



1.

$$y = 3^x + 2x$$

(a) Complete the table below, giving the values of  $y$  to 2 decimal places.

$x$	0	0.2	0.4	0.6	0.8	1
$y$	1	1.65	2.35	3.13	4.01	5

(2)

(b) Use the trapezium rule, with all the values of  $y$  from your table, to find an approximate

value for  $\int_0^1 (3^x + 2x) dx$ .

(4)

$$\begin{aligned} & b/ \quad 0.2 \left( \frac{1}{2} + 1.65 + 2.35 + 3.13 + 4.01 + \frac{5}{2} \right) \\ & \quad = \underline{\underline{2.828 \text{ units}^2}} \end{aligned}$$

2.

$$f(x) = 3x^3 - 5x^2 - 58x + 40$$

(a) Find the remainder when  $f(x)$  is divided by  $(x-3)$ .

(2)

Given that  $(x-5)$  is a factor of  $f(x)$ ,

(b) find all the solutions of  $f(x) = 0$ .

(5)

$$\begin{aligned} \text{a/ } f(3) &= 3(3)^3 - 5(3)^2 - 58(3) + 40 \\ &= \underline{\underline{-98}} \end{aligned}$$

$$\begin{array}{r} \text{b/} \\ x-5 \overline{) \begin{array}{r} 3x^2 + 10x - 8 \\ 3x^3 - 5x^2 - 58x + 40 \\ \underline{3x^3 - 15x^2} \\ 10x^2 - 58x \\ \underline{10x^2 - 50x} \\ -8x + 40 \\ \underline{-8x + 40} \\ 0 \end{array}} \end{array}$$

$$\begin{aligned} &(x-5)(3x^2 + 10x - 8) \\ &\underline{(x-5)(3x-2)(x+4)} = 0 \end{aligned}$$

$$\underline{\underline{x=5}} \quad \underline{\underline{x=\frac{2}{3}}} \quad \underline{\underline{x=-4}}$$



3.

 $y = x^2 - k\sqrt{x}$ , where  $k$  is a constant.(a) Find  $\frac{dy}{dx}$ .

(2)

(b) Given that  $y$  is decreasing at  $x = 4$ , find the set of possible values of  $k$ .

(2)

$$a) \quad y = x^2 - kx^{1/2}$$

$$\frac{dy}{dx} = 2x - \frac{1}{2}kx^{-1/2}$$

$$b) \quad \text{when } x = 4 \quad \frac{dy}{dx} < 0$$

$$2(4) - \frac{1}{2}k(4)^{-1/2} < 0$$

$$8 - \frac{1}{2}k\left(\frac{1}{2}\right) < 0$$

$$8 - \frac{1}{4}k < 0$$

$$8 < \frac{1}{4}k$$

$$\underline{\underline{32 < k}}$$

$$\underline{\underline{k > 32}}$$

4. (a) Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of  $(1+ax)^7$ , where  $a$  is a constant. Give each term in its simplest form. (4)

Given that the coefficient of  $x^2$  in this expansion is 525,

- (b) find the possible values of  $a$ . (2)

$$\begin{array}{cccc} & & 1 & 7 & 21 \\ a/ & & & & \\ & & (1)^7 & + 7(1)^6(ax) & + 21(1)^5(ax)^2 \\ & & 1 & + 7ax & + 21a^2x^2 \end{array}$$

$$\begin{array}{l} b/ \quad 21a^2 = 525 \\ \quad \quad a^2 = 25 \\ \quad \quad \underline{\underline{a = \pm 5}} \end{array}$$

5. (a) Given that  $5 \sin \theta = 2 \cos \theta$ , find the value of  $\tan \theta$ .

(1)

- (b) Solve, for  $0 \leq x < 360^\circ$ ,

$$5 \sin 2x = 2 \cos 2x,$$

giving your answers to 1 decimal place.

(5)

$$a/ \quad 5 \sin \theta = 2 \cos \theta$$

$$5 \tan \theta = 2$$

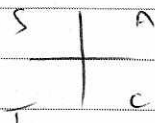
$$\underline{\underline{\tan \theta = 2/5}}$$

$$b/ \quad \tan(2x) = 2/5$$

$$2x = 21.80140949, 201.8014095,$$

$$381.8014095, 561.8014095$$

$$x = \underline{\underline{10.9, 100.9, 190.9, 280.9}}$$



6.

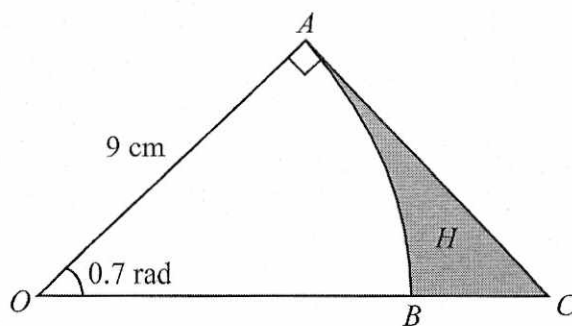


Figure 1

Figure 1 shows the sector  $OAB$  of a circle with centre  $O$ , radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc  $AB$ . (2)

(b) Find the area of the sector  $OAB$ . (2)

The line  $AC$  shown in Figure 1 is perpendicular to  $OA$ , and  $OBC$  is a straight line.

(c) Find the length of  $AC$ , giving your answer to 2 decimal places. (2)

The region  $H$  is bounded by the arc  $AB$  and the lines  $AC$  and  $CB$ .

(d) Find the area of  $H$ , giving your answer to 2 decimal places. (3)

$$\begin{aligned} \text{a) Arc Length} &= \theta r \\ &= 0.7 \times 9 \\ &= \underline{\underline{6.3 \text{ cm}}} \end{aligned}$$

$$\begin{aligned} \text{b) Sector Area} &= \frac{\theta}{2} r^2 \\ &= \frac{0.7}{2} \times 9^2 \\ &= 28.35 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{c) } \tan(\theta) &= \frac{o}{a} \\ \tan(0.7) &= \frac{x}{9} \\ 9 \tan(0.7) &= x \\ x &= 7.58 \text{ cm (2dp)} \end{aligned}$$



## Question 6 continued

$$d) \text{ Area of triangle} = \frac{1}{2} \times 9 \times 7.58$$

$$= 34.1 \text{ cm}^2 \text{ (51)}$$

$$34.1 - 28.35 = \underline{\underline{5.76 \text{ cm}^2}}$$



7. (a) Given that

$$2 \log_3(x-5) - \log_3(2x-13) = 1,$$

show that  $x^2 - 16x + 64 = 0$ .

(5)

(b) Hence, or otherwise, solve  $2 \log_3(x-5) - \log_3(2x-13) = 1$ .

(2)

$$a) \quad 2 \log_3(x-5) - \log_3(2x-13) = 1$$

$$\log_3(x-5)^2 - \log_3(2x-13) = 1$$

$$\log_3\left(\frac{(x-5)^2}{2x-13}\right) = 1$$

$$\frac{(x-5)^2}{2x-13} = 3$$

$$(x-5)^2 = 3(2x-13)$$

$$x^2 - 5x - 5x + 25 = 6x - 39$$

$$x^2 - 10x + 25 = 6x - 39$$

$$\underline{x^2 - 16x + 64 = 0}$$

$$b) \quad (x-8)(x-8) = 0$$

$$\underline{\underline{x=8}}$$

8.

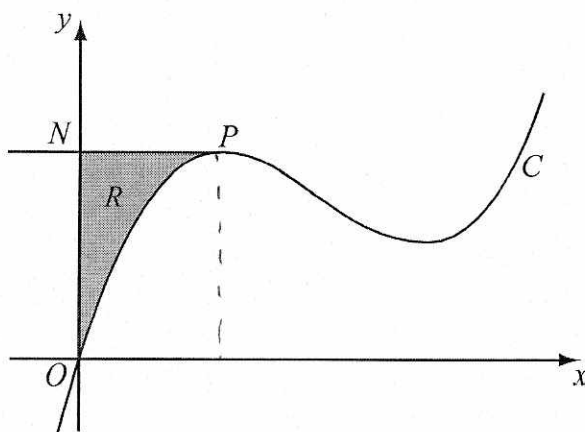


Figure 2

Figure 2 shows a sketch of part of the curve  $C$  with equation

$$y = x^3 - 10x^2 + kx,$$

where  $k$  is a constant.

The point  $P$  on  $C$  is the maximum turning point.

Given that the  $x$ -coordinate of  $P$  is 2,

(a) show that  $k = 28$ .

(3)

The line through  $P$  parallel to the  $x$ -axis cuts the  $y$ -axis at the point  $N$ .

The region  $R$  is bounded by  $C$ , the  $y$ -axis and  $PN$ , as shown shaded in Figure 2.

(b) Use calculus to find the exact area of  $R$ .

(6)

a/ turning point is where  $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 3x^2 - 20x + k$$

$$\frac{dy}{dx} = 0 \quad \text{when } x = 2$$

$$0 = 3(2)^2 - 20(2) + k$$

$$0 = 12 - 40 + k$$

$$\underline{\underline{k = 28}}$$

## Question 8 continued

$$y = x^3 - 10x^2 + 28x$$

when  $x = 2$

$$\begin{aligned} y &= (2)^3 - 10(2)^2 + 28(2) \\ &= 24 \end{aligned}$$

$$\text{Area of rectangle} = 2 \times 24 = 48 \text{ units}^2$$

$$\int_0^2 x^3 - 10x^2 + 28x \, dx$$

$$\left[ \frac{x^4}{4} - \frac{10x^3}{3} + 14x^2 + c \right]_0^2$$

$$\left[ \frac{(2)^4}{4} - \frac{10(2)^3}{3} + 14(2)^2 \right] - [0]$$

$$= \frac{100}{3} \text{ units}^2$$

$$R = 48 - \frac{100}{3} = \underline{\underline{\frac{44}{3} \text{ units}^2}}$$

9. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(a) Show that the predicted adult population at the end of Year 2 is 25 750. (1)

(b) Write down the common ratio of the geometric sequence. (1)

The model predicts that Year  $N$  will be the first year in which the adult population of the town exceeds 40 000.

(c) Show that

$$(N-1)\log 1.03 > \log 1.6 \quad (3)$$

(d) Find the value of  $N$ . (2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(e) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000. (3)

a/  $a = 25000 \quad r = 1.03$

$$25000 \times 1.03 = \underline{\underline{25750}}$$

b/  $1.03$

c/  ~~$40000 < \frac{N}{2} (2(25000) + (N-1))$~~

~~$$S_n = \frac{a(1-r^n)}{1-r}$$~~

~~$$40000 < \frac{25000(1-1.03^N)}{1-1.03}$$~~

~~$$-1200 < 25000(1-1.03^N)$$~~



Question 9 continued

$$\frac{-1200}{25000} < 1 - 1.03^N$$

$$\frac{1.03^N - 1200}{25000} < 1$$

$$1.03^N < 37000$$

$$u_n = ar^{n-1}$$

$$40000 < 25000 (1.03)^{N-1}$$

$$1.6 < 1.03^{N-1}$$

$$\log 1.6 < \log 1.03^{N-1}$$

$$\log 1.6 < (N-1) \log 1.03$$

$$15.9... < N-1$$

$$16.9... < N$$

$$\underline{N = 17}$$

$$e/ \quad S_n = \frac{a(1-r^n)}{1-r}$$

$$S_{10} = \frac{25000(1-1.03^{10})}{1-1.03}$$

$$= 1286596.98$$

$$\underline{\underline{\$287000}}$$

10. The circle  $C$  has centre  $A(2, 1)$  and passes through the point  $B(10, 7)$ .

(a) Find an equation for  $C$ .

(4)

The line  $l_1$  is the tangent to  $C$  at the point  $B$ .

(b) Find an equation for  $l_1$ .

(4)

The line  $l_2$  is parallel to  $l_1$  and passes through the mid-point of  $AB$ .

Given that  $l_2$  intersects  $C$  at the points  $P$  and  $Q$ ,

(c) find the length of  $PQ$ , giving your answer in its simplest surd form.

(3)

$$a) \quad (x-2)^2 + (y-1)^2 = r^2 \quad (10, 7)$$

$$\begin{aligned} (10-2)^2 + (7-1)^2 &= r^2 \\ 8^2 + 6^2 &= r^2 \\ 100 &= r^2 \end{aligned}$$

$$\underline{(x-2)^2 + (y-1)^2 = 100}$$

$$b) \quad \text{gradient of } AB: \quad \begin{matrix} x_1, y_1 & x_2, y_2 \\ (2, 1) & (10, 7) \end{matrix}$$

$$m = \frac{7-1}{10-2} = \frac{6}{8} = \frac{3}{4}$$

$$l_1 \text{ is perpendicular } \therefore m = -\frac{4}{3}$$

$$y = -\frac{4}{3}x + c \quad (10, 7)$$

$$7 = -\frac{4}{3}(10) + c$$

$$7 = -\frac{40}{3} + c$$

$$21 = -40 + 3c$$

$$61 = 3c$$

$$c = 61/3$$

Question 10 continued

Leave blank

$$y = -\frac{4}{3}x + \frac{61}{3}$$

c/ ~~2~~  $m = -\frac{4}{3}$

mid point of AB  $\left(\frac{2+10}{2}, \frac{1+7}{2}\right)$

$$\left(\underset{x}{6}, \underset{y}{4}\right)$$

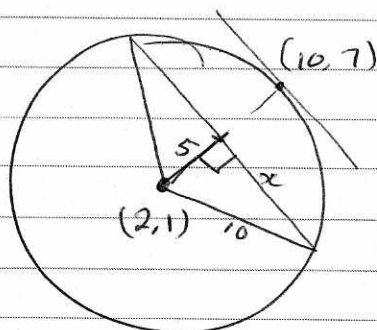
$$y = -\frac{4}{3}x + c$$

$$4 = -\frac{4}{3}(6) + c$$

$$4 = -8 + c$$

$$12 = c$$

$$y = -\frac{4}{3}x + 12$$



$$5^2 + x^2 = 10^2$$

$$25 + x^2 = 100$$

$$x^2 = 75$$

$$x = \sqrt{75}$$

$$= \sqrt{3 \times 25}$$

$$= \underline{5\sqrt{3}}$$

$$\Rightarrow 2 \times 5\sqrt{3} = \underline{\underline{10\sqrt{3}}}$$