Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 6 / 0 1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Tuesday 18 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Find $\int x^2 e^x \mathrm{d}x$. **(5)**

(b) Hence find the exact value of $\int_0^1 x^2 e^x \mathrm{d}x$. **(2)**

1a) $\int u \frac{dv}{dx}\, dx = uv - \int v \frac{du}{dx}\, dx$

$u = x^2 \qquad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 2x \qquad v = e^x$

$= x^2 e^x - \int 2x e^x \, dx$

$u = 2x \qquad \frac{dv}{dx} = e^x$

$\frac{du}{dx} = 2 \qquad v = e^x$

$= x^2 e^x - \left(2x e^x - \int 2e^x \, dx\right)$

$= x^2 e^x - 2x e^x + 2e^x + c$

b/ $\left[x^2 e^x - 2x e^x + 2e^x\right]_0^1$

$\left[e - 2e + 2e\right] - \left[2\right]$

$\underline{\underline{e - 2}}$

Leave blank

2. (a) Use the binomial expansion to show that

$$\sqrt{\left(\frac{1+x}{1-x}\right)} \approx 1 + x + \frac{1}{2}x^2, \quad |x| < 1$$

(6)

(b) Substitute $x = \frac{1}{26}$ into

$$\sqrt{\left(\frac{1+x}{1-x}\right)} = 1 + x + \frac{1}{2}x^2$$

to obtain an approximation to $\sqrt{3}$

Give your answer in the form $\frac{a}{b}$ where a and b are integers.

(3)

2a) $\sqrt{\frac{1+x}{1-x}} = (1+x)^{1/2}(1-x)^{-1/2}$

$= \left(1 + \tfrac{1}{2}x + \frac{(\frac{1}{2})(-\frac{1}{2})x^2}{2}\right)$

$\left(1 + (-\tfrac{1}{2})(-x) + \frac{(-\frac{1}{2})(-\frac{3}{2})x^2}{2}\right)$

$= \left(1 + \frac{1}{2}x - \frac{1}{8}x^2\right)\left(1 + \frac{1}{2}x + \frac{3}{8}x^2\right)$

$= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^2 \ldots$

$= 1 + x + \frac{1}{2}x^2$

b) $\sqrt{\frac{1+\frac{1}{26}}{1-\frac{1}{26}}} = \frac{3\sqrt{3}}{5}$

$\frac{3\sqrt{3}}{5} = 1 + \frac{1}{26} + \frac{1}{2}\left(\frac{1}{26}\right)^2$

$\frac{3\sqrt{3}}{5} = \frac{1405}{1352}$

$= \frac{7025}{4056}$

3.

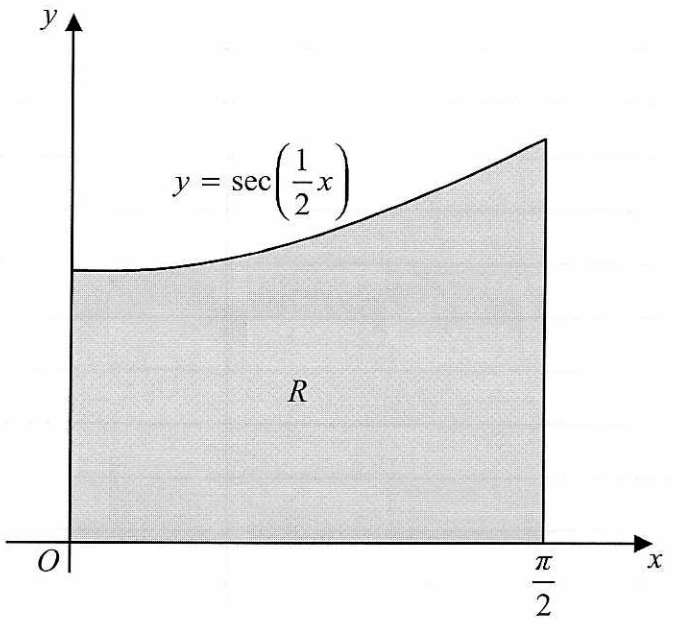

Figure 1

Figure 1 shows the finite region R bounded by the x-axis, the y-axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276	1.154701	1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places.
(1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R, giving your answer to 4 decimal places.
(3)

Region R is rotated through 2π radians about the x-axis.

(c) Use calculus to find the exact volume of the solid formed.
(4)

Leave blank

Question 3 continued

b/

$$\frac{\pi}{6}\left(\frac{1}{2} + 1.035276 + 1.154701 + \frac{1.414214}{2}\right)$$

$= 1.7787$ units^2

c/

$$\pi \int_0^{\frac{\pi}{2}} y^2 \; dx$$

$$\pi \int_0^{\pi/2} \sec^2(1/2 x) \; dx$$

$$\cancel{\pi \int_0^{\pi/2}}$$

$$\pi \left[2\tan\left(\tfrac{1}{2}x\right)\right]_0^{\pi/2}$$

$$\pi \left[(2) - (0)\right]$$

$\underline{\underline{2\pi}}$ units^3

Turn over

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4. A curve C has parametric equations

$$x = 2\sin t, \quad y = 1 - \cos 2t, \quad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $t = \frac{\pi}{6}$

(4)

(b) Find a cartesian equation for C in the form

$$y = \mathrm{f}(x), \quad -k \leqslant x \leqslant k,$$

stating the value of the constant k.

(3)

(c) Write down the range of $\mathrm{f}(x)$.

(2)

a) $\frac{dx}{dt} = 2\cos t \qquad \frac{dy}{dt} = 2\sin 2t$

$\frac{dy}{dx} = \frac{\cancel{2}\sin 2t}{\cancel{2}\cos t} = \frac{2\cancel{\cos t}\sin t}{\cancel{\cos t}} = 2\sin t$

where $t = \pi/6 \quad \frac{dy}{dx} = 1$

b) $y = 1 - \cos 2t$
$= 1 - (\cos^2 t - \sin^2 t)$
$= 1 - (1 - 2\sin^2 t)$
$= 2\sin^2 t$

$\boxed{x^2 = 4\sin^2 t}$

$y = \frac{1}{2}x^2 \qquad -2 \leq x \leq 2$

c) $0 \leq f(x) \leq 2$

Leave blank

5. (a) Use the substitution $x = u^2$, $u > 0$, to show that

$$\int \frac{1}{x(2\sqrt{x}-1)}\,dx = \int \frac{2}{u(2u-1)}\,du$$

(3)

(b) Hence show that

$$\int_1^9 \frac{1}{x(2\sqrt{x}-1)}\,dx = 2\ln\left(\frac{a}{b}\right)$$

where a and b are integers to be determined.

(7)

5a) $x = u^2$

$$\int \frac{1}{(u^2)(2\sqrt{(u^2)}-1)}\frac{dx}{du}\,du$$

$$\int \frac{1}{u^2(2u-1)}\frac{dx}{du}\,du$$

$$\int \frac{1}{u^2(2u-1)}\cdot 2u\,du \qquad \frac{dx}{du} = 2u$$

$$\int \frac{2}{u(2u-1)}\,du$$

b) $$\int_1^3 \frac{2}{u(2u-1)}\,du$$

$$\frac{2}{u(2u-1)} = \frac{A}{u} + \frac{B}{2u-1}$$

$$2 = A(2u-1) + B(u)$$

Let $u=0$ $\quad 2 = -A$

$A = -2$

Let $u=\frac{1}{2}$ $\quad 2 = \frac{1}{2}B$

$B = 4$

$$\int_1^3 \frac{-2}{u} + \frac{4}{2u-1}\,du$$

Leave blank

Question 5 continued

$$\left[-2\ln u + 2\ln(2u-1) \right]_1^3$$

$$[-2\ln 3 + 2\ln 5] - [-2\ln 1 + 2\ln 1]$$

$$-2\ln 3 + 2\ln 5$$

$$2(\ln 5 - \ln 3)$$

$$2\ln\frac{5}{3}$$

Leave blank

6. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at any time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta), \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$,

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

6a/ $\frac{d\theta}{dt} = \lambda(120 - \theta)$

$\int \frac{1}{120-\theta} \, d\theta = \int \lambda \, dt$

$-\ln(120-\theta) = \lambda t + c$

$\theta = 20 \quad t = 0$

$-\ln(100) = c$

$-\ln(120-\theta) = \lambda t + -\ln 100$

$\ln 100 - \ln(120-\theta) = \lambda t$

$\ln\left(\frac{100}{120-\theta}\right) = \lambda t$

$\frac{100}{120-\theta} = e^{\lambda t}$

Leave blank

Question 6 continued

$$100 = e^{\lambda t}(120 - \theta)$$

$$\frac{100}{e^{\lambda t}} = 120 - \theta$$

$$\theta = 120 - \frac{100}{e^{\lambda t}}$$

$$= 120 - 100e^{-\lambda t}$$

b) $100 = 120 - 100e^{-0.01t}$

$$\frac{1}{5} = e^{-0.01t}$$

$$\ln\frac{1}{5} = -0.01t$$

$t = 161$ seconds

Turn over

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7. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

A point Q lies on the curve.

The tangent to the curve at Q is parallel to the y-axis.

Given that the x coordinate of Q is negative,

(b) use your answer to part (a) to find the coordinates of Q.

(7)

7a/ $\cancel{2x}$ $u = 4x$ $v = y$

$\frac{du}{dx} = 4$ $\frac{dv}{dx} = \frac{dy}{dx}$

$2x + 4x\frac{dy}{dx} + 4y + 2y\frac{dy}{dx} = 0$

$4x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - 4y$

$\frac{dy}{dx}(4x + 2y) = -2x - 4y$

$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$

tangent is parallel to y axis $\therefore \frac{dy}{dx} = \infty$

$4x + 2y = 0 \longrightarrow 2y = -4x$

$y = -2x$

$x^2 + 4xy + y^2 + 27 = 0$

$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$

$x^2 - 8x^2 + 4x^2 + 27 = 0$

$-3x^2 + 27 = 0$

$3x^2 = 27$

$x^2 = 9$

Leave blank

Question 7 continued

$x = \pm 3$

a) x coordinate is negative $x = -3$

$y = -2(-3)$
$= 6$

~~(3, 6)~~ $(-3, 6)$

Leave blank

8. With respect to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point A lies on l and has coordinates $(3, -2, 6)$.

The point P has position vector $(-p\mathbf{i} + 2p\mathbf{k})$ relative to O, where p is a constant.

Given that vector $\overrightarrow{PA}$ is perpendicular to l,

(a) find the value of p.

(4)

Given also that B is a point on l such that $\angle BPA = 45°$,

(b) find the coordinates of the two possible positions of B.

(5)

a) $\overrightarrow{PA} = \begin{pmatrix} 3+p \\ -2 \\ 6-2p \end{pmatrix}$

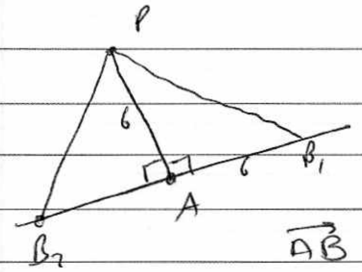
perpendicular $\therefore$ $a \cdot b = 0$

$2(3+p) - 2(2) - 1(6-2p) = 0$

$6 + 2p - 4 - 6 + 2p = 0$

$4p - 4 = 0$

$p = 1$

b) 

$B = \begin{pmatrix} 13 + 2\lambda \\ 8 + 2\lambda \\ 1 - \lambda \end{pmatrix}$

$\overrightarrow{AB} = \begin{pmatrix} 10+2\lambda \\ 10+2\lambda \\ -5-\lambda \end{pmatrix}$ $\overrightarrow{PA} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$

$\cancel{40 + 8\lambda - 20 - 4\lambda - 20 - 4\lambda = 0}$

$|PA| = \sqrt{4^2 + 2^2 + 4^2}$
$= 6$

Leave blank

Question 8 continued

$|AB| = 6$ (isosceles triangle)

$\cancel{\sqrt{(10+2\lambda)^2 + (10+2\lambda)^2 + (7-5-\lambda)^2} = 6}$

direction of $l = \sqrt{2^2 + 2^2 + 1^2}$

$= \sqrt{9}$

$= 3$

$\therefore$ B is $2 \times \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}$ away from A

$\begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$

$\begin{matrix} 3 \\ -2 \\ 6 \end{matrix} + \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$ or $\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \\ -2 \end{pmatrix}$

$\underline{\underline{\begin{pmatrix} 7 \\ 2 \\ 4 \end{pmatrix}}}$ $\cancel{\begin{matrix} 7 \; -1 \\ -6 \\ 4 \end{matrix}}$ $\underline{\underline{\begin{pmatrix} -1 \\ -6 \\ 8 \end{pmatrix}}}$