Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 23 January 2012 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$ **(4)**

(b) $\dfrac{\sin 4x}{x^3}$ **(5)**

1a) $u = x^2$ $v = \ln(3x)$

$\frac{du}{dx} = 2x$ $\frac{dv}{dx} = \frac{1}{3x} \times 3 = \frac{1}{x}$

$\frac{dy}{dx} = x^2\left(\frac{1}{x}\right) + \ln(3x)(2x)$

$= x + 2x\ln(3x)$

b) $u = \sin 4x$ $v = x^3$

$\frac{du}{dx} = 4\cos 4x$ $\frac{dv}{dx} = 3x^2$

$\frac{dy}{dx} = \frac{x^3(4\cos 4x) - \sin(4x)(3x^2)}{(x^3)^2}$

$= \frac{4x^3\cos(4x) - 3x^2\sin(4x)}{x^6}$

$= \frac{4x\cos(4x) - 3\sin(4x)}{x^4}$

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Question 1 continued

(Total 9 marks)

Q1

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2.

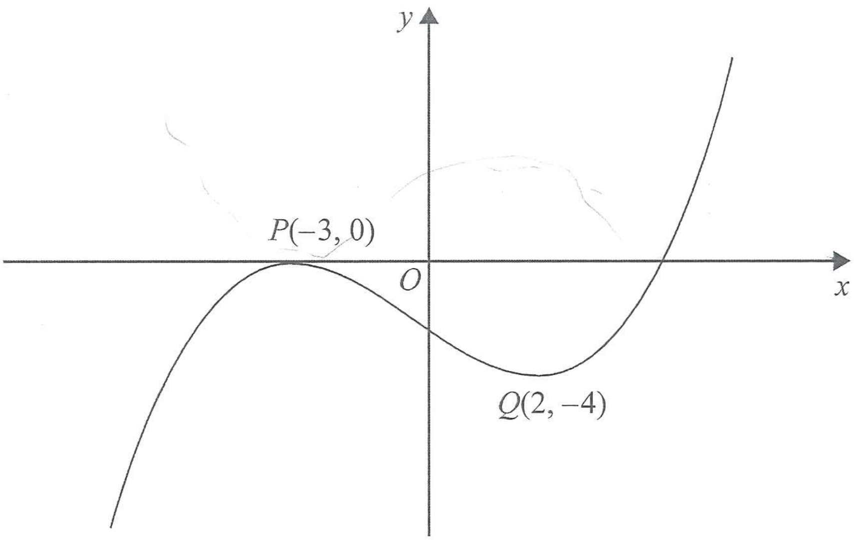


Figure 1

Figure 1 shows the graph of equation $y = \mathrm{f}(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

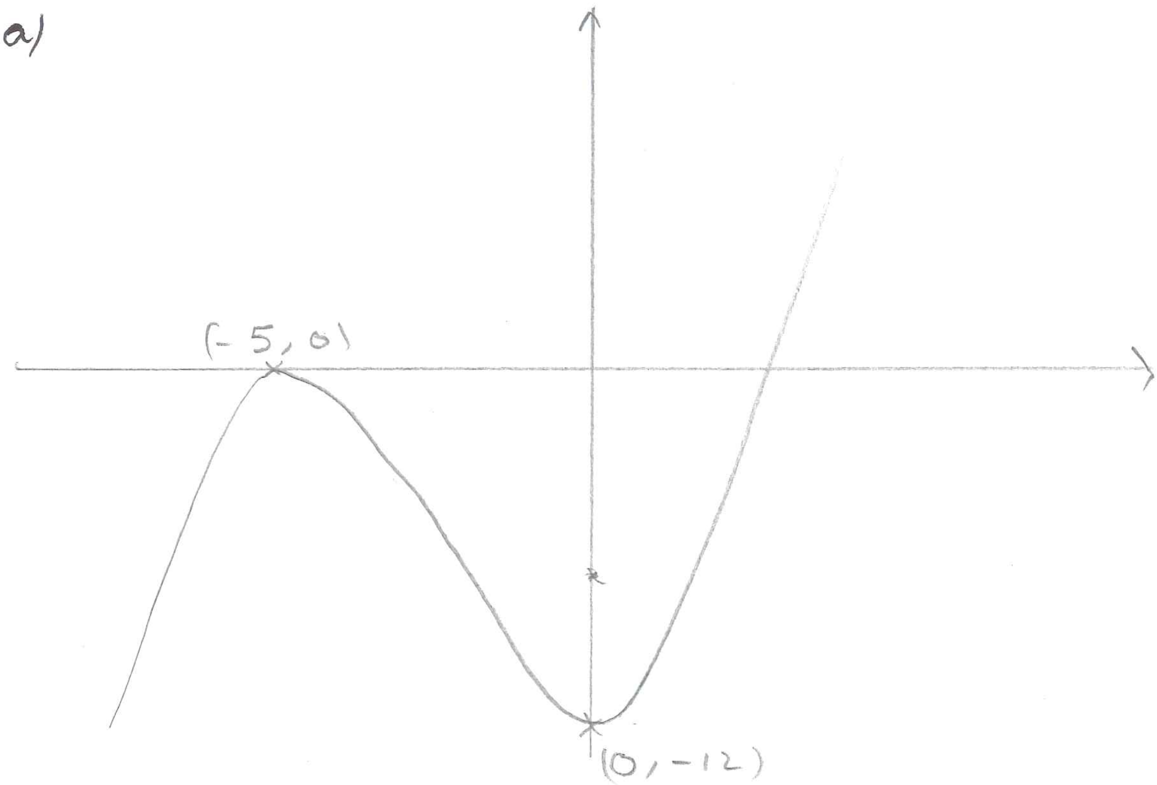
Sketch, on separate diagrams, the graphs of

(a) $y = 3\mathrm{f}(x + 2)$ **(3)**

(b) $y = |\mathrm{f}(x)|$ **(3)**

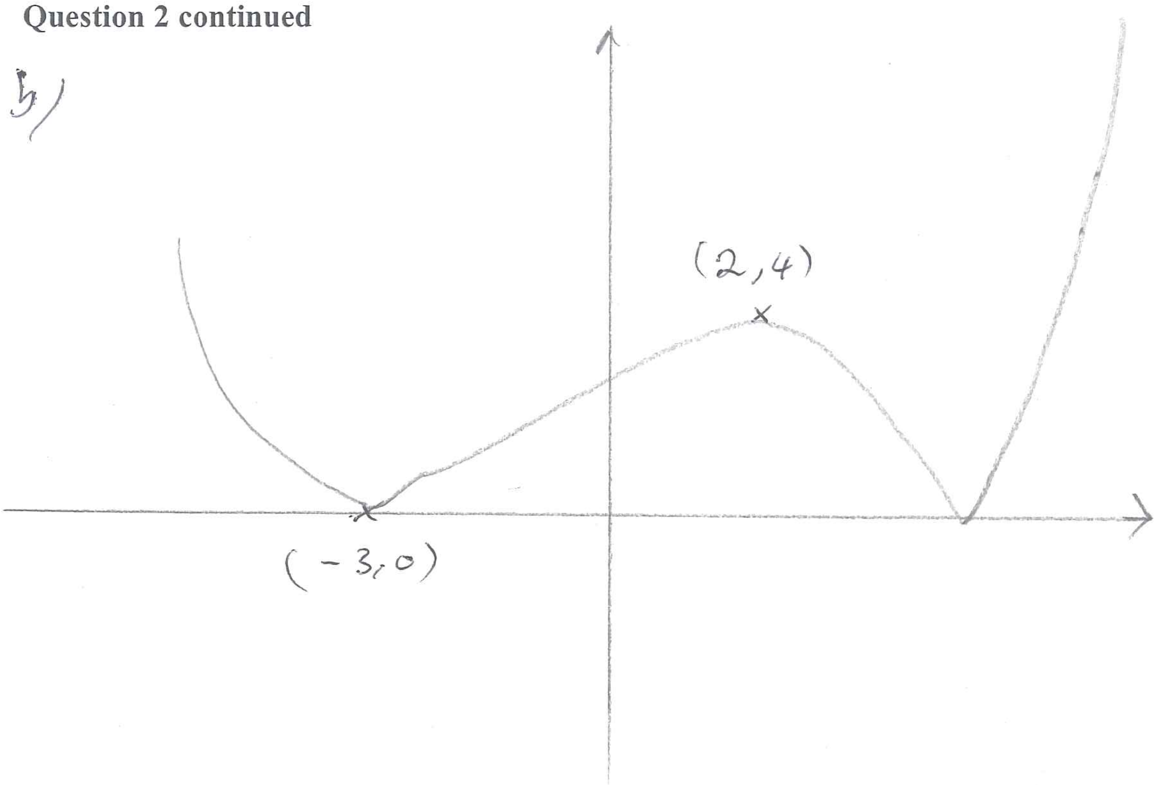
On each diagram, show the coordinates of any stationary points.

a)



Question 2 continued

b)



Q2

(Total 6 marks)

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3. The area, A mm^2, of a bacterial culture growing in milk, t hours after midday, is given by

$$A = 20e^{1.5t}, \quad t \geqslant 0$$

(a) Write down the area of the culture at midday.

(1)

(b) Find the time at which the area of the culture is twice its area at midday. Give your answer to the nearest minute.

(5)

3a) 20 mm^2

b) $A = 40$

$40 = 20e^{1.5t}$

$2 = e^{1.5t}$

$\ln 2 = 1.5t$

$t = \dfrac{\ln 2}{1.5}$

= 27 mins 44 secs

= 28 Minutes after midday

12.28 pm

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Question 3 continued

Q3

(Total 6 marks)

Turn over

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4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

$$\frac{dx}{dy} = 2\sec^2\left(y + \frac{\pi}{12}\right)$$

$$= 8$$

$$\frac{dy}{dx} = \frac{1}{8}$$

Gradient of the normal $= -8$

when $y = \pi/4$ $\quad x = 2\tan\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$

$$= 2\sqrt{3}$$

$$y = mx + c$$

$$\frac{\pi}{4} = -8(2\sqrt{3}) + c$$

$$\frac{\pi}{4} + 16\sqrt{3} = c$$

$$y = -8x + \frac{\pi}{4} + 16\sqrt{3}$$

Leave blank

Question 4 continued

Q4

(Total 7 marks)

Leave blank

5. Solve, for $0 \leqslant \theta \leqslant 180°$,

$$2\cot^2 3\theta = 7\operatorname{cosec} 3\theta - 5$$

Give your answers in degrees to 1 decimal place.

(10)

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$2(\operatorname{cosec}^2 3\theta - 1) = 7\operatorname{cosec} 3\theta - 5$

$2\operatorname{cosec}^2 3\theta - 2 = 7\operatorname{cosec} 3\theta - 5$

$2\operatorname{cosec}^2 3\theta - 7\operatorname{cosec} 3\theta + 3 = 0$

$(2\operatorname{cosec} 3\theta - 1)(\operatorname{cosec} 3\theta - 3) = 0$

$\operatorname{cosec} 3\theta = \frac{1}{2}$ $\quad$ $\operatorname{cosec} 3\theta = 3$

$\sin 3\theta = 2$ $\quad$ $\sin 3\theta = \frac{1}{3}$

NO SOLUTIONS $\quad$ $3\theta = 19.4712206\,3,\ 160.5287794,\ 379.4712206,\ 520.5287794$

$\theta = 6.5°,\ 53.5°,\ 126.5°,\ 173.5°$

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Question 5 continued

Q5

(Total 10 marks)

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6. $$f(x) = x^2 - 3x + 2\cos\left(\tfrac{1}{2}x\right), \quad 0 \leqslant x \leqslant \pi$$

(a) Show that the equation $f(x) = 0$ has a solution in the interval $0.8 < x < 0.9$

(2)

The curve with equation $y = f(x)$ has a minimum point P.

(b) Show that the x-coordinate of P is the solution of the equation

$$x = \frac{3 + \sin\left(\tfrac{1}{2}x\right)}{2}$$

(4)

(c) Using the iteration formula

$$x_{n+1} = \frac{3 + \sin\left(\tfrac{1}{2}x_n\right)}{2}, \quad x_0 = 2$$

find the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(d) By choosing a suitable interval, show that the x-coordinate of P is 1.9078 correct to 4 decimal places.

(3)

6a) $f(0.8) = 0.0821219880 1$
$f(0.9) = -0.0891057952 9$

change of sign ∴ solution lies between 0.8 and 0.9

b) Min. point where $\frac{dy}{dx} = 0$

$f'(x) = 2x - 3 - ~~\cos~~ \sin\left(\frac{1}{2}x\right)$

$0 = 2x - 3 - \sin\left(\frac{1}{2}x\right)$

$\frac{3 + \sin\left(\frac{1}{2}x\right)}{2} = \frac{2x}{2}$

$x = \frac{3 + \sin\left(\frac{1}{2}x\right)}{2}$

Question 6 continued

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c/ $x_0 = 2$

$x_1 = 1.921$

$x_2 = 1.910$

$x_3 = 1.908$

d/ $f'(x) = 2x - 3 - \sin(\frac{1}{2}x)$

$f'(1.90775) = -1.63 \times 10^{-4}$

$f'(1.90785) = 7.66 \times 10^{-6}$

change of sign $\therefore$ x coordinate of P is 1.9078 to 4dp.

Turn over

Leave blank

Question 6 continued

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Question 6 continued

Q6

(Total 12 marks)

Turn over

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7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2+7x-4} - \frac{1}{x+4}, \qquad x \in \mathbb{R}, x > \frac{1}{2}$$

(a) Show that $f(x) = \frac{1}{2x-1}$ **(4)**

(b) Find $f^{-1}(x)$ **(3)**

(c) Find the domain of f^{-1} **(1)**

$$g(x) = \ln(x+1)$$

(d) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e. **(4)**

a) $\frac{3(x+1)}{(2x-1)(x+4)} - \frac{1}{x+4}$

$\frac{3(x+1)}{(2x-1)(x+4)} - \frac{1(2x-1)}{(2x-1)(x+4)}$

$\frac{3(x+1) - 1(2x-1)}{(2x-1)(x+4)}$

$\frac{3x+3-2x+1}{(2x-1)(x+4)}$

$\frac{(x+4)}{(2x-1)(x+4)}$

$\frac{1}{2x-1}$

b) $y = \frac{1}{2x-1}$

$x = \frac{1}{2y-1}$

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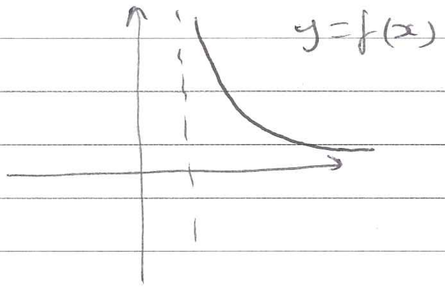
Question 7 continued

$x(2y-1) = 1$

$2xy - x = 1$

$2xy = 1 + x$

$y = \frac{1+x}{2x}$

c) 

$x > 0$

d) $fg(x) = \frac{1}{2\ln(x+1) - 1}$

$\frac{1}{7} = \frac{1}{2\ln(x+1) - 1}$

$7 = 2\ln(x+1) - 1$

$8 = 2\ln(x+1)$

$4 = \ln(x+1)$

$e^4 = x + 1$

$x = e^4 - 1$

Leave blank

Question 7 continued

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Question 7 continued

Q7

(Total 12 marks)

Turn over

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8. (a) Starting from the formulae for $\sin(A+B)$ and $\cos(A+B)$, prove that

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

(4)

(b) Deduce that

$$\tan\left(\theta + \frac{\pi}{6}\right) = \frac{1+\sqrt{3}\tan\theta}{\sqrt{3}-\tan\theta}$$

(3)

(c) Hence, or otherwise, solve, for $0 \leqslant \theta \leqslant \pi$,

$$1+\sqrt{3}\tan\theta = (\sqrt{3}-\tan\theta)\tan(\pi-\theta)$$

Give your answers as multiples of π.

(6)

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$ $\div \cos A$

$\tan(A+B) = \frac{\tan A\cos B + \sin B}{\cos B - \tan A\sin B}$ $\div \cos B$

$= \frac{\tan A + \tan B}{1 - \tan A\tan B}$

b) $\tan(\theta + \frac{\pi}{6}) = \frac{\tan\theta + \tan\pi/6}{1 - \tan\theta\tan\pi/6}$

$= \frac{\tan\theta + \frac{\sqrt{3}}{3}}{1 - \tan\theta\left(\frac{\sqrt{3}}{3}\right)}$ $\times\sqrt{3}$ $\times\sqrt{3}$

$= \frac{\sqrt{3}\tan\theta + 1}{\sqrt{3} - \tan\theta}$

Leave blank

Question 8 continued

c) $1 + \sqrt{3}\tan\theta = (\sqrt{3} - \tan\theta)\tan(\pi - \theta)$

$$\frac{1 + \sqrt{3}\tan\theta}{\sqrt{3} - \tan\theta} = \tan(\pi - \theta)$$

$$\tan\left(\theta + \frac{\pi}{6}\right) = \tan(\pi - \theta)$$

$$\theta + \frac{\pi}{6} = \pi - \theta$$

$$2\theta = \frac{5\pi}{6}$$

$$\theta = \frac{5\pi}{12}, \frac{11\pi}{12}$$

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Question 8 continued

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Question 8 continued

Leave blank

Question 8 continued

Q8

(Total 13 marks)

TOTAL FOR PAPER: 75 MARKS

END