Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	5	/	0	1	Signature	

Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3

Advanced

Wednesday 20 January 2010 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer to each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 9 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1. Express

$$\frac{x+1}{3x^2-3} - \frac{1}{3x+1}$$

as a single fraction in its simplest form.

(4)

$$\frac{x+1}{3(x^2-1)} - \frac{1}{3x+1}$$

$$\frac{1 \ \cancel{x+1}}{3(\cancel{x+1})(x-1)} - \frac{1}{3x+1}$$

$$\frac{1}{3(x-1)} - \frac{1}{(3x+1)}$$

$$\frac{1(3x+1)}{3(x-1)(3x+1)} - \frac{3(x-1)}{3(x-1)(3x+1)}$$

$$\frac{3x+1-3x+3}{3(x-1)(3x+1)}$$

$$\frac{4}{3(x-1)(3x+1)}$$

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Question 1 continued

Q1

(Total 4 marks)

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2.

$$f(x) = x^3 + 2x^2 - 3x - 11$$

(a) Show that f(x) = 0 can be rearranged as

$$x = \sqrt{\left(\frac{3x+11}{x+2}\right)}, \quad x \neq -2.$$

(2)

The equation f(x) = 0 has one positive root α.

The iterative formula $x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)}$ is used to find an approximation to α.

(b) Taking $x_1 = 0$, find, to 3 decimal places, the values of x_2, x_3 and x_4.

(3)

(c) Show that $\alpha = 2.057$ correct to 3 decimal places.

(3)

a) $x^3 + 2x^2 - 3x - 11 = 0$

$x^3 + 2x^2 = 3x + 11$

$x^2(x+2) = 3x + 11$

$x^2 = \frac{3x+11}{x+2}$

$x = \sqrt{\frac{3x+11}{x+2}}$

b) $x_{n+1} = \sqrt{\frac{3(x_n) + 11}{(x_n) + 2}}$

$x_1 = 0$

$x_2 = 2.345$

$x_3 = 2.037$

$x_4 = 2.059$

c) Lower bound = 2.0565 upper bound = 2.0575

$f(2.0565) = -0.01378163788$

$f(2.0575) = 4.140109375 \times 10^{-3}$

change of sign $\therefore \alpha = 2.057$ to 3 d.p.

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Question 2 continued

Q2

(Total 8 marks)

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3. (a) Express $5\cos x - 3\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.
(4)

(b) Hence, or otherwise, solve the equation

$$5\cos x - 3\sin x = 4$$

for $0 \leqslant x < 2\pi$, giving your answers to 2 decimal places.
(5)

3a/
$R\cos(~~x~~ x+\alpha) = R\cos x \cos\alpha - R\sin x \sin\alpha$

~~5 cos α~~ $R\cos\alpha = 5$
$R\sin\alpha = 3$

$\tan\alpha = 3/5$
$\alpha = 0.5404195003$

$R^2 = 5^2 + 3^2$
$R = \sqrt{34}$

$\sqrt{34}\cos(x + 0.5404195003)$

b/ $\sqrt{34}\cos(x + 0.5404195003) = 4$
$\cos(x + 0.5404195003) = \frac{4}{\sqrt{34}}$
$x + 0.5404195003 = 0.8148269...$
$5.468358...$

$x = 0.27, 4.93$

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Question 3 continued

Q3

(Total 9 marks)

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4. (i) Given that $y = \dfrac{\ln(x^2+1)}{x}$, find $\dfrac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$.

(5)

i/ $u = \ln(x^2+1)$

$v = x$

$\dfrac{du}{dx} = \dfrac{2x}{x^2+1}$

$\dfrac{dv}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{\dfrac{2x^2}{x^2+1} - \ln(x^2+1)}{x^2}$

ii/ $x = \tan y$

$\dfrac{dx}{dy} = \sec^2 y$

$\dfrac{dx}{dy} = 1 + \tan^2 y$

$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

$\dfrac{\cos^2 x}{\cos^2 x} + \dfrac{\sin^2 x}{\cos^2 x} = \dfrac{1}{\cos^2 x}$

$1 + \tan^2 x = \sec^2 x$

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Question 4 continued

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Question 4 continued

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Question 4 continued

Q4

(Total 9 marks)

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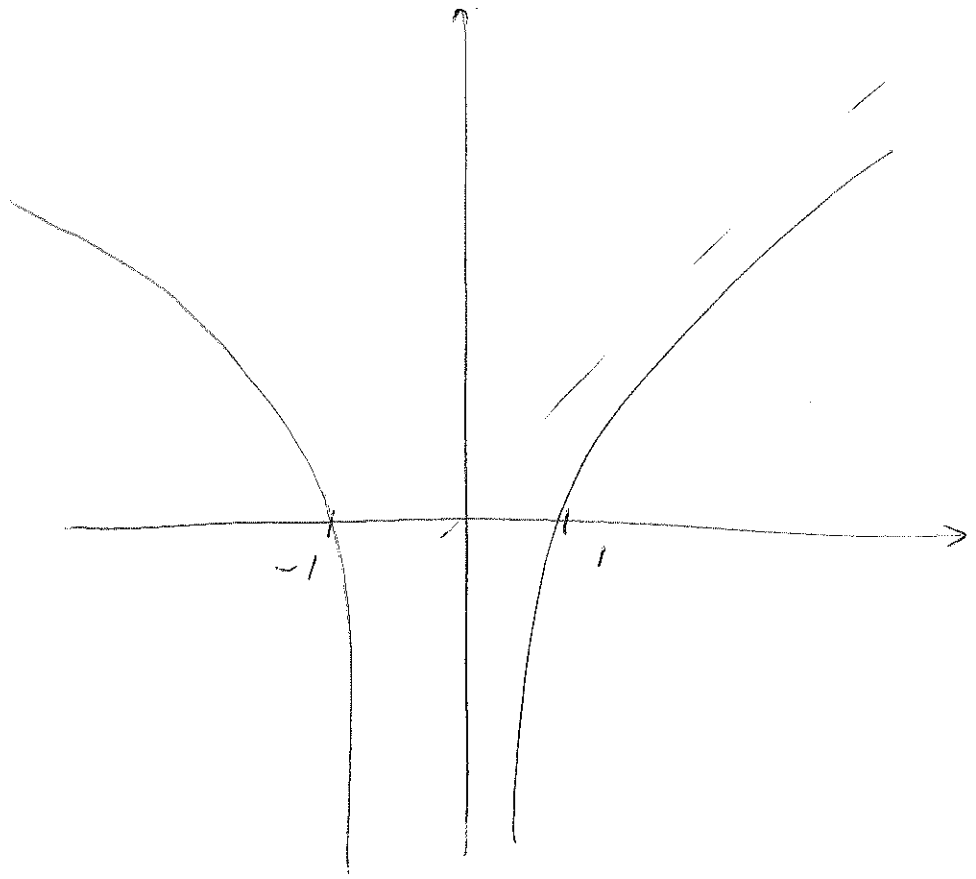
5. Sketch the graph of $y = \ln|x|$, stating the coordinates of any points of intersection with the axes.

(3)



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Question 5 continued

Q5

(Total 3 marks)

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6.

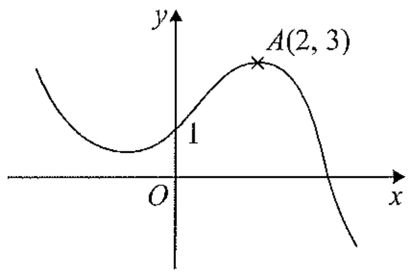


Figure 1

Figure 1 shows a sketch of the graph of $y = \mathrm{f}(x)$.

The graph intersects the y-axis at the point (0, 1) and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = \mathrm{f}(-x) + 1$,

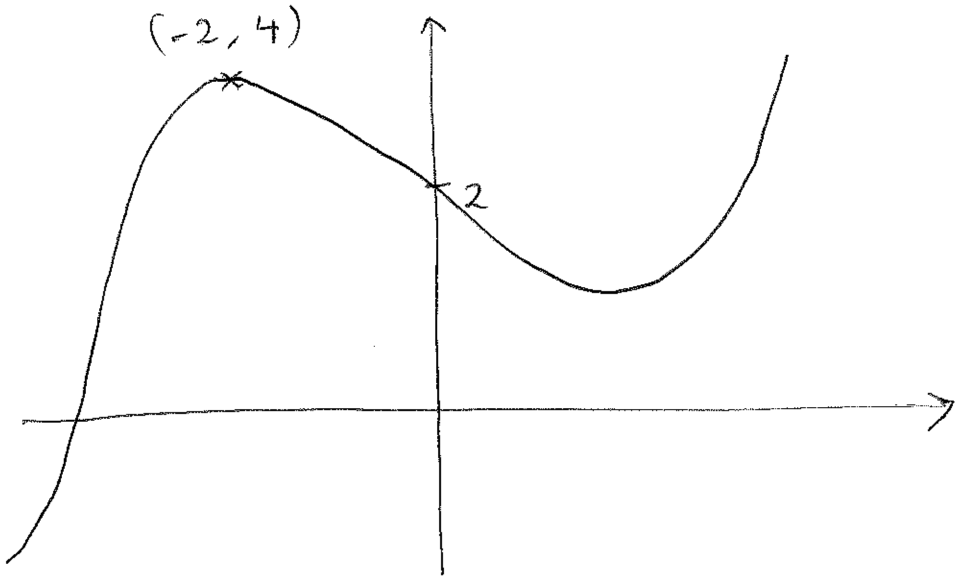
(ii) $y = \mathrm{f}(x + 2) + 3$,

(iii) $y = 2\mathrm{f}(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

(9)

i/



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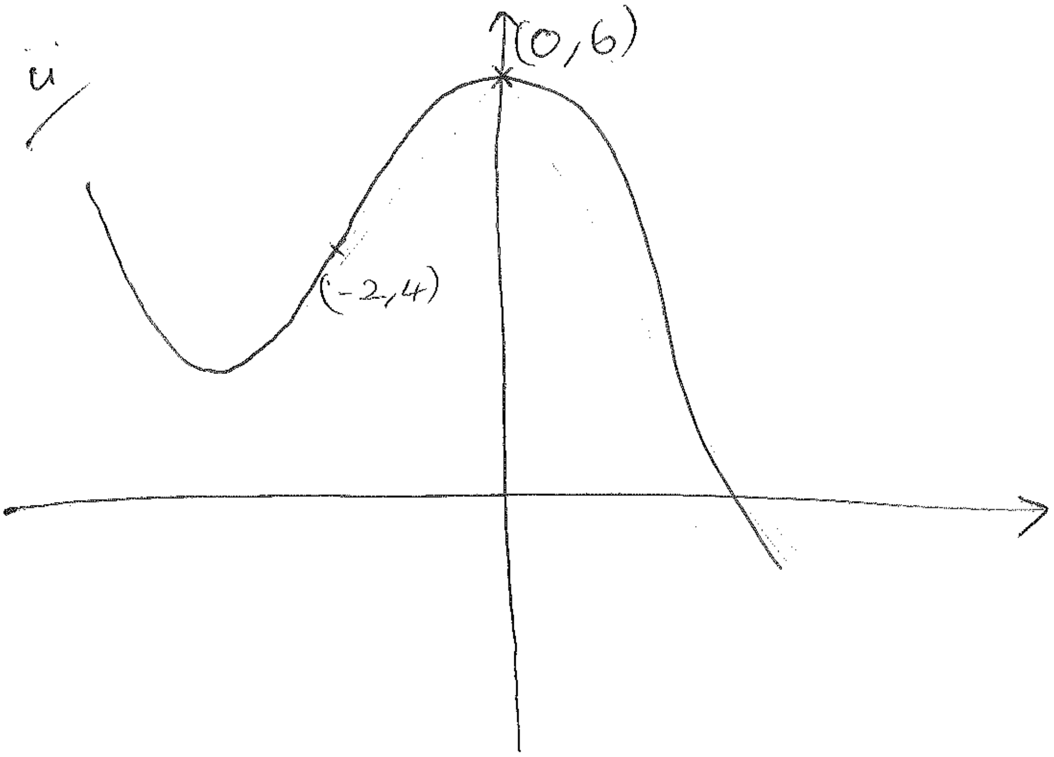
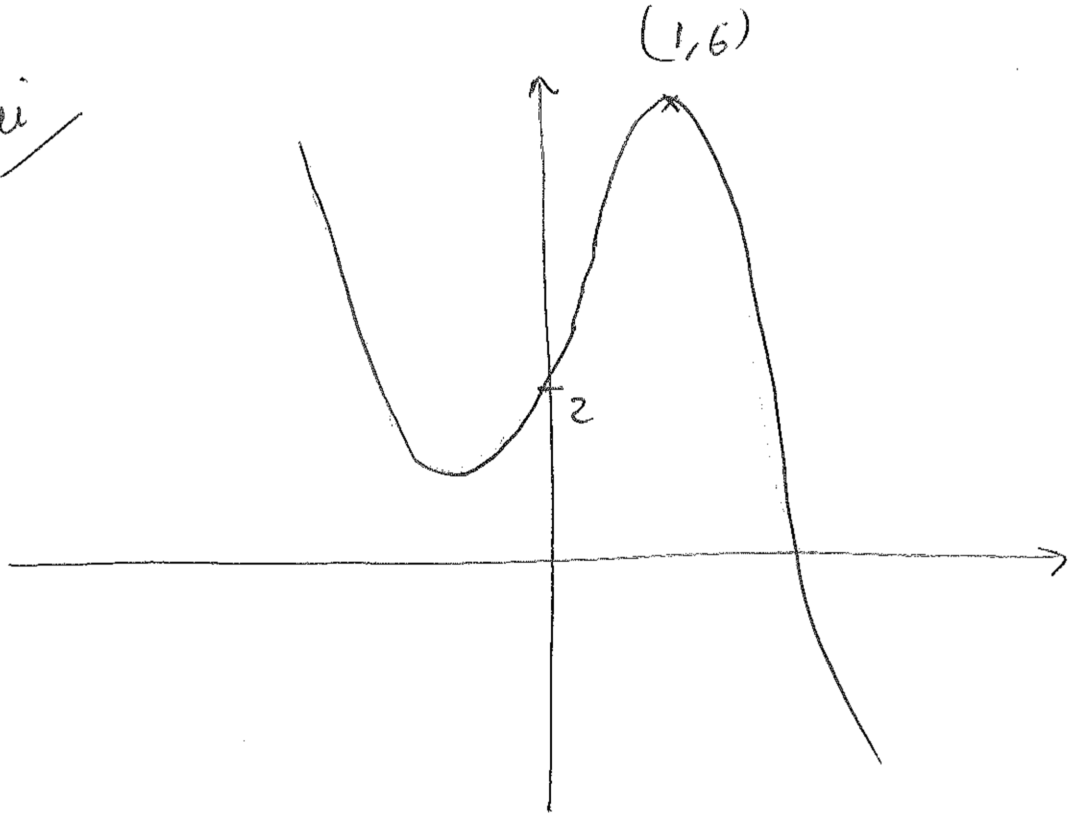
Question 6 continued

ii



iii



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Question 6 continued

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Question 6 continued

Q6

(Total 9 marks)

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7. (a) By writing $\sec x$ as $\frac{1}{\cos x}$, show that $\frac{d(\sec x)}{dx} = \sec x \tan x$.

(3)

Given that $y = e^{2x} \sec 3x$,

(b) find $\frac{dy}{dx}$.

(4)

The curve with equation $y = e^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

a) $\sec x$

$\frac{1}{\cos x}$

$y = (\cos x)^{-1}$

$\frac{dy}{dx} = -1(\cos x)^{-2} \cdot -\sin x$

$= \sin x (\cos x)^{-2}$

$= \frac{\sin x}{\cos^2 x}$

$= \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x}$

$= \tan x \sec x$

$= \sec x \tan x$

b) $u = e^{2x}$ $\quad v = \sec 3x$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = 3 \sec 3x \tan 3x$

$\frac{dy}{dx} = 3e^{2x} \sec 3x \tan 3x + 2e^{2x} \sec 3x$

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Question 7 continued

c/ turning point where $\frac{dy}{dx} = 0$

$3e^{2x} \sec 3x \tan 3x + 2e^{2x} \sec 3x = 0$

$(e^{2x})(\sec 3x)(3 \tan 3x + 2) = 0$

$\sec 3x = 0$ $\quad \tan 3x = -2/3$

$\cos 3x = 0$ $\quad 3x = -0.5880026035$

$3x = \frac{1}{2}\pi, -\frac{1}{2}\pi$ $\quad x = -0.1960008678$

$x = \frac{1}{6}\pi, -\frac{1}{6}\pi$

$\underline{a = -0.196}$ $\quad b = e^{2(-0.196)} \sec(3(-0.196))$

$b = \underline{0.812}$

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Question 7 continued

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Question 7 continued

Q7

(Total 11 marks)

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8. Solve

$$\operatorname{cosec}^2 2x - \cot 2x = 1$$

for $0 \leqslant x \leqslant 180°$.

(7)

$\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x}$

$1 + \cot^2 x = \operatorname{cosec}^2 x$

$\operatorname{cosec}^2 2x - \cot 2x = 1$

$1 + \cot^2 2x - \cot 2x = 1$

$\cot^2 2x - \cot 2x = 0$

$\cot 2x(\cot 2x - 1) = 0$

$\cot 2x = 0 \quad \cot 2x = 1$

$\tan 2x = \infty \quad \tan 2x = 1$

$2x = 90, 270 \quad 2x = 45, 225$

$x = 22.5, 45, 112.5, 135$

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Question 8 continued

Q8

(Total 7 marks)

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9. (i) Find the exact solutions to the equations

(a) $\ln(3x-7) = 5$ **(3)**

(b) $3^x e^{7x+2} = 15$ **(5)**

(ii) The functions f and g are defined by

$$f(x) = e^{2x} + 3, \qquad x \in \mathbb{R}$$
$$g(x) = \ln(x-1), \qquad x \in \mathbb{R},\ x > 1$$

(a) Find f^{-1} and state its domain. **(4)**

(b) Find fg and state its range. **(3)**

9i a/ $\ln(3x-7) = 5$

$3x - 7 = e^5$

$3x = e^5 + 7$

$x = \dfrac{e^5 + 7}{3}$

b/ $3^x e^{7x+2} = 15$

$\ln(3^x e^{7x+2}) = \ln 15$

$\ln 3^x + \ln e^{7x+2} = \ln 15$

$x \ln 3 + 7x + 2 = \ln 15$

$x(\ln 3 + 7) = \ln(15) - 2$

$x = \dfrac{\ln(15) - 2}{\ln(3) + 7}$

ii/ a/ $f(x) = e^{2x} + 3$

$y = e^{2x} + 3$

$y - 3 = e^{2x}$

$\ln(y-3) = 2x$

$x = \frac{1}{2}\ln(y-3)$

$f^{-1}(x) = \frac{1}{2}\ln(x-3)$ $\quad x > 3$

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Question 9 continued

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Question 9 continued

b) $f(x) = e^{2x} + 3$

$g(x) = \ln(x-1)$

$fg(x) = e^{2\ln(x-1)} + 3$

$= e^{\ln(x-1)^2} + 3$

$= (x-1)^2 + 3$

$fg(x) > 3$

N 3 5 3 8 1 A 0 2 5 2 8

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Question 9 continued

N 3 5 3 8 1 A 0 2 7 2 8

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Question 9 continued

Q9

(Total 15 marks)

TOTAL FOR PAPER: 75 MARKS

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