Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1 R	Signature	

Paper Reference(s)

6666/01R

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

1. Express in partial fractions

$$\frac{5x+3}{(2x+1)(x+1)^2}$$

(4)

Leave blank

$$\frac{5x+3}{(2x+1)(x+1)^2} = \frac{A}{2x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$5x+3 = A(x+1)^2 + B(2x+1)(x+1) + C(2x+1)$$

Let $x = -1$

$-2 = -C$

$C = 2$

Let $x = -\frac{1}{2}$

$\frac{1}{2} = \frac{1}{4}A$

$A = 2$

Let $x = 0$

$3 = A + B + C$

$3 = 2 + B + 2$

$B = -1$

$$\frac{2}{2x+1} - \frac{1}{x+1} + \frac{2}{(x+1)^2}$$

Leave blank

2. The curve C has equation

$$3^{x-1} + \underset{u}{x}\underset{v}{y} - y^2 + 5 = 0$$

Show that $\frac{dy}{dx}$ at the point (1, 3) on the curve C can be written in the form $\frac{1}{\lambda}\ln(\mu e^3)$,

where λ and μ are integers to be found.

$\frac{du}{dx} = 1 \quad \frac{dv}{dx} = \frac{dy}{dx}$

(7)

2)

$$3^{x-1}\ln 3 + x\frac{dy}{dx} + y - 2y\frac{dy}{dx} = 0$$

$\underset{x}{(1,}\underset{y}{3)}$

$$\ln 3 + \frac{dy}{dx} + 3 - 6\frac{dy}{dx} = 0$$

$$3 + \ln 3 = 5\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{3}{5} + \frac{1}{5}\ln 3$$

$$= \frac{1}{5}(3 + \ln 3)$$

$$= \frac{1}{5}(3\ln e + \ln 3)$$

$$= \frac{1}{5}(\ln e^3 + \ln 3)$$

$$= \frac{1}{5}(\ln 3e^3)$$

$$\underline{\underline{\lambda = 5}} \quad \underline{\underline{\mu = 3}}$$

Leave blank

3. Using the substitution $u = 2 + \sqrt{(2x + 1)}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2 + \sqrt{(2x + 1)}} \, \mathrm{d}x$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

$u = 2 + (2x+1)^{1/2}$

$\frac{du}{dx} = (2x+1)^{-1/2}$

when $x = 4$ when $x = 0$

$u = 2 + \sqrt{2(4)+1}$ $u = 2 + \sqrt{2(0)+1}$

$= 5$ $= 3$

$\int_3^5 \frac{1}{u} \frac{dx}{du} \, du$

$\int_3^5 \frac{1}{u}(2x+1)^{1/2} \, du$ $\boxed{\frac{dx}{du} = \frac{1}{(2x+1)^{-1/2}}}$

$\int_3^5 \frac{1}{u}(u-2) \, du$ $\boxed{u-2 = \sqrt{2x+1}}$

$\int_3^5 \frac{u-2}{u} \, du$

$\int_3^5 1 - \frac{2}{u} \, du$

$\left[u - 2\ln u \right]_3^5$

$(5 - 2\ln 5) - (3 - 2\ln 3)$

$2 - 2\ln 5 + 2\ln 3$

$2 + 2(\ln 3 - \ln 5) = 2 + 2\ln\left(\frac{3}{5}\right)$

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4. (a) Find the binomial expansion of

$$\sqrt[3]{(8-9x)}, \qquad |x| < \frac{8}{9}$$

in ascending powers of x, up to and including the term in x^3. Give each coefficient as a simplified fraction.

(6)

(b) Use your expansion to estimate an approximate value for $\sqrt[3]{7100}$, giving your answer to 4 decimal places. State the value of x, which you use in your expansion, and show all your working.

(3)

$$(8-9x)^{\frac{1}{3}}$$

$$8^{\frac{1}{3}}\left(1-\frac{9}{8}x\right)^{\frac{1}{3}}$$

$$2\left(1-\frac{9}{8}x\right)^{1/3}$$

$$2\left(1+\frac{1}{3}\left(-\frac{9}{8}x\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2}\left(-\frac{9}{8}x\right)^2+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{5}{3}\right)}{6}\left(-\frac{9}{8}x\right)^3\right)$$

$$2\left(1 \cancel{+} -\frac{3}{8}x-\frac{9}{64}x^2-\frac{45}{512}x^3\right)$$

$$2-\frac{3}{4}x-\frac{9}{32}x^2-\frac{45}{256}x^3$$

b/ $\boxed{x=0.1}$

$$\sqrt[3]{7.1} = 2-\frac{3}{4}(0.1)-\frac{9}{32}(0.1)^2-\frac{45}{256}(0.1)^3$$

$$= 1.922011719$$

$$\sqrt[3]{7.1} \times \sqrt[3]{1000} = \sqrt{7100}$$

$$\sqrt[3]{7.1} \times 10 = \sqrt{7100}$$

$$\sqrt[3]{7100} = 1.922011719 \times 10$$

$$= 19.22011719$$

$$= 19.2201\cancel{2} \quad 4dp$$

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5.

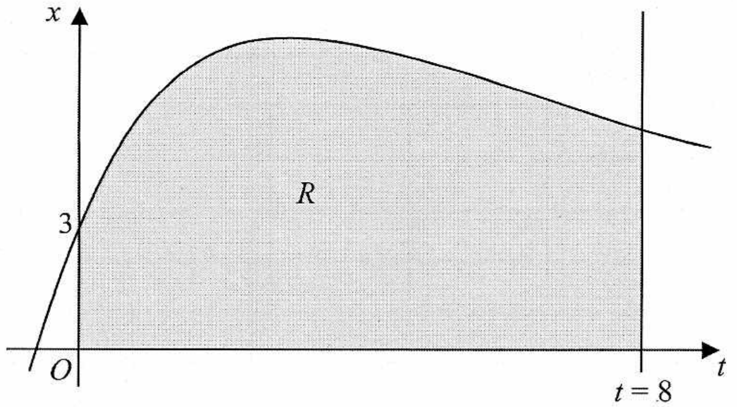

Figure 1

Figure 1 shows part of the curve with equation $x = 4te^{-\frac{1}{3}t} + 3$. The finite region R shown shaded in Figure 1 is bounded by the curve, the x-axis, the t-axis and the line $t = 8$.

(a) Complete the table with the value of x corresponding to $t = 6$, giving your answer to 3 decimal places.

t	0	2	4	6	8
x	3	7.107	7.218	6.248	5.223

(1)

(b) Use the trapezium rule with all the values of x in the completed table to obtain an estimate for the area of the region R, giving your answer to 2 decimal places.

(3)

(c) Use calculus to find the exact value for the area of R.

(6)

(d) Find the difference between the values obtained in part (b) and part (c), giving your answer to 2 decimal places.

(1)

b/ $2\left(\frac{3}{2} + 7.107 + 7.218 + 6.248 + \frac{5.223}{2}\right)$

$= 49.37$ units^2

c/ $\int_0^8 4te^{-\frac{1}{3}t} + 3 \; dt$

$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$

P 4 2 9 5 4 A 0 1 4 2 8

Question 5 continued

$\frac{dv}{dt} = e^{-\frac{1}{3}t}$ $\quad u = 4t$

$v = -3e^{-\frac{1}{3}t}$ $\quad \frac{du}{dt} = 4$

$-12te^{-\frac{1}{3}t} - \int -12e^{-\frac{1}{3}t}\,dt$

$-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} + c$

$\left[-12te^{-\frac{1}{3}t} - 36e^{-\frac{1}{3}t} + 3t\right]_0^8$

$\left(-96e^{-8/3} - 36e^{-8/3} + 24\right) - (-36)$

$-132e^{-8/3} + 60$ units^2

d/ $-13e^{-8/3} + 60 - 49.37$

$= \underline{\underline{1.46}}$ units^2

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6. Relative to a fixed origin O, the point A has position vector $21\mathbf{i} - 17\mathbf{j} + 6\mathbf{k}$ and the point B has position vector $25\mathbf{i} - 14\mathbf{j} + 18\mathbf{k}$.

The line l has vector equation

$$\mathbf{r} = \begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix}$$

where a, b and c are constants and λ is a parameter.

Given that the point A lies on the line l,

(a) find the value of a.

(3)

Given also that the vector $\overrightarrow{AB}$ is perpendicular to l,

(b) find the values of b and c,

(5)

(c) find the distance AB.

(2)

The image of the point B after reflection in the line l is the point B'.

(d) Find the position vector of the point B'.

(2)

a/ $\begin{pmatrix} a \\ b \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} 6 \\ c \\ -1 \end{pmatrix} = \begin{pmatrix} 21 \\ -17 \\ 6 \end{pmatrix}$

k// $10 - \lambda = 6$

$\lambda = 4$

i// $a + 6(4) = 21$

$\underline{\underline{a = -3}}$

b/ perpendicular $a.b = 0$

$\overrightarrow{AB} = \begin{pmatrix} 4 \\ 3 \\ 12 \end{pmatrix}$ $\quad 4(6) + 3(c) + 12(-1) = 0$

$24 + 3c - 12 = 0$

$3c = -12$

$\underline{\underline{c = -4}}$

Leave blank

Question 6 continued

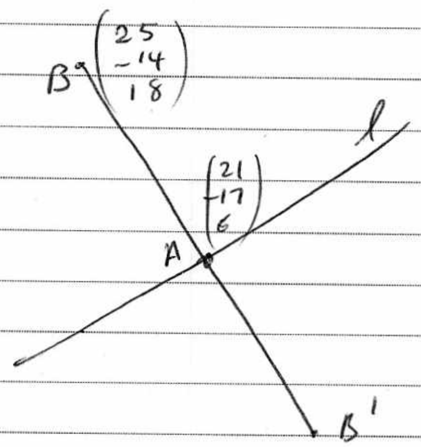
ii/ $b + 4(-4) = -17$

$b + -16 = -17$

$\underline{b = -1}$

c/ $\sqrt{4^2 + 3^2 + 12^2} = \underline{13 \text{ units}}$

d/



$B' = \begin{pmatrix} 17 \\ -20 \\ -6 \end{pmatrix}$

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7.

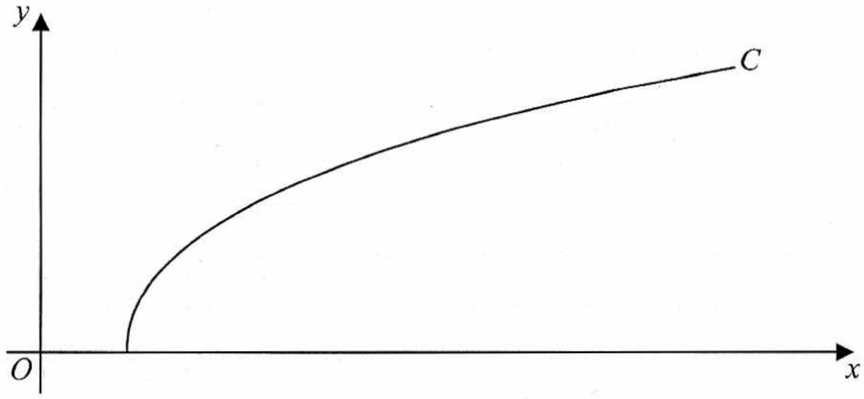

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 27\sec^3 t, \;\; y = 3\tan t, \qquad 0 \leqslant t \leqslant \frac{\pi}{3}$$

(a) Find the gradient of the curve C at the point where $t = \dfrac{\pi}{6}$

(4)

(b) Show that the cartesian equation of C may be written in the form

$$y = (x^{\frac{2}{3}} - 9)^{\frac{1}{2}}, \qquad\qquad a \leqslant x \leqslant b$$

stating the values of a and b.

(3)

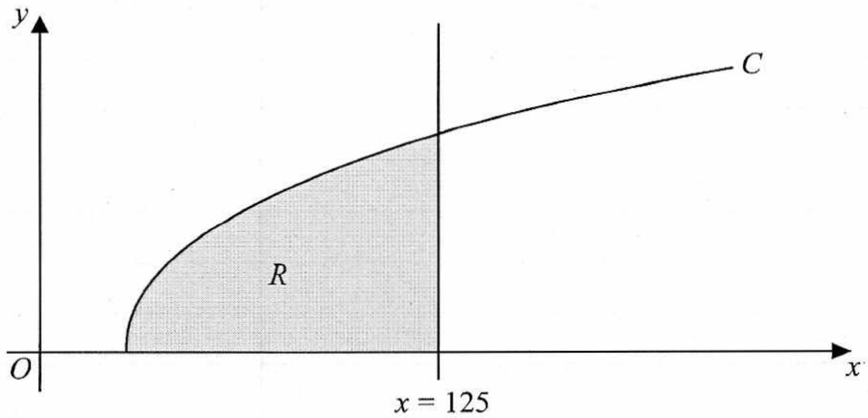

Figure 3

The finite region R which is bounded by the curve C, the x-axis and the line $x = 125$ is shown shaded in Figure 3. This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(c) Use calculus to find the exact value of the volume of the solid of revolution.

(5)

Leave blank

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Leave blank

Question 7 continued

a/ $x = 27(\sec t)^3$

$\frac{dx}{dt} = 81(\sec^2 t)(\sec t)(\tan t)$

$y = 3\tan t$

$\frac{dy}{dt} = 3\sec^2 t$

$\frac{dy}{dx} = \frac{3\sec^2 t}{81(\sec^2 t)(\sec t)(\tan t)}$

$= \frac{1}{27\sec t\tan t}$

$= \frac{\cos t}{27\tan t}$ when $t = \pi/6$

$= \frac{1}{18}$

b/ $x = 27\sec^3 t$

$x^{1/3} = 3\sec t$

$x^{2/3} = 9\sec^2 t$

$y = 3\tan t$

$y^2 = 9\tan^2 t$

$y^2 = 9(\sec^2 t - 1)$

$y^2 = 9\sec^2 t - 9$

$y^2 + 9 = 9\sec^2 t$

$x^{2/3} = y^2 + 9$

$y^2 = x^{2/3} - 9$

$y = (x^{2/3} - 9)^{1/2}$

when $t = 0$ $x = 27$ $a = 27$

$t = \pi/3$ $x = 216$ $b = 216$

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Question 7 continued

Crosses x when $y=0$

$0 = 3\tan t$

$0 = \tan t$

$t = 0$

$x = \dfrac{27}{(\cos 0)^3}$

$= 27$

$\pi \int_{27}^{125} x^{\frac{2}{3}} - 9 \quad dx$ $\quad \boxed{\pi \int y^2 \, dx}$

$\pi \left[\frac{3}{5} x^{\frac{5}{3}} - 9x \right]_{27}^{125}$

$\pi \left[750 - - \frac{486}{5} \right)$

$= \underline{\underline{847.2\pi}} \text{ units}^3$

(Total 12 marks)

Q7

P 4 2 9 5 4 A 0 2 5 2 8

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8. In an experiment testing solid rocket fuel, some fuel is burned and the waste products are collected. Throughout the experiment the sum of the masses of the unburned fuel and waste products remains constant.

Let x be the mass of waste products, in kg, at time t minutes after the start of the experiment. It is known that at time t minutes, the rate of increase of the mass of waste products, in kg per minute, is k times the mass of unburned fuel remaining, where k is a positive constant.

The differential equation connecting x and t may be written in the form

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k(M - x), \text{ where } M \text{ is a constant.}$$

(a) Explain, in the context of the problem, what $\frac{\mathrm{d}x}{\mathrm{d}t}$ and M represent. **(2)**

Given that initially the mass of waste products is zero,

(b) solve the differential equation, expressing x in terms of k, M and t. **(6)**

Given also that $x = \frac{1}{2}M$ when $t = \ln 4$,

(c) find the value of x when $t = \ln 9$, expressing x in terms of M, in its simplest form. **(4)**

a/ $\frac{dx}{dt}$ is the rate of change in mass of waste products

M is the mass of unburned fuel at $t=0$

b/ $t0$ ~~M~~ $x=0$

$$\int \frac{1}{M-x}\,dx = \int k\,dt$$

$$-\ln(M-x) = kt + C$$

$[t=0\ x=0]$ $\quad -\ln M = C$

$$-\ln(M-x) = kt - \ln M$$

Leave blank

Question 8 continued

$$\ln M - \ln(M-x) = kt$$

$$\ln\left(\frac{M}{M-x}\right) = kt$$

$$\frac{M}{M-x} = e^{kt}$$

$$M = (M-x)e^{kt}$$

$$M = Me^{kt} - xe^{kt}$$

$$xe^{kt} = Me^{kt} - M$$

$$x = \frac{Me^{kt} - M}{e^{kt}}$$

c) $x = \frac{1}{2}M \quad t = \ln 4$

$$\frac{1}{2}M = \frac{Me^{k\ln 4} - M}{e^{k\ln 4}}$$

$$\boxed{e^{\ln 4^{k}}}$$

$$\frac{1}{2}Me^{k\ln 4} = Me^{k\ln 4} - M$$

$$\frac{1}{2}e^{k\ln 4} = e^{k\ln 4} - 1$$

$$1 = \frac{1}{2}e^{k\ln 4}$$

$$1 = \frac{1}{2}\cdot 4^{k}$$

$$2 = 4^{k}$$

$$k = 1/2$$

$$x = \frac{Me^{\frac{1}{2}\ln 9} - M}{e^{\frac{1}{2}\ln 9}} \qquad k = 1/2 \quad t = \ln 9$$

$$= \frac{3m - m}{3} \qquad \boxed{e^{\ln 9^{1/2}} = e^{\ln 3} = 3}$$

$$= \underline{\underline{\frac{2}{3}M}}$$

P 4 2 9 5 4 A 0 2 7 2 8