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Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 16 June 2015 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Find the binomial expansion of

$$(4+5x)^{\frac{1}{2}}, \quad |x| < \frac{4}{5}$$

in ascending powers of x, up to and including the term in x^2.
Give each coefficient in its simplest form.

(5)

(b) Find the exact value of $(4+5x)^{\frac{1}{2}}$ when $x = \frac{1}{10}$

Give your answer in the form $k\sqrt{2}$, where k is a constant to be determined.

(1)

(c) Substitute $x = \frac{1}{10}$ into your binomial expansion from part (a) and hence find an approximate value for $\sqrt{2}$

Give your answer in the form $\frac{p}{q}$ where p and q are integers.

(2)

1a) $4^{\frac{1}{2}}\left(1+\frac{5}{4}x\right)^{1/2}$

$2\left(1+\frac{5}{4}x\right)^{1/2}$

$2\left(1+\frac{1}{2}\left(\frac{5}{4}x\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}\left(\frac{5}{4}x\right)^2\right)$

$= 2\left(1+\frac{5}{8}x-\frac{25}{128}x^2\right)$

$= 2+\frac{5}{4}x-\frac{25}{64}x^2$

b/ $\left(4+5\left(\frac{1}{10}\right)\right)^{\frac{1}{2}}$

$\left(\frac{9}{2}\right)^{\frac{1}{2}} = \frac{3}{\sqrt{2}}$

$= \frac{3\sqrt{2}}{2}$ $\quad k = 3/2$

c/ $\frac{3}{2}\sqrt{2} = 2+\frac{5}{4}\left(\frac{1}{10}\right)-\frac{25}{64}\left(\frac{1}{10}\right)^2$

$\frac{3}{2}\sqrt{2} = \frac{543}{256}$

$\sqrt{2} = \frac{181}{128}$

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2. The curve C has equation

$$x^2 - 3xy - 4y^2 + 64 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$

(Solutions based entirely on graphical or numerical methods are not acceptable.)

(6)

$u = -3x \quad v = y$

$\frac{du}{dx} = -3 \quad \frac{dv}{dx} = \frac{dy}{dx}$

2a) $2x - 3x\frac{dy}{dx} - 3y - 8y\frac{dy}{dx} = 0$

$2x - 3y = 3x\frac{dy}{dx} + 8y\frac{dy}{dx}$

$2x - 3y = \frac{dy}{dx}(3x + 8y)$

$\frac{dy}{dx} = \frac{2x - 3y}{3x + 8y}$

b) $\frac{2x - 3y}{3x + 8y} = 0$

$2x - 3y = 0$ (1)

$x^2 - 3xy - 4y^2 + 64 = 0$ (2)

$2x = 3y$

$x = \frac{3}{2}y$

$\left(\frac{3}{2}y\right)^2 - \cancel{\frac{3y}{2}}\ 3\left(\frac{3}{2}y\right)(y) - 4y^2 + 64 = 0$

$\frac{9}{4}y^2 - \frac{9}{2}y^2 - 4y^2 + 64 = 0$

$64 = \frac{25}{4}y^2$

$y^2 = \frac{256}{25}$

$y = \pm\frac{16}{5}$

P 4 4 8 2 7 A 0 4 3 2

Question 2 continued

$x = \frac{3}{2}\left(\frac{16}{5}\right)$ or $x = \frac{3}{2}\left(\frac{-16}{5}\right)$

$= \frac{48}{10}$ $= \frac{-48}{10}$

$= \frac{24}{5}$ $= \frac{-24}{5}$

$\left(\frac{24}{5}, \frac{16}{5}\right)$ and $\cancel{\frac{-16}{5}, \frac{24}{}}$

$\left(\frac{-24}{5}, \frac{-16}{5}\right)$

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3.

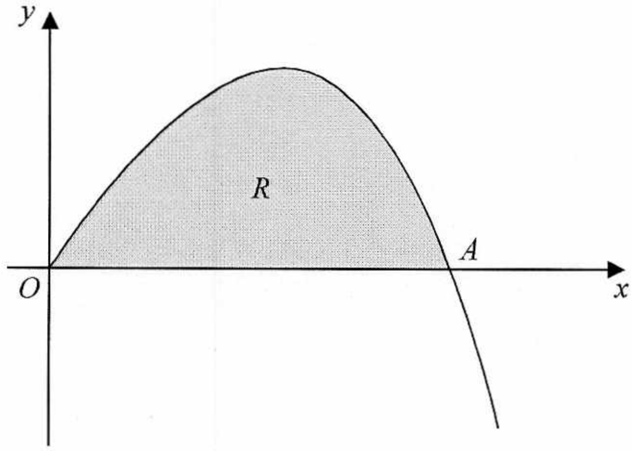

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find

$$\int xe^{\frac{1}{2}x}\,dx$$

(3)

The finite region R, shown shaded in Figure 1, is bounded by the x-axis and the curve with equation

$$y = 4x - xe^{\frac{1}{2}x},\ x \geqslant 0$$

(c) Find, by integration, the exact value for the area of R.
Give your answer in terms of $\ln 2$

(3)

a/ crosses x when $y=0$

$0 = 4x - xe^{\frac{1}{2}x}$

$= x(4 - e^{\frac{1}{2}x})$

$x=0$ $\quad 4 - e^{\frac{1}{2}x} = 0$

$e^{\frac{1}{2}x} = 4$

$\frac{1}{2}x = \ln 4$

$x = 2\ln 4 = \underline{4\ln 2}$

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Question 3 continued

$$\int x\, e^{\frac{1}{2}x}\, dx$$

$$\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx}\, dx$$

$$\boxed{\begin{array}{ll} u = x & \frac{dv}{dx} = e^{\frac{1}{2}x} \\ \frac{du}{dx} = 1 & v = 2e^{\frac{1}{2}x} \end{array}}$$

$$= 2xe^{\frac{1}{2}x} - \int 2e^{\frac{1}{2}x}\, dx$$

$$= 2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x} + c$$

c/

$$\int_0^{4\ln 2} 4x - xe^{\frac{1}{2}x}\, dx$$

$$\left[2x^2 - 2xe^{\frac{1}{2}x} + 4e^{\frac{1}{2}x}\right]_0^{4\ln 2}$$

$$\left[2(4\ln 2)^2 - 2(4\ln 2)e^{\frac{1}{2}(4\ln 2)} + 4e^{\frac{1}{2}(4\ln 2)}\right]$$

$$- \left[0 - 0 + 4\right]$$

$$\left[2(4\ln 2)^2 - (8\ln 2)(4) + 16\right] - \left[4\right]$$

$$2(4\ln 2)^2 - \overset{32}{\cancel{16}}\ln 2 + 16 - 4$$

$$2(4\ln 2)^2 - \overset{32}{\cancel{16}}\ln 2 + 12$$

Leave blank

4. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$$

where λ and μ are scalar parameters and p is a constant.

The lines l_1 and l_2 intersect at the point A.

(a) Find the coordinates of A.

(2)

(b) Find the value of the constant p.

(3)

(c) Find the acute angle between l_1 and l_2, giving your answer in degrees to 2 decimal places.

(3)

The point B lies on l_2 where $\mu = 1$

(d) Find the shortest distance from the point B to the line l_1, giving your answer to 3 significant figures.

(3)

4a) i)) $5 = 8 + 3\mu$

$-3 = 3\mu$

$\underline{\mu = -1}$

$$\begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} - 1 \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \underline{\begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix}}$$

b) j)) $-3 + \lambda = 1$

$\lambda = 4$

k)) $p + 4(-3) = -2 - 1(-5)$

$p - 12 = 3$

$\underline{p = 15}$

P 4 4 8 2 7 A 0 1 2 3 2

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Question 4 continued

c/ $\cos\theta = \dfrac{a.b}{|a|\,|b|}$

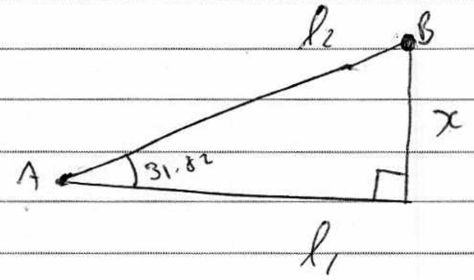
$a.b = 0(3) + 1(4) - 3(-5)$
$= \cancel{8}\ 19$

$|a| = \sqrt{0^2 + 1^2 + 3^2}$ $\quad$ $|b| = \sqrt{3^2 + 4^2 + 5^2}$
$= \sqrt{10}$ $\quad$ $= \sqrt{50}$

$\cos\theta = \dfrac{19}{\sqrt{10}\cdot\sqrt{50}}$

$\theta = 31.82^\circ$ 2dp

d/ B: $\begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + 1 \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ 9 \\ -7 \end{pmatrix}$



$AB = \sqrt{6^2 + 8^2 + 10^2}$
$= \sqrt{200}$

$\sin(31.82) = \dfrac{x}{\sqrt{200}}$

$x = 7.46$ m (3sf)

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5. A curve C has parametric equations

$$x = 4t + 3, \quad y = 4t + 8 + \frac{5}{2t}, \quad t \neq 0$$

$4t + 8 + \frac{5}{2}t^{-1}$

(a) Find the value of $\frac{dy}{dx}$ at the point on C where $t = 2$, giving your answer as a fraction in its simplest form.

(3)

(b) Show that the cartesian equation of the curve C can be written in the form

$$y = \frac{x^2 + ax + b}{x - 3}, \quad x \neq 3$$

where a and b are integers to be determined.

(3)

a/ $\frac{dy}{dt} = 4 - \frac{5}{2}t^{-2}$

$\frac{dx}{dt} = 4$

$\frac{dy}{dx} = \frac{4 - \frac{5}{2}t^{-2}}{4}$

when $t=2$ $\frac{dy}{dx} = \frac{27}{32}$

b/ $x = 4t + 3$

$x - 3 = 4t$

$t = \frac{x-3}{4}$

$y = 4\left(\frac{x-3}{4}\right) + 8 + \frac{5}{2\left(\frac{x-3}{4}\right)}$

$= x - 3 + 8 + \frac{5}{\frac{x-3}{2}}$

$= x + 5 + \frac{10}{x-3}$

P 4 4 8 2 7 A 0 1 6 3 2

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Question 5 continued

$$y = \frac{(x+5)(x-3)}{(x-3)} + \frac{10}{(x-3)}$$

$$= \frac{x^2 + 2x - 15 + 10}{x - 3}$$

$$= \frac{x^2 + 2x - 5}{x - 3}$$

$a = 2 \quad b = -5$

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6.

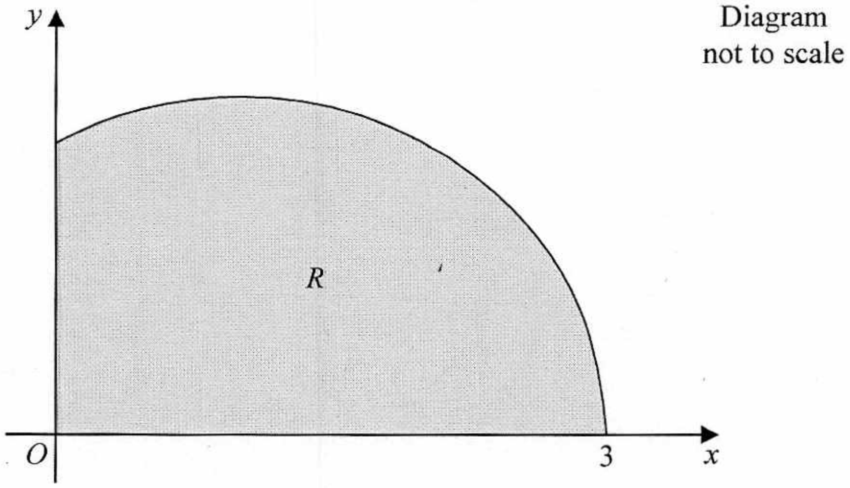

Figure 2

Figure 2 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leqslant x \leqslant 3$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis, and the y-axis.

(a) Use the substitution $x = 1 + 2\sin\theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)}\,dx = k\int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2\theta\,d\theta$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R.

(3)

$\int_0^3 \sqrt{(3-x)(x+1)}\ dx$ $\qquad x = 1 + 2\sin\theta$

$3 = 1+2\sin\theta$

$\int_0^3 \sqrt{(3-1-2\sin\theta)(1+2\sin\theta+1)}\ \frac{dx}{d\theta}\ d\theta$

$0 = 1+2\sin\theta$

$\int_{\cancel{0}}^{\cancel{3}} \sqrt{(2-2\sin\theta)(2+2\sin\theta)}\ \frac{dx}{d\theta}\ d\theta$

$2 = 2\sin\theta$ $\qquad 0 = 1 + 2\sin\theta$

$\theta = \frac{1}{2}\pi$ $\qquad \theta = -\frac{1}{6}\pi$

P 4 4 8 2 7 A 0 2 0 3 2

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Question 6 continued

$$\frac{dx}{d\theta} = 2\cos\theta$$

$$\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} \sqrt{(2-2\sin\theta)(2+2\sin\theta)} \quad 2\cos\theta \quad d\theta$$

$$\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} \sqrt{4-4\sin^2\theta} \quad (2\cos\theta) \quad d\theta$$

$$\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} \sqrt{4\cos^2\theta} \quad (2\cos\theta) \quad d\theta$$

$$\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} 2\cos\theta \cdot 2\cos\theta \quad d\theta$$

$$\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} 4\cos^2\theta \quad d\theta.$$

$$4\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} \cos^2\theta \quad d\theta.$$

b/ $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta + 1 = 2\cos^2\theta$

$\frac{1}{2}\cos 2\theta + \frac{1}{2} = \cos^2\theta.$

$$4\int_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{2}\cos 2\theta + \frac{1}{2} \quad d\theta$$

$$4\left[\frac{1}{4}\sin 2\theta + \frac{1}{2}\theta\right]_{-\frac{1}{6}\pi}^{\frac{1}{2}\pi}$$

$$4\left[\left(0 + \frac{1}{4}\pi\right) - \left(-\frac{\sqrt{3}}{8} - \frac{1}{12}\pi\right)\right]$$

Leave blank

Question 6 continued

$4\left(\frac{1}{4}\pi + \frac{\sqrt{3}}{8} + \frac{1}{12}\pi\right)$

$4\left(\frac{1}{3}\pi + \frac{\sqrt{3}}{8}\right)$

$\frac{4}{3}\pi + \frac{\sqrt{3}}{2}$

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7. (a) Express $\dfrac{2}{P(P-2)}$ in partial fractions.

(3)

A team of biologists is studying a population of a particular species of animal.

The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{2}P(P-2)\cos 2t, \quad t \geqslant 0$$

where P is the population in thousands, and t is the time measured in years since the start of the study.

Given that $P = 3$ when $t = 0$,

(b) solve this differential equation to show that

$$P = \frac{6}{3 - e^{\frac{1}{2}\sin 2t}}$$

(7)

(c) find the time taken for the population to reach 4000 for the first time. Give your answer in years to 3 significant figures.

(3)

7a) $\dfrac{2}{P(P-2)} = \dfrac{A}{P} + \dfrac{B}{P-2}$

$2 = A(P-2) + B(P)$

Let $P = 0$

$2 = -2A$

$A = -1$

Let $P = 2$

$2 = 2B$

$B = 1$

$\dfrac{1}{P-2} - \dfrac{1}{P}$

P 4 4 8 2 7 A 0 2 4 3 2

Leave blank

Question 7 continued

b/ $\frac{dP}{dt} = \frac{1}{2} P(P-2)\cos 2t$

$\int \frac{2}{P(P-2)}\, dP = \int \cos 2t \; dt$

$\int \frac{1}{P-2} - \frac{1}{P}\, dP = \int \cos 2t \; dt$

$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t + C$

$P = 3 \quad t = 0$

$\ln 1 - \ln 3 = \frac{1}{2}\sin 0 + C$

$-\ln 3 = C$

$\ln(P-2) - \ln P = \frac{1}{2}\sin 2t - \ln 3$

$\ln\left(\frac{P-2}{P}\right) = \frac{1}{2}\sin 2t - \ln 3$

$\frac{P-2}{P} = e^{\frac{1}{2}\sin 2t - \ln 3}$

$\frac{P-2}{P} = \frac{e^{1/2 \sin 2t}}{e^{\ln 3}}$

$\frac{P-2}{P} = \frac{e^{1/2\sin 2t}}{3}$

$\frac{P-2}{P} = 1/3\, e^{1/2 \sin 2t}$

$P - 2 = \frac{1}{3} P e^{1/2\sin 2t}$

$P - \frac{1}{3}Pe^{1/2\sin 2t} = 2$

$P(1 - \frac{1}{3}e^{1/2\sin 2t}) = 2$

$P = \frac{2}{1 - \frac{1}{3}e^{1/2\sin 2t}}$

P 4 4 8 2 7 A 0 2 5 3 2

Turn over

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Question 7 continued

$$P = \frac{6}{3 - e^{1/2 \sin 2t}}$$

c/

$$4 = \frac{6}{3 - e^{1/2 \sin 2t}}$$

$$12 - 4e^{1/2 \sin 2t} = 6$$

$$6 = 4e^{1/2 \sin 2t}$$

$$\frac{3}{2} = e^{1/2 \sin 2t}$$

$$\ln\left(\frac{3}{2}\right) = \frac{1}{2} \sin 2t$$

$$2 \ln\left(\frac{3}{2}\right) = \sin 2t$$

$t = 0.473$ years.

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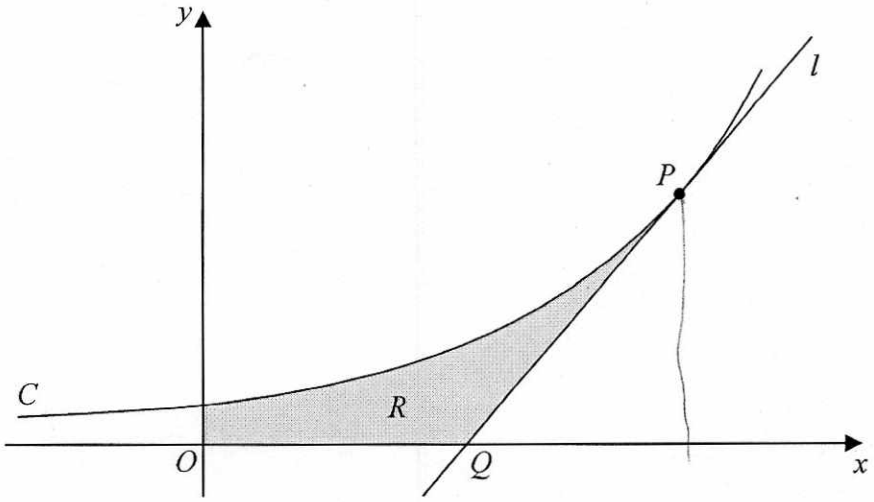

Diagram not to scale

Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = 3^x$$

The point P lies on C and has coordinates (2, 9).

The line l is a tangent to C at P. The line l cuts the x-axis at the point Q.

(a) Find the exact value of the x coordinate of Q.

(4)

The finite region R, shown shaded in Figure 3, is bounded by the curve C, the x-axis, the y-axis and the line l. This region R is rotated through 360° about the x-axis.

(b) Use integration to find the exact value of the volume of the solid generated.

Give your answer in the form $\frac{p}{q}$ where p and q are exact constants.

[*You may assume the formula* $V = \frac{1}{3}\pi r^2 h$ *for the volume of a cone.*]

(6)

8 a) $y = 3^x$

$\frac{dy}{dx} = 3^x \ln 3$

when $x = 2$

$\frac{dy}{dx} = 9 \ln 3$

$y = mx + c$

P 4 4 8 2 7 A 0 2 8 3 2

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Question 8 continued

$9 = (9\ln 3)(2) + c$

$9 = 18\ln 3 + c$

$c = 9 - 18\ln 3$

$y = (9\ln 3)(x) + 9 - 18\ln 3$

crosses x when $y=0$

$0 = (9\ln 3)x + 9 - 18\ln 3$

$x = \dfrac{18\ln 3 - 9}{9\ln 3} = \dfrac{2\ln 3 - 1}{\ln 3}$

$= \underline{2 - \dfrac{1}{\ln 3}}$

b) $\pi\int_0^2 y^2\,dx - \frac{1}{3}\pi r^2 h$

$\pi\int_0^2 3^{2x}\,dx - \frac{1}{3}\pi r^2 h$

$\pi\left[\dfrac{1}{2\ln 3}\cdot 3^{2x}\right]_0^2 - \frac{1}{3}\pi r^2 h$

$\pi\left[\dfrac{1}{2\ln 3}\cdot 81 - \dfrac{1}{2\ln 3}\right] - \frac{1}{3}\pi r^2 h$

$\dfrac{81\pi}{2\ln 3} - \dfrac{\pi}{2\ln 3} - \dfrac{1}{3}\pi(9)^2\left(2 - \left(2 - \dfrac{1}{\ln 3}\right)\right)$

$\dfrac{80\pi}{2\ln 3} - \dfrac{1}{3}\pi(81)\left(\dfrac{1}{\ln 3}\right)$

$\dfrac{80\pi}{2\ln 3} - \dfrac{27\pi}{\ln 3} = \dfrac{80\pi}{2\ln 3} - \dfrac{54\pi}{2\ln 3}$

$\dfrac{26\pi}{2\ln 3} = \underline{\dfrac{13\pi}{\ln 3}}$

P 4 4 8 2 7 A 0 3 1 3 2