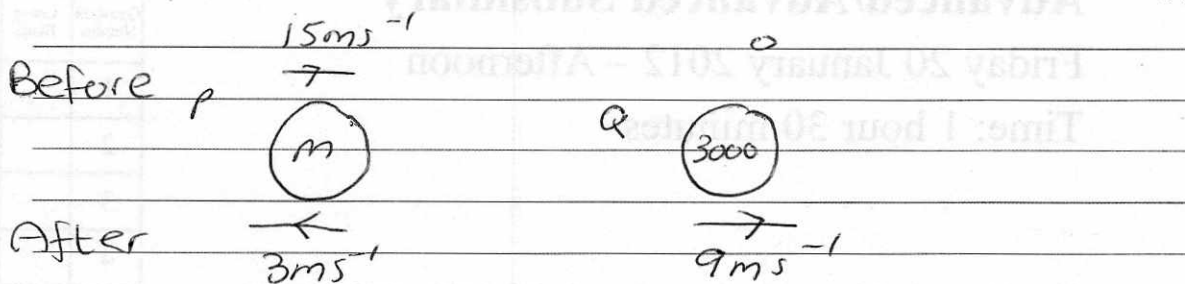


1. A railway truck P , of mass m kg, is moving along a straight horizontal track with speed 15 m s^{-1} . Truck P collides with a truck Q of mass 3000 kg, which is at rest on the same track. Immediately after the collision the speed of P is 3 m s^{-1} and the speed of Q is 9 m s^{-1} . The direction of motion of P is reversed by the collision.

Modelling the trucks as particles, find

- (a) the magnitude of the impulse exerted by P on Q , (2)
- (b) the value of m . (3)



a/

$$I = mv - mu$$

$$= 3000(9) - 3000(0)$$

$$= \underline{\underline{27000 \text{ N s}}}$$

b/

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m(15) + 3000(0) = m(-3) + 3000(9)$$

$$15m = -3m + 27000$$

$$18m = 27000$$

$$m = \underline{\underline{1500 \text{ kg}}}$$

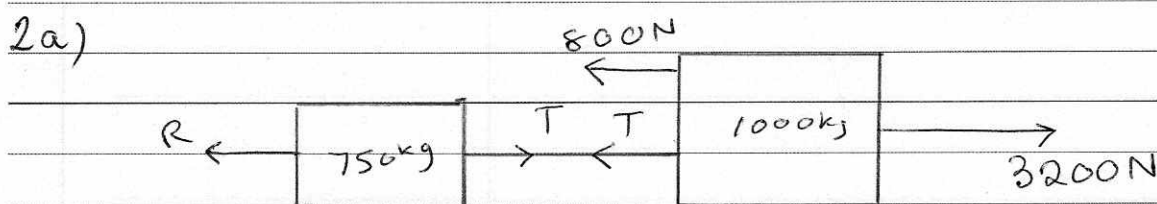


2. A car of mass 1000 kg is towing a caravan of mass 750 kg along a straight horizontal road. The caravan is connected to the car by a tow-bar which is parallel to the direction of motion of the car and the caravan. The tow-bar is modelled as a light rod. The engine of the car provides a constant driving force of 3200 N. The resistances to the motion of the car and the caravan are modelled as constant forces of magnitude 800 newtons and R newtons respectively.

Given that the acceleration of the car and the caravan is 0.88 ms^{-2} ,

(a) show that $R=860$, (3)

(b) find the tension in the tow-bar. (3)



whole system

$$3200 - 800 - R = 1750(0.88)$$

$$3200 - 800 - R = 1540$$

$$R = 3200 - 800 - 1540$$

$$= \underline{\underline{860 \text{ N}}}$$

b/ Caravan:

$$T - 860 = 750(0.88)$$

$$T - 860 = 660$$

$$T = \underline{\underline{1520 \text{ N}}}$$



3. Three forces F_1 , F_2 and F_3 acting on a particle P are given by

$$F_1 = (7i - 9j) \text{ N}$$

$$F_2 = (5i + 6j) \text{ N}$$

$$F_3 = (pi + qj) \text{ N}$$

where p and q are constants.

Given that P is in equilibrium,

(a) find the value of p and the value of q .

(3)

The force F_3 is now removed. The resultant of F_1 and F_2 is R .
Find

(b) the magnitude of R ,

(2)

(c) the angle, to the nearest degree, that the direction of R makes with j .

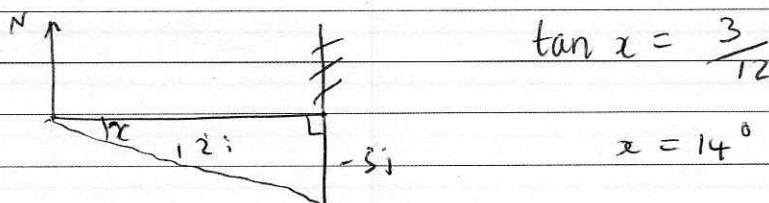
(3)

$$\begin{aligned} 3a) \quad i// \quad 7 + 5 + p &= 0 \\ & \underline{p = -12} \\ j// \quad -9 + 6 + q &= 0 \\ & \underline{q = 3} \end{aligned}$$

$$b/ \quad F_1 + F_2 = 12i - 3j$$

$$\sqrt{12^2 + 3^2} = \underline{\underline{3\sqrt{17}}}$$

c/



$$90 + 14 = \underline{\underline{104}} \text{ (nearest degree)}$$



4.

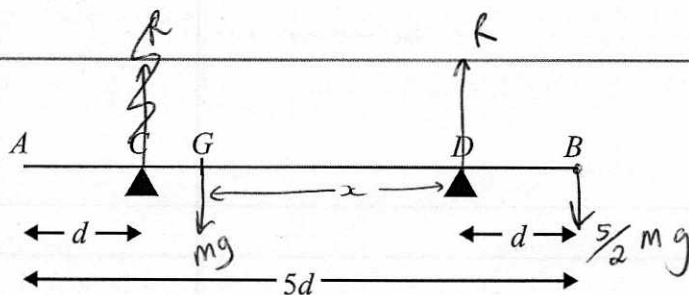


Figure 1

A non-uniform rod AB , of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D , where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G . A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D . $\rightarrow R_C = 0$

(a) Show that $GD = \frac{5}{2}d$. (4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod. (5)

a/ ~~Let $CG = x$ and $GD = 3d - x$~~

~~Taking moments about C:~~

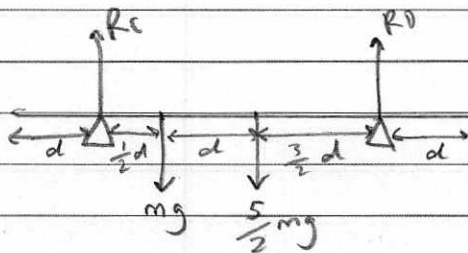
~~$4d \left(\frac{5}{2}mg \right) = x$~~

Taking moments about D:

$d \left(\frac{5}{2}mg \right) = x (mg)$

$x = \frac{5}{2}d$

b/



Taking moments about C:



Question 4 continued

$$\frac{1}{2} \text{ d mg} + \frac{3}{2} \text{ d} \left(\frac{5}{2} \text{ mg} \right) = 3 \text{ d R}_0.$$

$$\frac{1}{2} \text{ mg} + \frac{15}{4} \text{ mg} = 3 \text{ R}_0$$

$$\frac{17}{4} \text{ mg} = 3 \text{ R}_0$$

$$\text{R mg} = \frac{17}{12} \text{ mg}$$

$$\frac{m = \frac{17}{12} \text{ mg}}{12 \text{ g}}$$



5. A stone is projected vertically upwards from a point A with speed $u \text{ m s}^{-1}$. After projection the stone moves freely under gravity until it returns to A . The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$, (3)

(b) find the greatest height above A reached by the stone, (2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A . (6)

a/ $s = 0$

$u = ?$

$v =$

$a = -9.8$

$t = 3\frac{4}{7}$

$$s = ut + \frac{1}{2} at^2$$

$$0 = u(3\frac{4}{7}) + \frac{1}{2}(-9.8)(3\frac{4}{7})^2$$

$$= \frac{25}{7}u - \frac{125}{2}$$

$$\frac{125}{2} = \frac{25}{7}u$$

$$u = 17.5 \text{ m s}^{-1}$$

b/ $s = ?$

$u = 17.5$

$v = 0$

$a = -9.8$

t

$$v^2 = u^2 + 2as$$

$$0 = (17.5)^2 + 2(-9.8)s$$

$$s = 15.6 \text{ m (3sf)}$$



Question 5 continued

$$\begin{aligned}
 c/ \quad s &= 6.6 \\
 u &= 17.5 \\
 v &= \\
 a &= -9.8 \\
 t &= ?
 \end{aligned}$$

$$s = ut + \frac{1}{2} at^2$$

$$6.6 = \cancel{17.5t} + \frac{1}{2}(-9.8)(t)^2$$

$$\cancel{6.6} = \cancel{4.9t^2}$$

$$6.6 = 17.5t - 4.9t^2$$

$$4.9t^2 - 17.5t + 6.6 = 0$$

$$a = 4.9 \quad b = -17.5 \quad c = 6.6$$

$$t = \frac{-(-17.5) \pm \sqrt{(-17.5)^2 - 4(4.9)(6.6)}}{2(4.9)}$$

$$= \frac{22}{7} \quad \text{and} \quad \frac{3}{7}$$

$$= \frac{22}{7} - \frac{3}{7} = \frac{19}{7} \text{ seconds.}$$

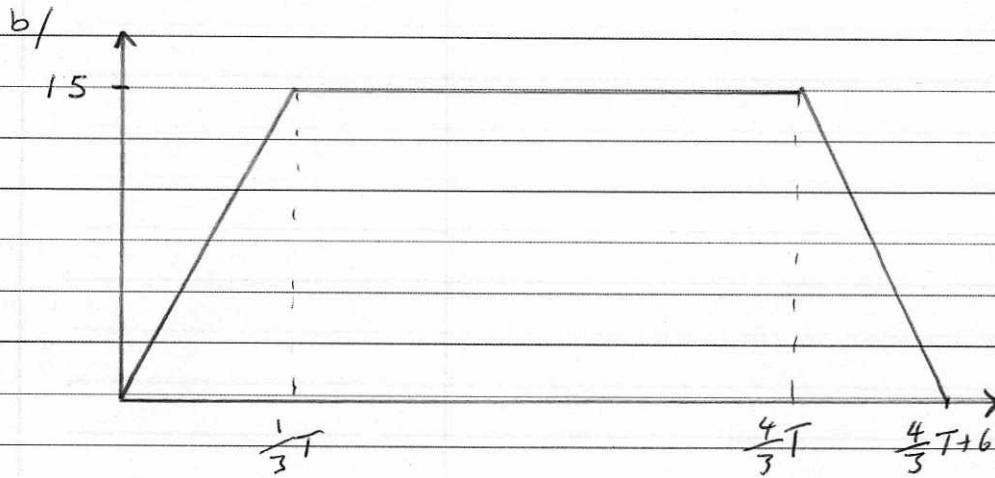
$$= \underline{\underline{2.71 \text{ seconds (5s)}}}$$



6. A car moves along a straight horizontal road from a point A to a point B , where $AB = 885$ m. The car accelerates from rest at A to a speed of 15 ms^{-1} at a constant rate $a \text{ ms}^{-2}$. The time for which the car accelerates is $\frac{1}{3}T$ seconds. The car maintains the speed of 15 ms^{-1} for T seconds. The car then decelerates at a constant rate of 2.5 ms^{-2} stopping at B .

- (a) Find the time for which the car decelerates. (2)
- (b) Sketch a speed-time graph for the motion of the car. (2)
- (c) Find the value of T . (4)
- (d) Find the value of a . (2)
- (e) Sketch an acceleration-time graph for the motion of the car. (3)

6a/ $\frac{15}{2.5} = 6$ seconds



c/ $15 \times \frac{\frac{1}{3}T + \frac{4}{3}T + 6}{2} = 885$

$\frac{5}{3}T + 6 = 59$

$\frac{5}{3}T = 53$

$\frac{5}{3}T = 53$

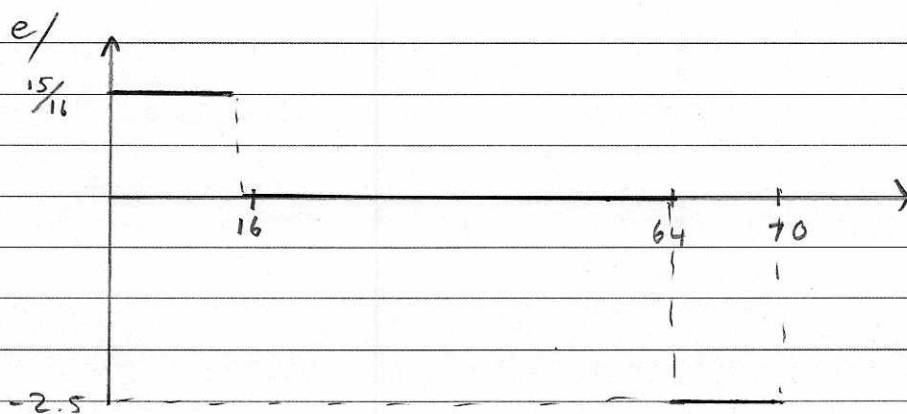
$T = \underline{\underline{48 \text{ seconds}}}$



Question 6 continued

$$e/d) \quad a = \frac{15}{\frac{1}{3}(48)}$$

$$= \frac{15}{16} = \underline{\underline{0.94 \text{ ms}^{-2}}}$$



7. [In this question, the unit vectors \mathbf{i} and \mathbf{j} are due east and due north respectively. Position vectors are relative to a fixed origin O .]

A boat P is moving with constant velocity $(-4\mathbf{i} + 8\mathbf{j}) \text{ km h}^{-1}$.

- (a) Calculate the speed of P .

(2)

When $t = 0$, the boat P has position vector $(2\mathbf{i} - 8\mathbf{j}) \text{ km}$. At time t hours, the position vector of P is $\mathbf{p} \text{ km}$.

- (b) Write down \mathbf{p} in terms of t .

(1)

A second boat Q is also moving with constant velocity. At time t hours, the position vector of Q is $\mathbf{q} \text{ km}$, where

$$\mathbf{q} = 18\mathbf{i} + 12\mathbf{j} - t(6\mathbf{i} + 8\mathbf{j})$$

Find

- (c) the value of t when P is due west of Q ,

(3)

- (d) the distance between P and Q when P is due west of Q .

(3)

$$7a) \quad \sqrt{4^2 + 8^2} = \underline{\underline{8.94 \text{ ms}^{-1}}} \quad (3 \text{ sf})$$

b/

$$\mathbf{p} = 2\mathbf{i} - 8\mathbf{j} + t(-4\mathbf{i} + 8\mathbf{j})$$

c/ due west. $\mathbf{j} = 0$

$$-8 + 8t = 12 - 8t$$

$$16t = 20$$

$$t = \frac{5}{4}$$

$$\begin{aligned} d/ \quad \mathbf{p} &= 2\mathbf{i} - 8\mathbf{j} + \frac{5}{4}(-4\mathbf{i} + 8\mathbf{j}) \\ &= 2\mathbf{i} - 8\mathbf{j} - 5\mathbf{i} + 10\mathbf{j} \\ &= -3\mathbf{i} + 2\mathbf{j} \end{aligned}$$

$$\begin{aligned} \mathbf{q} &= 18\mathbf{i} + 12\mathbf{j} - \frac{5}{4}(6\mathbf{i} + 8\mathbf{j}) \\ &= \frac{21}{2}\mathbf{i} + 2\mathbf{j} \end{aligned}$$



Question 7 continued

$$d = \frac{27}{2} \dots 3$$

$$= \frac{27}{2} = 13.5 \text{ km}$$



8.

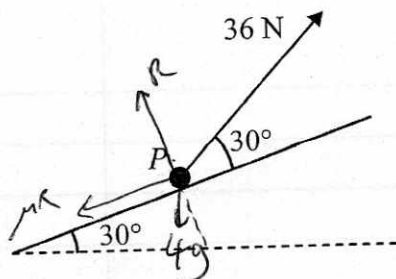


Figure 2

A particle P of mass 4 kg is moving up a fixed rough plane at a constant speed of 16 ms^{-1} under the action of a force of magnitude 36 N . The plane is inclined at 30° to the horizontal. The force acts in the vertical plane containing the line of greatest slope of the plane through P , and acts at 30° to the inclined plane, as shown in Figure 2. The coefficient of friction between P and the plane is μ . Find

(a) the magnitude of the normal reaction between P and the plane, (4)

(b) the value of μ . (5)

The force of magnitude 36 N is removed.

(c) Find the distance that P travels between the instant when the force is removed and the instant when it comes to rest. (5)

a) Resolving perp to plane:

$$R + 36 \sin 30 = 4g \cos 30$$

$$R = 4g \cos 30 - 36 \sin 30$$

$$= \underline{15.9\text{ N (3sf)}}$$

by Resolving parallel to plane:

$$36 \cos 30 = 4g \sin 30 + \mu(15.9)$$

$$\mu = \frac{36 \cos 30 - 4g \sin 30}{15.9}$$

$$= \underline{0.726\text{ 3sf}}$$



Question 8 continued

$$s = ?$$

$$u = 16$$

$$v = 0$$

$$a = +26 \rightarrow -11.1$$

$$t$$

$$F = ma$$

$$0 = 4g \sin$$

$$[R = 4g \cos 30]$$

$$0 - 0.76(4g \cos 30) - 4g \sin 30 = 4a$$

$$a = -1.26 (3st)$$

$$a = -11.1 \text{ ms}^{-2}$$

$$v^2 = u^2 + 2as$$

$$0 = 16^2 + 2(-11.1)s$$

$$s = 11.6 \text{ m}$$

