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Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1 R	Signature	

Paper Reference(s)

6665/01R

Edexcel GCE

Core Mathematics C3

Advanced

Thursday 13 June 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination
Mathematical Formulae (Pink)

Items included with question papers
Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Express

$$\frac{3x+5}{x^2+x-12} - \frac{2}{x-3}$$

as a single fraction in its simplest form.

(4)

$$\frac{3x+5}{(x+4)(x-3)} - \frac{2}{x-3}$$

$$\frac{3x+5}{(x+4)(x-3)} - \frac{2(x+4)}{(x+4)(x-3)}$$

$$\frac{3x+5-2x-8}{(x+4)(x-3)}$$

$$\frac{(x-3)}{(x+4)(x-3)}$$

$$\frac{1}{x+4}$$

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2.

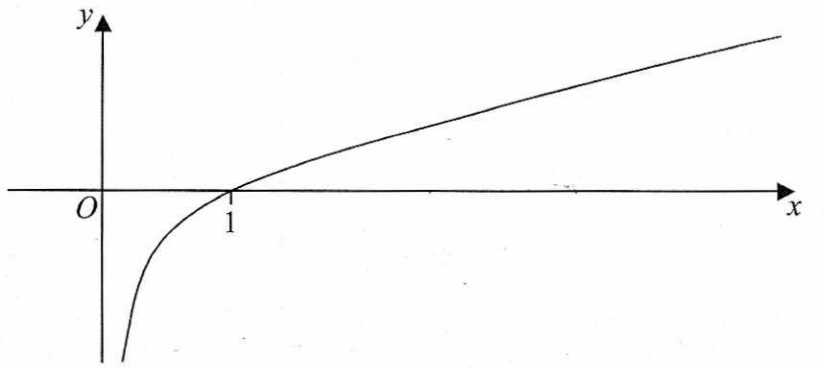


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$, $x > 0$, where f is an increasing function of x. The curve crosses the x-axis at the point $(1, 0)$ and the line $x = 0$ is an asymptote to the curve.

On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, $x > 0$

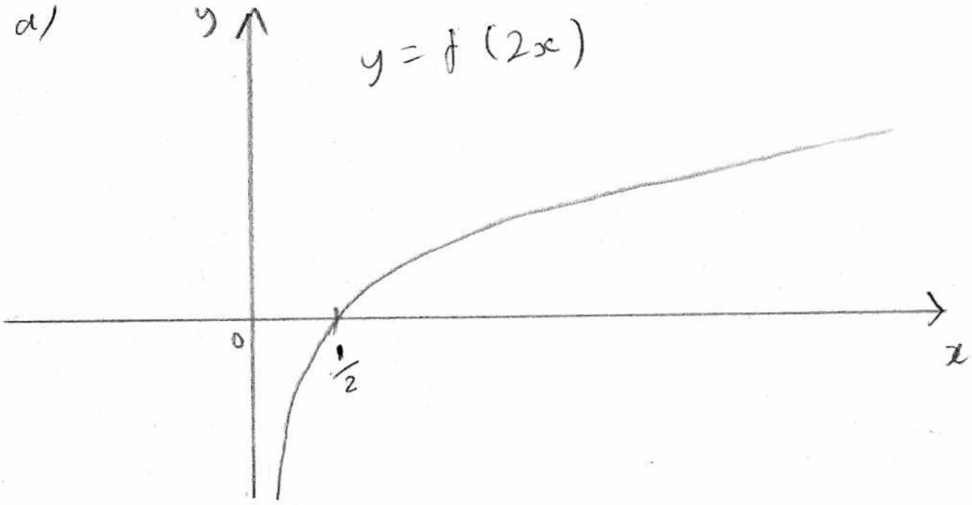
(2)

(b) $y = |f(x)|$, $x > 0$

(3)

Indicate clearly on each sketch the coordinates of the point at which the curve crosses or meets the x-axis.

a)



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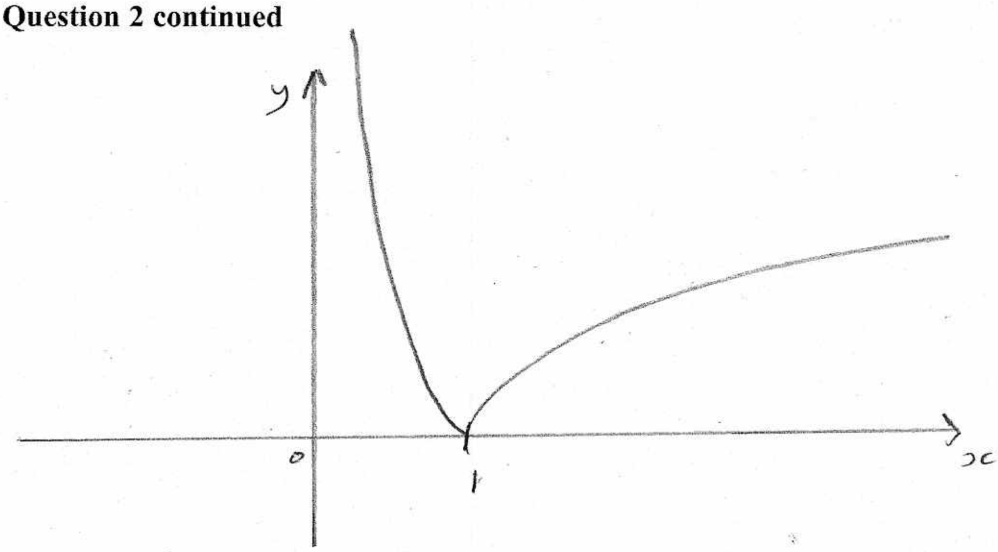
Question 2 continued



Q2

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3. $$f(x) = 7\cos x + \sin x$$

Given that $f(x) = R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(a) find the exact value of R and the value of α to one decimal place. **(3)**

(b) Hence solve the equation

$$7\cos x + \sin x = 5$$

for $0 \leqslant x < 360°$, giving your answers to one decimal place. **(5)**

(c) State the values of k for which the equation

$$7\cos x + \sin x = k$$

has only one solution in the interval $0 \leqslant x < 360°$ **(2)**

$R\cos(x - \alpha) = R\cos x \cos\alpha + R\sin x \sin\alpha$

$R\cos\alpha = 7$

$R\sin\alpha = 1$

$\tan\alpha = 1/7$

$\alpha = 8.1°$

$R = \sqrt{7^2 + 1^2}$

$= \sqrt{50}$

b/ $\sqrt{50}\cos(x - 8.1) = 5$

$\cos(x - 8.1) = \frac{\sqrt{2}}{2}$

$x - 8.1 = 45, 315$

$x = 53.1°, 323.1°$

c/ $k = \sqrt{50}$ or $-\sqrt{50}$

Leave blank

4. The functions f and g are defined by

$$f : x \mapsto 2|x| + 3, \qquad x \in \mathbb{R},$$

$$g : x \mapsto 3 - 4x, \qquad x \in \mathbb{R}$$

(a) State the range of f. **(2)**

(b) Find fg(1). **(2)**

(c) Find g^{-1}, the inverse function of g. **(2)**

(d) Solve the equation

$$gg(x) + [g(x)]^2 = 0$$

(5)

4a) $f(x) \geqslant 3$

b) $g(1) = 3 - 4(1)$
$= -1$

$f(-1) = 2|(-1)| + 3$
$= 5$

$f(g)(1) = 5$

c) $y = 3 - 4x$
$x = 3 - 4y$
$4y = 3 - x$
$y = \frac{3 - x}{4}$

$g^{-1}(x) = \frac{3 - x}{4}$

d)
$3 - 4(3 - 4x) + (3 - 4x)^2 = 0$
$3 - 12 + 16x + 9 - 24x + 16x^2 = 0$
$16x^2 - 8x = 0$
$8x(2x - 1) = 0$

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Question 4 continued

$x = 0$ or $x = 1/2$

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5. (a) Differentiate

$$\frac{\cos 2x}{\sqrt{x}}$$

with respect to x.

(3)

(b) Show that $\frac{d}{dx}(\sec^2 3x)$ can be written in the form

$$\mu(\tan 3x + \tan^3 3x)$$

where μ is a constant.

(3)

(c) Given $x = 2\sin\left(\frac{y}{3}\right)$, find $\frac{dy}{dx}$ in terms of x, simplifying your answer.

(4)

a/ $u = \cos 2x \qquad v = x^{\frac{1}{2}}$

$\frac{du}{dx} = -2\sin 2x \qquad \frac{dv}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{-2x^{\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{1}{2}}\cos 2x}{x}$$

$$= -2x^{-\frac{1}{2}}\sin 2x - \frac{1}{2}x^{-\frac{3}{2}}\cos 2x$$

b/ $y = (\sec 3x)^2$

$\frac{dy}{dx} = 2(\sec 3x)(3\sec 3x\tan 3x)$

$= 6\sec^2 3x\tan 3x$

$= 6(\tan^2 3x + 1)(\tan 3x)$

$= 6(\tan^3 3x + \tan 3x)$

$= 6(\tan 3x + \tan^3 3x)$

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Question 5 continued

c) $x = 2\sin(y/3)$

$\frac{dx}{dy} = \frac{2}{3}\cos(y/3)$

$x = 2\sin(y/3)$

$x^2 = 4\sin^2(y/3)$

$x^2 = 4(1 - \cos^2 y/3)$

$x^2 = 4 - 4\cos^2(y/3)$

$4\cos^2(y/3) = 4 - x^2$

$\cos^2 y/3 = 1 - 1/4\, x^2$

$\cos y/3 = \sqrt{1 - 1/4\, x^2}$

$\frac{dx}{dy} = \frac{2}{3}\sqrt{1 - 1/4\, x^2}$

$\frac{dy}{dx} = \frac{3}{2}(1 - 1/4\, x^2)^{-1/2}$

$= \frac{3}{2(1 - 1/4\, x^2)^{1/2}}$

$= \frac{3}{4^{1/2}(1 - 1/4\, x^2)^{1/2}}$

$= \frac{3}{(4 - x^2)^{1/2}}$

$= \frac{3}{(4 - x^2)^{1/2}}$

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6. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec}2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leqslant \theta < 2\pi$, the equation

$$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta$$

You must show all your working. Give your answers in terms of π.

(6)

i/ $\sin 2x = 2\sin x \cos x$

$\operatorname{cosec} 2x = \dfrac{1}{2\sin x \cos x}$

$= \frac{1}{2}\operatorname{cosec} x \sec x$

$\lambda = \frac{1}{2}$

ii/ $3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$

$3\sec^2\theta + 3\sec\theta = 2\sec^2\theta - 2$

$\sec^2\theta + 3\sec\theta + 2 = 0$

$(\sec\theta + 1)(\sec\theta + 2) = 0$

$\sec\theta = -1$ $\quad$ $\sec\theta = -2$

$\cos\theta = -1$ $\quad$ $\cos\theta = -\frac{1}{2}$

$\theta = \pi$ $\quad$ $\theta = \frac{2}{3}\pi, \frac{4}{3}\pi$

$[2\pi - \frac{2}{3}\pi]$

$\theta = \frac{2}{3}\pi, \pi, \frac{4}{3}\pi$

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7.

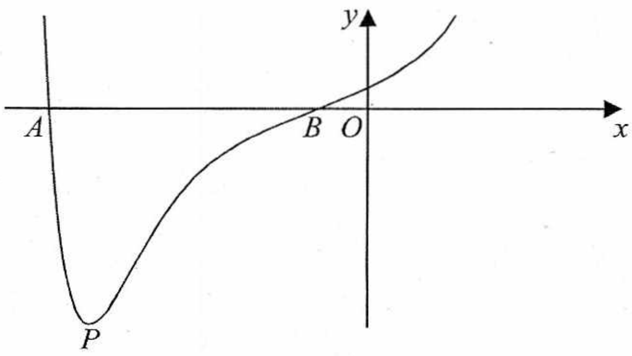

Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x^2 + 3x + 1)\mathrm{e}^{x^2}$$

The curve cuts the x-axis at points A and B as shown in Figure 2.

(a) Calculate the x coordinate of A and the x coordinate of B, giving your answers to 3 decimal places.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

The curve has a minimum turning point at the point P as shown in Figure 2.

(c) Show that the x coordinate of P is the solution of

$$x = -\frac{3(2x^2 + 1)}{2(x^2 + 2)}$$

(3)

(d) Use the iteration formula

$$x_{n+1} = -\frac{3(2x_n^2 + 1)}{2(x_n^2 + 2)}, \qquad \text{with } x_0 = -2.4,$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

The x coordinate of P is α.

(e) By choosing a suitable interval, prove that $\alpha = -2.43$ to 2 decimal places.

(2)

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Question 7 continued

a) Crosses x when $y=0$

$$0 = (x^2 + 3x + 1)\, e^{x^2}$$
$$0 = x^2 + 3x + 1$$

$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(1)}}{2(1)}$$

$$= \underline{\underline{-0.382}} \text{ and } \underline{\underline{-2.618}}$$

b) $u = x^2 + 3x + 1$ $\quad v = e^{x^2}$

$\frac{du}{dx} = 2x + 3$ $\quad \frac{dv}{dx} = 2x\, e^{x^2}$

$$f'(x) = 2x e^{x^2}(x^2 + 3x + 1) + e^{x^2}(2x + 3)$$
$$= e^{x^2}\left(2x(x^2 + 3x + 1) + (2x + 3)\right)$$
$$= e^{x^2}(2x^3 + 6x^2 + 2x + 2x + 3)$$
$$= e^{x^2}(2x^3 + 6x^2 + 4x + 3)$$

c) min point where $f'(x) = 0$

e^{x^2} cannot $= 0$

$$0 = 2x^3 + 6x^2 + 4x + 3$$
$$0 = 6x^2 + 3 + 2x^3 + 4x$$
$$0 = 3(2x^2 + 1) \cancel{+ 2(2x^2 + 4}$$
$$+ 2x(x^2 + 2)$$
$$-2x(x^2 + 2) = 3(2x^2 + 1)$$
$$x = \frac{-3(2x^2 + 1)}{\cancel{x}\, 2(x^2 + 2)}$$

Turn over

P 4 2 9 5 3 A 0 2 3 2 8

Question ~~4~~ 7 continued

$x_0 = -2.4$
$x_1 = -2.420$
$x_2 = -2.427$
$x_3 = -2.430$

e) $f'(-2.425) = 22.45537861$

$f'(-2.435) = -15.0254096$

Change of sign $\therefore$ $\alpha = -2.43$ to 2dp.

(Total 9 marks)

Q4

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8.

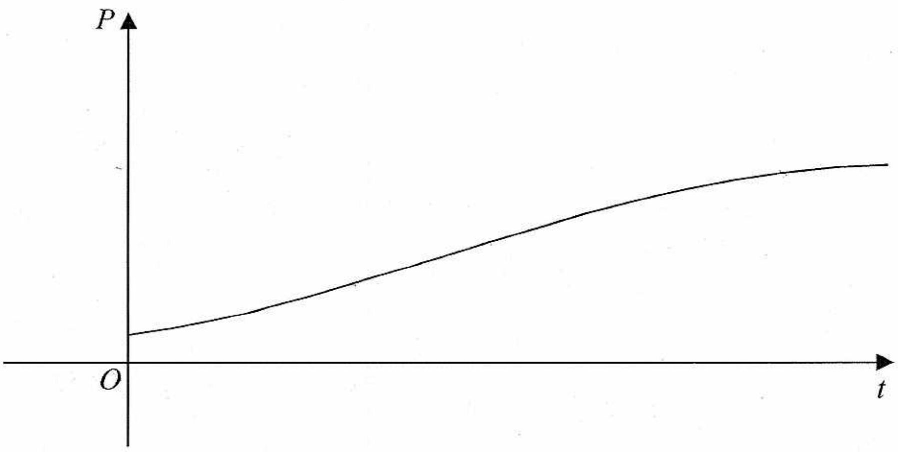

Figure 3

The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \qquad t \geqslant 0,$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(a) find the population at the start of the study,

(2)

(b) find a value for the expected upper limit of the population.

(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(c) calculate the value of k to 3 decimal places.

(5)

Using this value for k,

(d) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.

(2)

(e) Find, using $\frac{\mathrm{d}P}{\mathrm{d}t}$, the rate at which the population is growing at 10 years from the start of the study.

(3)

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Question 8 continued

a/ $t=0 \qquad P = 1000$

b/ 8000

c/ $t=3 \qquad P=2500$

$$2500 = \frac{8000}{1+7e^{-3k}}$$

$$1 + 7e^{-3k} = \frac{16}{5}$$

$$7e^{-3k} = \frac{11}{5}$$

$$e^{-3k} = \frac{11}{35}$$

$$-3k = \ln\left(\frac{11}{35}\right)$$

$$k = -\frac{1}{3}\ln\left(\frac{11}{35}\right)$$

$$= 0.386 \quad (3dp)$$

d/ $t=10$

$$P = \frac{8000}{1+7e^{-k(10)}}$$

$$= 6970 \quad (3sf)$$

e/ $P = 8000\left(1+7e^{-kt}\right)^{-1}$

$$\frac{dP}{dt} = -8000\left(1+7e^{-kt}\right)^{-2}\left(-7ke^{-kt}\right)$$

$$= \frac{56000ke^{-kt}}{\left(1+7e^{-kt}\right)^2}$$

$t=10 \qquad \frac{dP}{dt} = \underline{\underline{346}}$ people / year (3sf)

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