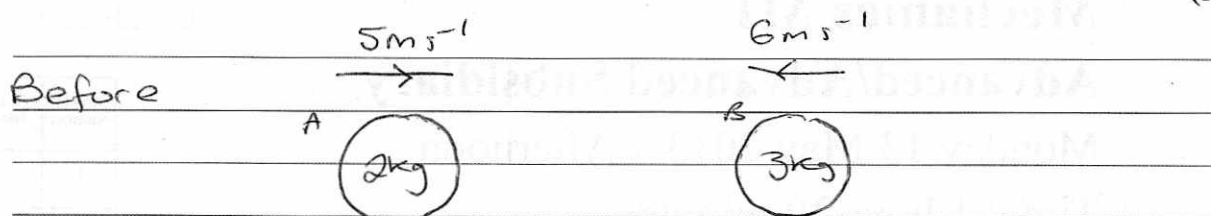


1. Two particles A and B , of mass 2 kg and 3 kg respectively, are moving towards each other in opposite directions along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately before the collision the speed of A is 5 m s^{-1} and the speed of B is 6 m s^{-1} . The magnitude of the impulse exerted on B by A is 14 N s . Find

(a) the speed of A immediately after the collision, (3)

(b) the speed of B immediately after the collision. (3)



a/

$$I = mv - mu$$

$$|14| = 2(v) - 2(5)$$

$$|14| = 2v - 10$$

$$-14 = 2v - 10$$

$$14 = 2v - 10$$

$$-4 = 2v$$

$$24 = 2v$$

$$v = -2\text{ m s}^{-1}$$

$$v = 12 \quad \times$$

$$\text{speed} = \underline{\underline{2\text{ m s}^{-1}}}$$

b/ $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$2(5) + 3(-6) = 2(-2) + 3(v_2)$$

$$10 - 18 = -4 + 3v_2$$

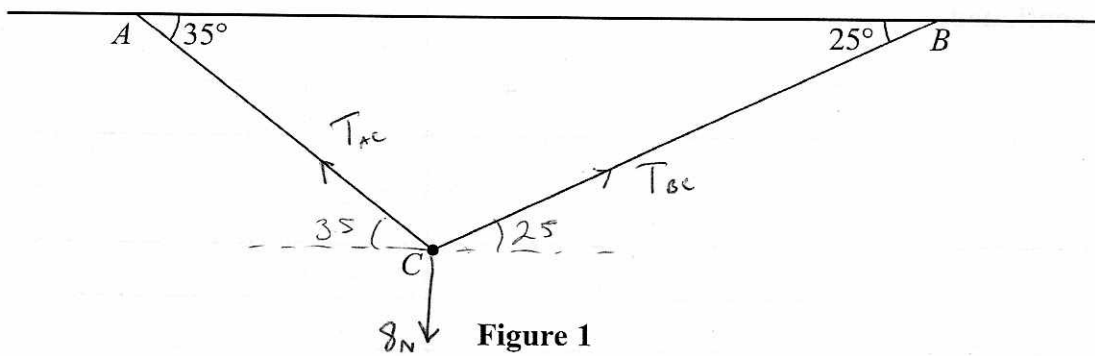
$$-4 = 3v_2$$

$$v_2 = \frac{-4}{3}\text{ m s}^{-1}$$

$$\text{Speed} = \underline{\underline{\frac{4}{3}\text{ m s}^{-1}}}$$



2.



A particle of weight 8 N is attached at C to the ends of two light inextensible strings AC and BC. The other ends, A and B, are attached to a fixed horizontal ceiling. The particle hangs at rest in equilibrium, with the strings in a vertical plane. The string AC is inclined at 35° to the horizontal and the string BC is inclined at 25° to the horizontal, as shown in Figure 1. Find

(i) the tension in the string AC,

(ii) the tension in the string BC.

(8)

← = →

$$2i) \quad T_{AC} \cos 35 = T_{BC} \cos 25$$

$$T_{AC} = \frac{T_{BC} \cos 25}{\cos 35}$$

$$\uparrow = \downarrow \quad T_{AC} \sin 35 + T_{BC} \sin 25 = 8$$

$$T_{BC} \frac{\cos 25}{\cos 35} \cdot \sin 35 + T_{BC} \sin 25 = 8$$

$$T_{BC} \left(\frac{\cos 25}{\cos 35} \cdot \sin 35 + \sin 25 \right) = 8$$

$$T_{BC} = 7.57 \text{ N (3sf)}$$

$$T_{AC} = \frac{7.57 \cos 25}{\cos 35}$$

$$= 8.37 \text{ N (3sf)}$$



3.

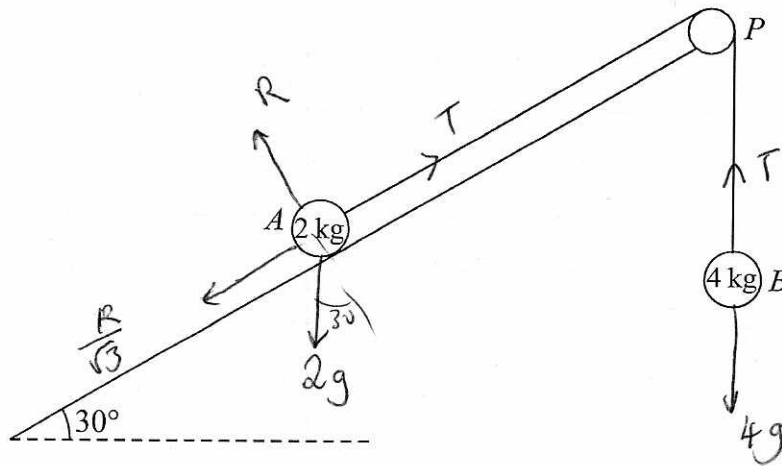


Figure 2

A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B , of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P . The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs freely below P , as shown in Figure 2. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

Find the tension in the string immediately after the particles are released.

(9)

$$A: R = 2g \cos 30$$

$$F_r = \frac{2g \cos 30}{\sqrt{3}} = g$$

$$A: F = ma$$

$$T - g - 2g \sin 30 = 2a$$

$$B: F = ma$$

$$4g - T = 4a \quad (1)$$

$$T - g - 2g \sin 30 = 2a$$

$$2T - 2g - 4g \sin 30 = 4a \quad (2)$$

$$4g - T = 2T - 2g - 4g \sin 30$$

$$8g = 3T$$

$$T = \frac{8}{3}g \text{ N}$$



4. At time $t = 0$, two balls A and B are projected vertically upwards. The ball A is projected vertically upwards with speed 2 m s^{-1} from a point 50 m above the horizontal ground. The ball B is projected vertically upwards from the ground with speed 20 m s^{-1} . At time $t = T$ seconds, the two balls are at the same vertical height, h metres, above the ground. The balls are modelled as particles moving freely under gravity. Find

(a) the value of T , (5)

(b) the value of h . (2)

A:	B:
$s = s - 50$	$s = s$
$u = 2$	$u = 20$
$v =$	$v =$
$a = -9.8$	$a = -9.8$
$t = T$	$t = T$

$s = ut + \frac{1}{2}at^2$	$s = ut + \frac{1}{2}at^2$
$s - 50 = 2(T) + \frac{1}{2}(-9.8)T^2$	$s = 20T + \frac{1}{2}(-9.8)T^2$
$s = 2T + \frac{1}{2}(-9.8)T^2 + 50$	

$$2T + \frac{1}{2}(-9.8)T^2 + 50 = 20T + \frac{1}{2}(-9.8)T^2$$

$$2T + 50 = 20T$$

$$50 = 18T$$

$$T = \frac{50}{18} \text{ seconds}$$

$$= \frac{25}{9} \text{ seconds}$$

$$= \underline{\underline{2.78 \text{ (3sf)} \text{ seconds}}}$$

b) $s = 20\left(\frac{25}{9}\right) + \frac{1}{2}(-9.8)\left(\frac{25}{9}\right)^2$

$$s = \underline{\underline{17.7 \text{ m}}} \quad \text{3sf}$$



5.

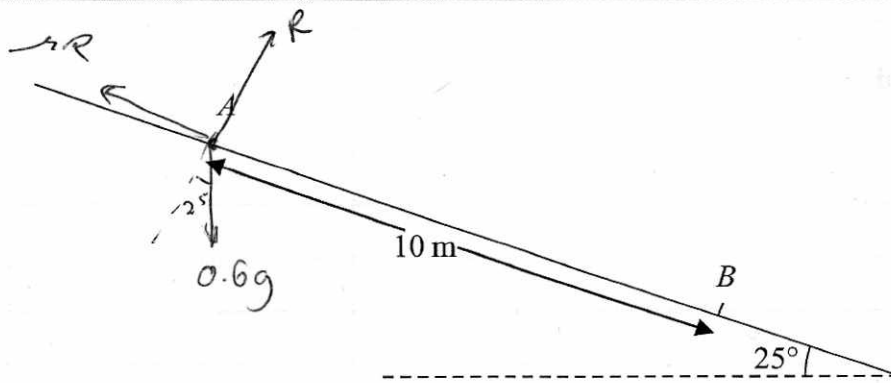


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B , where $AB = 10 \text{ m}$, as shown in Figure 3. The speed of P at A is 2 m s^{-1} . The particle P takes 3.5 s to move from A to B . Find

- (a) the speed of P at B , (3)
- (b) the acceleration of P , (2)
- (c) the coefficient of friction between P and the plane. (5)

5a/

$$s = 10$$

$$u = 2$$

$$v$$

$$a$$

$$t = 3.5$$

$$v^2 = u^2 + 2as$$

$$= (2)^2 + 2(a)(10)$$

$$\frac{v+u}{2} \cdot t = s$$

$$\frac{(v+2)(3.5)}{2} = 10$$

$$v = \frac{26}{1} \text{ m s}^{-1}$$

b/

$$v = u + at$$

$$\frac{26}{1} = 2 + a(3.5)$$

$$(3.71)(3.5)$$

$$a = \frac{24}{49} \text{ m s}^{-2}$$

$$(0.490) \text{ 3sr}$$



Question 5 continued

$$c/ \quad \uparrow = \downarrow$$

$$R = 0.6g \cos 25$$

$$F = ma.$$

$$0.6g \sin 25 - \mu (0.6g \cos 25) = 0.6 \left(\frac{24}{49} \right)$$

$$\frac{0.6g \sin 25 - 0.6 \left(\frac{24}{49} \right)}{0.6g \cos 25} = \mu$$

$$\mu = \underline{\underline{0.411}} \quad 3 \text{ sf}$$



6. [In this question \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively. Position vectors are given with respect to a fixed origin O .]

A ship S is moving with constant velocity $(3\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$. At time $t = 0$, the position vector of S is $(-4\mathbf{i} + 2\mathbf{j}) \text{ km}$.

(a) Find the position vector of S at time t hours. (2)

A ship T is moving with constant velocity $(-2\mathbf{i} + n\mathbf{j}) \text{ km h}^{-1}$. At time $t = 0$, the position vector of T is $(6\mathbf{i} + \mathbf{j}) \text{ km}$. The two ships meet at the point P .

(b) Find the value of n . (5)

(c) Find the distance OP . (4)

$$\begin{aligned} \text{a/} \quad \mathbf{r} &= \mathbf{r}_0 + \mathbf{v}t \\ &= (-4\mathbf{i} + 2\mathbf{j}) + t(3\mathbf{i} + 3\mathbf{j}) \end{aligned}$$

$$\begin{aligned} \text{b/} \quad \mathbf{r}_T &= \mathbf{r}_0 + \mathbf{v}t \\ &= (6\mathbf{i} + \mathbf{j}) + t(-2\mathbf{i} + n\mathbf{j}) \end{aligned}$$

ships meet $\mathbf{r}_S = \mathbf{r}_T$

$$(-4\mathbf{i} + 2\mathbf{j}) + t(3\mathbf{i} + 3\mathbf{j}) = (6\mathbf{i} + \mathbf{j}) + t(-2\mathbf{i} + n\mathbf{j})$$

$$\begin{aligned} \text{i//} \quad -4 + 3t &= 6 - 2t \\ 5t &= 10 \\ t &= 2. \end{aligned}$$

$$\begin{aligned} \text{ii//} \quad 2 + 3t &= 1 + nt && [t=2] \\ 2 + 3(2) &= 1 + 2n \\ 8 &= 1 + 2n \\ 7 &= 2n \\ n &= \underline{\underline{7/2}} \end{aligned}$$



Question 6 continued

c/

$$p = -4i + 2j + 2(3i + 3j)$$

$$= -2i + 8j$$

$$|\vec{OP}| = \sqrt{2^2 + 8^2}$$

$$= \underline{\underline{\sqrt{68}}} \quad (8.25 \text{ km } 35^\circ)$$



7.

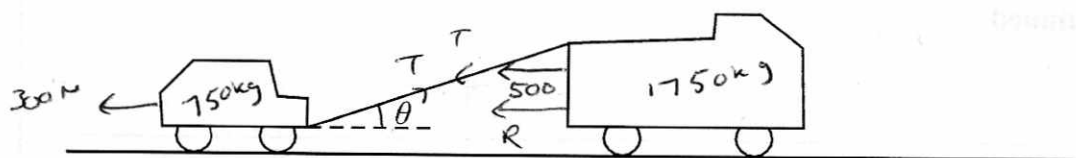


Figure 4

A truck of mass 1750 kg is towing a car of mass 750 kg along a straight horizontal road. The two vehicles are joined by a light towbar which is inclined at an angle θ to the road, as shown in Figure 4. The vehicles are travelling at 20 m s^{-1} as they enter a zone where the speed limit is 14 m s^{-1} . The truck's brakes are applied to give a constant braking force on the truck. The distance travelled between the instant when the brakes are applied and the instant when the speed of each vehicle is 14 m s^{-1} is 100 m.

- (a) Find the deceleration of the truck and the car. (3)

The constant braking force on the truck has magnitude R newtons. The truck and the car also experience constant resistances to motion of 500 N and 300 N respectively. Given that $\cos \theta = 0.9$, find

- (b) the force in the towbar, (4)

- (c) the value of R . (4)

a/ $s = 100$
 $u = 20$
 $v = 14$
 $a = ?$
 $t =$

$$v^2 = u^2 + 2as$$

$$(14)^2 = (20)^2 + 2(a)(100)$$

$$a = -1.02 \text{ m s}^{-2}$$

b/ Car : $F = ma$
 ~~$T = 300$~~
 $T(\cos \theta) - 300 = 750(-1.02)$
 $T(0.9) = -465$
 $T = -517 \text{ N (3sf)}$



Question 7 continued

c/ Truck $F=ma$

$$-500 - R - T \cos \theta = 1750(-1.02)$$

$$-500 - R + 465 = -1785$$

$$\underline{\underline{R = 1750 \text{ N}}}$$



8.

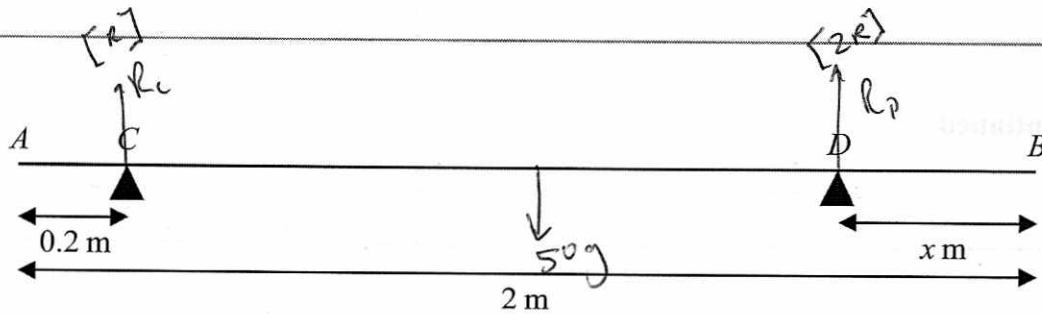


Figure 5

A uniform rod AB has length 2 m and mass 50 kg. The rod is in equilibrium in a horizontal position, resting on two smooth supports at C and D , where $AC = 0.2$ metres and $DB = x$ metres, as shown in Figure 5. Given that the magnitude of the reaction on the rod at D is twice the magnitude of the reaction on the rod at C ,

(a) find the value of x .

(6)

The support at D is now moved to the point E on the rod, where $EB = 0.4$ metres. A particle of mass m kg is placed on the rod at B , and the rod remains in equilibrium in a horizontal position. Given that the magnitude of the reaction on the rod at E is four times the magnitude of the reaction on the rod at C ,

(b) find the value of m .

(7)

a/ Forces up = Forces down

$$3R = 50g$$

$$R = \frac{50}{3}g$$

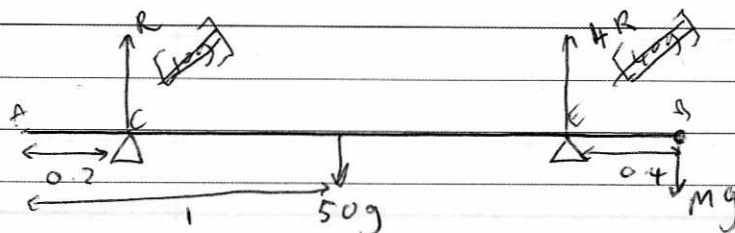
Taking moments about B:

$$x \times 2 \left(\frac{50}{3}g \right) + 1.8 \left(\frac{50}{3}g \right) = 1 (50g)$$

$$x = \frac{50g - 30g}{\frac{100}{3}g}$$

$$= 0.6 \text{ m}$$

b/



Question 8 continued

Forces up = forces down

$$5R = 50g + mg$$

$$R = \frac{50g + mg}{5}$$

Taking moments about C:

$$0.8(50g) + 1.8mg = 1.4 \left(4 \frac{50g + mg}{5} \right)$$

$$40g + 1.8mg = \frac{28}{25} (50g + mg)$$

$$40g + 1.8mg = 56g + \frac{28}{25}mg$$

$$\frac{17}{25}mg = 16g$$

$$m = 23.5 \text{ kg (3sf)}$$

