Centre No.						Paper Reference	Surname	Initial(s)
Candidate No.						6 6 6 5 / 0 1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Monday 24 January 2011 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. (a) Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.
Give the value of α to 3 decimal places.
(3)

(b) Hence write down the minimum value of $7\cos x - 24\sin x$.
(1)

(c) Solve, for $0 \leqslant x < 2\pi$, the equation

$$7\cos x - 24\sin x = 10$$

giving your answers to 2 decimal places.
(5)

$R\cos(\overset{x}{A} + \overset{\alpha}{B}) = R\cos \overset{x}{A} \cos \overset{\alpha}{B} - R\sin \overset{x}{A} \sin \overset{\alpha}{B}$

$R\cos(x + \alpha) = 7\cos x - 24\sin x$

$R\cos\alpha = 7$

$R\sin\alpha = 24$

$\tan\alpha = \frac{24}{7}$

$\alpha = 1.287$ (3dp)

$R^2 = 24^2 + 7^2$

$R = 25$

$25\cos(x + 1.287)$

b) -25

c) $25\cos(x + 1.287) = 10$

$\cos(x + 1.287) = 10/25$

$x + 1.287 = 1.159279481,$
$5.123905826,$
7.442464788

$x = 3.84, 6.16$ (2dp)

Leave blank

2. (a) Express

$$\frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)}$$

as a single fraction in its simplest form.

(4)

Given that

$$f(x) = \frac{4x-1}{2(x-1)} - \frac{3}{2(x-1)(2x-1)} - 2, \qquad x > 1,$$

(b) show that

$$f(x) = \frac{3}{2x-1}$$

(2)

(c) Hence differentiate $f(x)$ and find $f'(2)$.

(3)

2a) $\frac{(4x-1)(2x-1)}{2(x-1)(2x-1)} - \frac{3}{2(x-1)(2x-1)}$

$\frac{(4x-1)(2x-1) - 3}{2(x-1)(2x-1)}$

$\frac{8x^2-6x+1-3}{2(x-1)(2x-1)}$

$\frac{8x^2-6x-2}{2(x-1)(2x-1)}$

$\frac{4x^2-3x-1}{(x-1)(2x-1)}$

$\frac{(4x+1)(x-1)}{(x-1)(2x-1)}$

$\frac{4x+1}{2x-1}$

Question 2 continued

b/ $\frac{4x+1}{2x-1} - 2$

$\frac{4x+1}{(2x-1)} - \frac{2(2x-1)}{(2x-1)}$

$\frac{4x+1-4x+2}{2x-1}$

$\frac{3}{2x-1}$

c/ $f(x) = 3(2x-1)^{-1}$

$f'(x) = -3(2x-1)^{-2} \times \cancel{4}2$

$= -6(2x-1)^{-2}$

$f'(2) = -6(2(2)-1)^{-2}$

$= -2/3$

Leave blank

Leave blank

3. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leqslant \theta < 360°$.

(6)

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= (1 - \sin^2\theta) - \sin^2\theta$
$= 1 - 2\sin^2\theta$

$2(1 - 2\sin^2\theta) = 1 - 2\sin\theta$
$2 - 4\sin^2\theta = 1 - 2\sin\theta$

$0 = 4\sin^2\theta - 2\sin\theta - 1$
~~$\theta = ($~~

$a = 4 \quad b = -2 \quad c = -1$

$\sin\theta = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-1)}}{2(4)}$
$= \dfrac{2 \pm \sqrt{20}}{8}$
$= \dfrac{1 \pm \sqrt{5}}{4}$

+ve $\qquad$ -ve.

$\theta = 54°, 126° \qquad \theta = -18$
$\theta = 198°, 342°$

4. Joan brings a cup of hot tea into a room and places the cup on a table. At time t minutes after Joan places the cup on the table, the temperature, θ°C, of the tea is modelled by the equation

$$\theta = 20 + Ae^{-kt},$$

where A and k are positive constants.

Given that the initial temperature of the tea was 90°C,

(a) find the value of A.

(2)

The tea takes 5 minutes to decrease in temperature from 90°C to 55°C.

(b) Show that $k = \frac{1}{5}\ln 2$.

(3)

(c) Find the rate at which the temperature of the tea is decreasing at the instant when $t = 10$. Give your answer, in °C per minute, to 3 decimal places.

(3)

a) $90 = 20 + Ae^{0}$

$\underline{A = 70^{\circ}}$

$55 = 20 + 70e^{~~-k(5)~~}$

$35 = 70e^{-5k}$

$0.5 = e^{-5k}$

$\ln 0.5 = -5k$

$-\frac{1}{5}\ln 0.5 = k$

$-\frac{1}{5}\ln\frac{1}{2} = k$

$-\frac{1}{5}\ln 2^{-1} = k$

$\underline{\frac{1}{5}\ln 2 = k}$

c) $\frac{d\theta}{dt} = -k(70)e^{-kt}$

when $t = 10$

$\frac{d\theta}{dt} = -2.42601513\,2$

$= 2.~~43~~\,426$ °C per minute 3dp

2.426

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5.

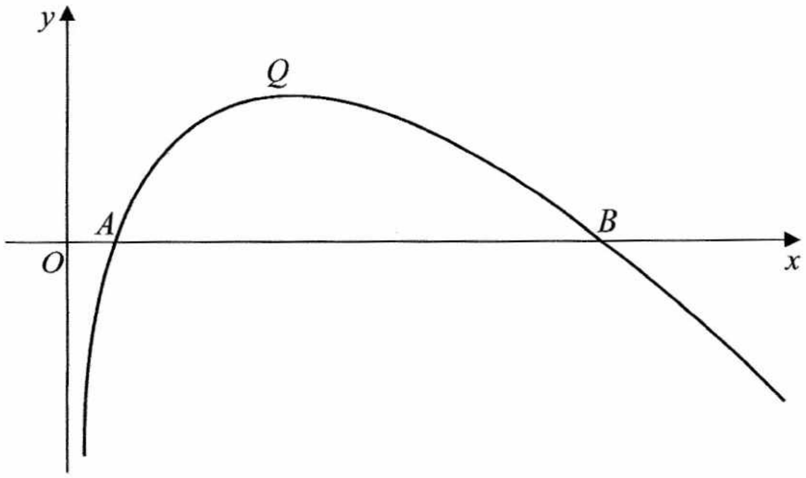

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (8 - x)\ln x, \quad x > 0$$

The curve cuts the x-axis at the points A and B and has a maximum turning point at Q, as shown in Figure 1.

(a) Write down the coordinates of A and the coordinates of B.

(2)

(b) Find $\mathrm{f}'(x)$.

(3)

(c) Show that the x-coordinate of Q lies between 3.5 and 3.6

(2)

(d) Show that the x-coordinate of Q is the solution of

$$x = \frac{8}{1 + \ln x}$$

(3)

To find an approximation for the x-coordinate of Q, the iteration formula

$$x_{n+1} = \frac{8}{1 + \ln x_n}$$

is used.

(e) Taking $x_0 = 3.55$, find the values of x_1, x_2 and x_3.
Give your answers to 3 decimal places.

(3)

Question 5 continued

a) A: $(1,0)$

B: $(8,0)$

b) $f(x) = (8-x)(\ln x)$

$u = 8 - x \quad v = \ln x$

$\frac{du}{dx} = -1 \quad \frac{dv}{dx} = \frac{1}{x}$

~~at~~ $f'(x) = \frac{8-x}{x} - \ln x$

d) Q is where $\frac{dy}{dx} = 0$

$\frac{8-x}{x} - \ln x = 0$

$8 - x - x\ln x = 0$

$8 = x + x\ln x$

$8 = x(1 + \ln x)$

$\frac{8}{1+\ln x} = x$

c) $f'(3.5) = 0.0329513172$

$f'(3.6) = -0.0587116232\,4$

change of sign $\therefore$ Q lies between 3.5 and 3.6

e) $(x_1) = \frac{8}{1+\ln(x_0)} =$

$x_0 = 3.55$

$x_1 = 3.529$

$x_2 = 3.538$

$x_3 = 3.534$

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Leave blank

6. The function f is defined by

$$\text{f}: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R},\ x \neq 5$$

(a) Find $\text{f}^{-1}(x)$.

(3)

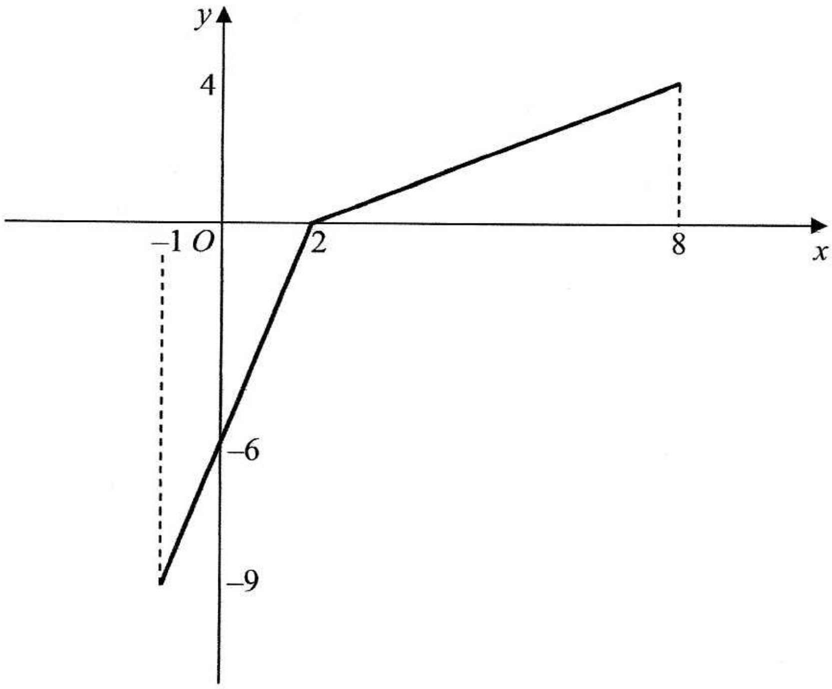

Figure 2

The function g has domain $-1 \leqslant x \leqslant 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = \text{g}(x)$.

(b) Write down the range of g.

(1)

(c) Find gg(2).

(2)

(d) Find fg(8).

(2)

(e) On separate diagrams, sketch the graph with equation

(i) $y = |\text{g}(x)|$,

(ii) $y = \text{g}^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(f) State the domain of the inverse function g^{-1}.

(1)

Leave blank

Question 6 continued

a) $y = \frac{3-2x}{x-5}$

$x = \frac{3-2y}{y-5}$

$x(y-5) = 3-2y$

$xy - 5x = 3 - 2y$

$xy + 2y = 3 + 5x$

$y(x+2) = 3 + 5x$

$y = \frac{3+5x}{x+2}$

$f^{-1}(x) = \frac{3+5x}{x+2}$

b) $-9 \leqslant g(x) \leqslant 4$

c) $g(2) = 0$

$g(0) = \underline{\underline{-6}}$

$\therefore \boxed{gg(2) = -6}$

d) $g(8) = 4$

$f(4) = \frac{3-2(4)}{(4)-5}$

$= \underline{\underline{5}}$

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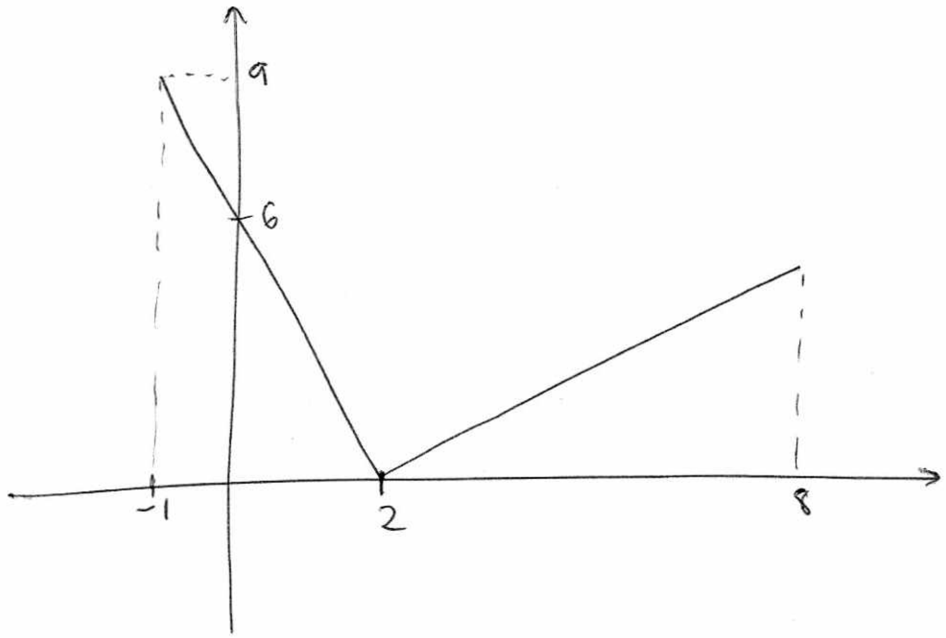
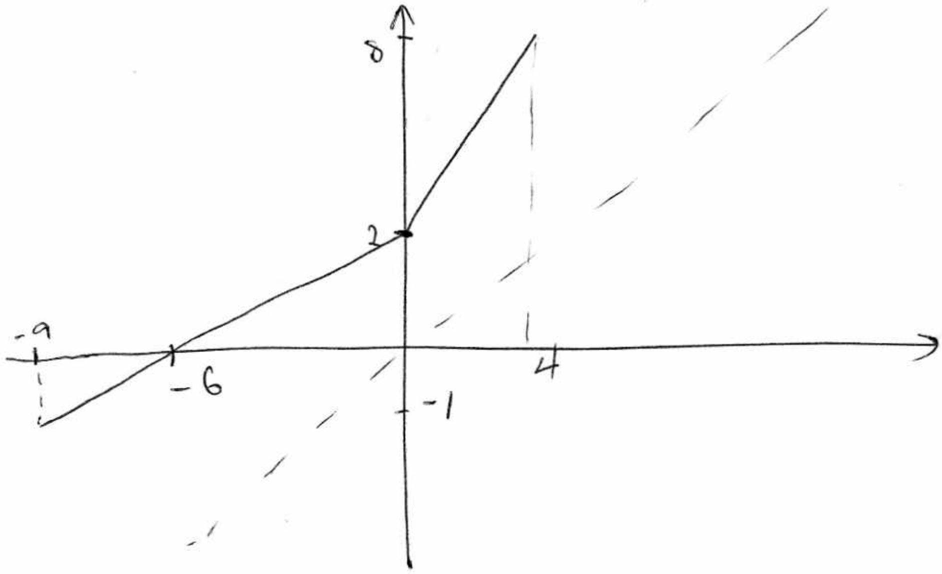
Question 6 continued

e i)





f) $-9 \leq x \leq 4$

Leave blank

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.
Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

$u = 3 + \sin 2x \qquad v = 2 + \cos 2x$

$\frac{du}{dx} = 2\cos 2x \qquad \frac{dv}{dx} = -2\sin 2x$

$$\frac{dy}{dx} = \frac{(2+\cos 2x)(2\cos 2x) - (3+\sin 2x)(-2\sin 2x)}{(2+\cos 2x)^2}$$

$$= \frac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2+\cos 2x)}$$

$$= \frac{4\cos 2x + 6\sin 2x + (2\cos^2 2x + 2\sin^2 2x)}{2+\cos 2x}$$

$$= \frac{4\cos 2x + 6\sin 2x + 2}{(2+\cos 2x)^2}$$

$$= \frac{6\sin 2x + 4\cos 2x + 2}{(2+\cos 2x)^2}$$

b) when $x = \pi/2 \qquad y = \frac{3 + \sin(2\frac{\pi}{2})}{2 + \cos(2\pi/2)}$

$= 3$

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Question 7 continued

when $x = \pi/2$

$$\frac{dy}{dx} = \frac{6\sin\left(2\frac{\pi}{2}\right) + 4\cos\left(2\frac{\pi}{2}\right) + 2}{\left(2 + \cos\left(2\frac{\pi}{2}\right)\right)^2}$$

$$= -2$$

$y = mx + c$

$3 = -2(\pi/2) + c$

$3 = -\pi + c$

$3 + \pi = c$

$\underline{\underline{y = -2x + (3 + \pi)}}$

Leave blank

8. (a) Given that

$$\frac{d}{dx}(\cos x) = -\sin x$$

show that $\frac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y$$

(b) find $\frac{dx}{dy}$ in terms of y.

(2)

(c) Hence find $\frac{dy}{dx}$ in terms of x.

(4)

$\sec x = \frac{1}{\cos x} = (\cos x)^{-1}$

$y = (\cos x)^{-1}$

$\frac{dy}{dx} = -(\cos x)^{-2}(-\sin x)$

$= (\cos x)^{-2}(\sin x)$

$= \frac{\sin x}{\cos^2 x}$

$= \frac{\sin x}{\cos x} \times \frac{1}{\cos x}$

$= \tan x \times \sec x$

$= \sec x \tan x$

b/ $x = \sec 2y$

$\frac{dx}{dy} = 2 \sec 2y \tan 2y$

c/ $\frac{dy}{dx} = \frac{1}{2 \sec 2y \tan 2y}$

Question 8 continued

$= \frac{1}{2} \cdot \frac{1}{\sec 2y} \cdot \frac{1}{\tan 2y}$

$= \cancel{\frac{1}{2x}}$

$= \frac{1}{2} \cdot \frac{1}{x} \cdot \frac{1}{\tan 2y}$

$= \frac{1}{2x} \cdot \frac{1}{\tan 2y}$

$$\boxed{\begin{aligned} \tan^2 x + 1 &= \sec^2 x \\ \tan^2 x &= \sec^2 x - 1 \\ \tan x &= \sqrt{\sec^2 x - 1} \end{aligned}}$$

$= \frac{1}{2x} \cdot \frac{1}{\sqrt{\sec^2 2y - 1}}$

$= \frac{1}{2x} \cdot \frac{1}{\sqrt{x^2 - 1}}$

$= \frac{1}{2x\sqrt{x^2-1}}$

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