

1. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$

Giving your answers to 3 significant figures where appropriate, find

- (a) the 20th term of the series, (2)
- (b) the sum of the first 20 terms of the series, (2)
- (c) the sum to infinity of the series. (2)

$$\begin{aligned} \text{a)} \quad u_n &= ar^{n-1} \\ u_{20} &= ar^{19} \\ &= (360) \left(\frac{7}{8}\right)^{19} \\ &= 28.5 \quad (3\text{sf}) \end{aligned}$$

$$\begin{aligned} \text{b)} \quad S_n &= \frac{a(1-r^n)}{1-r} \\ S_{20} &= \frac{(360) \left(1 - \left(\frac{7}{8}\right)^{20}\right)}{1 - \frac{7}{8}} \\ &= 2680 \quad (3\text{sf}) \end{aligned}$$

$$\begin{aligned} \text{c)} \quad S_\infty &= \frac{a}{1-r} \\ &= \frac{360}{1 - \frac{7}{8}} \\ &= 2880 \end{aligned}$$



2. A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C .

(4)

$$(x - a)^2 + (y - b)^2 = r^2$$

$$(x + 1)^2 + (y - 7)^2 = r^2$$

$$(0 + 1)^2 + (0 - 7)^2 = r^2$$

$$1 + 49 = r^2$$

$$50 = r^2$$

$$(x + 1)^2 + (y - 7)^2 = 50$$



3. (a) Find the first 4 terms of the binomial expansion, in ascending powers of x , of

$$\left(1 + \frac{x}{4}\right)^8$$

giving each term in its simplest form.

(4)

- (b) Use your expansion to estimate the value of $(1.025)^8$, giving your answer to 4 decimal places.

(3)

a) $1 \quad 8 \quad 28 \quad 56$

$$1(1)^8 + 8(1)^7\left(\frac{x}{4}\right) + 28(1)^6\left(\frac{x}{4}\right)^2 + 56(1)^5\left(\frac{x}{4}\right)^3$$

$$1 + 2x + \frac{7}{4}x^2 + \frac{7}{8}x^3$$

b) $1 + \frac{x}{4} = 1.025$

$$x = 0.1$$

$$1 + 2(0.1) + \frac{7}{4}(0.1)^2 + \frac{7}{8}(0.1)^3$$

$$= 1.218375$$

$$= 1.2184 \text{ (4dp)}$$



4. Given that $y = 3x^2$,

(a) show that $\log_3 y = 1 + 2\log_3 x$

(3)

(b) Hence, or otherwise, solve the equation

$$1 + 2\log_3 x = \log_3(28x - 9)$$

(3)

a) $y = 3x^2$

$$\log_3 y = \log_3 3x^2$$

$$\log_3 y = \log_3 3 + \log_3 x^2$$

$$\log_3 y = 1 + 2\log_3 x$$

b)

$$\log_3 y = \log_3(28x - 9)$$

$$y = 28x - 9$$

$$y = 3x^2$$

$$3x^2 = 28x - 9$$

$$3x^2 - 28x + 9 = 0$$

$$(3x - 1)(x - 9) = 0$$

$$\underline{x = \frac{1}{3}} \quad \underline{x = 9}$$



5. $f(x) = x^3 + ax^2 + bx + 3$, where a and b are constants.

Given that when $f(x)$ is divided by $(x+2)$ the remainder is 7,

(a) show that $2a - b = 6$

(2)

Given also that when $f(x)$ is divided by $(x-1)$ the remainder is 4,

(b) find the value of a and the value of b .

(4)

$$a) f(-2) = 7$$

$$(-2)^3 + a(-2)^2 + b(-2) + 3 = 7$$

$$-8 + 4a - 2b + 3 = 7$$

$$4a - 2b = 12$$

$$2a - b = 6$$

$$b) f(1) = 4$$

$$(1)^3 + a(1)^2 + b(1) + 3 = 4$$

$$1 + a + b + 3 = 4$$

$$a + b = 0$$

$$2a - b = 6$$

$$+ \quad + \quad +$$

$$a + b = 0$$

$$3a = 6$$

$$a = 2$$

$$b = -2$$



6.

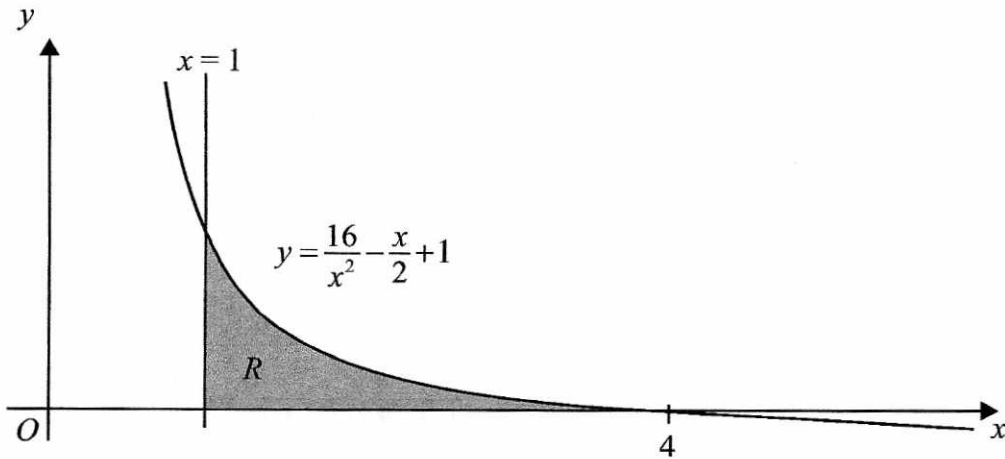


Figure 1

Figure 1 shows the graph of the curve with equation

$$y = \frac{16}{x^2} - \frac{x}{2} + 1, \quad x > 0$$

The finite region R , bounded by the lines $x = 1$, the x -axis and the curve, is shown shaded in Figure 1. The curve crosses the x -axis at the point $(4, 0)$.

(a) Complete the table with the values of y corresponding to $x = 2$ and 2.5

x	1	1.5	2	2.5	3	3.5	4
y	16.5	7.361	4	2.31	1.278	0.556	0

(2)

(b) Use the trapezium rule with all the values in the completed table to find an approximate value for the area of R , giving your answer to 2 decimal places.

(4)

(c) Use integration to find the exact value for the area of R .

(5)

$$b/ \quad 0.5 \left(\frac{16.5}{2} + 7.361 + 4 + 2.31 + 1.278 + 0.556 \right)$$

$$= 11.88 \text{ (2dp)}$$

$$c/ \quad y = 16x^{-2} - \frac{1}{2}x + 1$$

$$\int y \, dx = \frac{16x^{-1}}{-1} - \frac{\frac{1}{2}x^2}{2} + x + C$$



Question 6 continued

$$\left[-16x^{-1} - \frac{1}{4}x^2 + x + c \right]_1^4$$

$$[-4] - \left[-\frac{61}{4} \right]$$

$$= \frac{45}{4} \text{ units}^2$$



7.

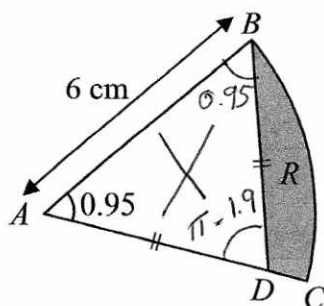


Figure 2

Figure 2 shows ABC , a sector of a circle of radius 6 cm with centre A . Given that the size of angle BAC is 0.95 radians, find

- (a) the length of the arc BC , (2)
- (b) the area of the sector ABC . (2)

The point D lies on the line AC and is such that $AD = BD$. The region R , shown shaded in Figure 2, is bounded by the lines CD , DB and the arc BC .

- (c) Show that the length of AD is 5.16 cm to 3 significant figures. (2)

Find

- (d) the perimeter of R , (2)
- (e) the area of R , giving your answer to 2 significant figures. (4)

$$\begin{aligned}
 \text{a/ Arc Length} &= \theta \times r \\
 &= 0.95 \times 6 \\
 &= \underline{\underline{5.7 \text{ cm}}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b/ Sector Area} &= \frac{\theta}{2} \times r^2 \\
 &= \frac{0.95}{2} \times 6^2 \\
 &= \underline{\underline{17.1 \text{ cm}^2}}
 \end{aligned}$$



Question 7 continued

$$c) \frac{a}{\sin(0.95)} = \frac{6}{\sin(\pi - 1.9)}$$

$$\begin{aligned} a &= \frac{6}{\sin(\pi - 1.9)} \times \sin(0.95) \\ &= 5.157447508 \\ &= 5.16 \text{ cm (3sf)} \end{aligned}$$

$$\begin{aligned} d) \text{ perimeter} &= 5.7 + (6 - 5.16) + 5.16 \\ &= \underline{\underline{11.7 \text{ cm}}} \end{aligned}$$

$$\begin{aligned} e) \text{ area} &= 17.1 - \left(\frac{1}{2} (6)(5.16) \sin(0.95) \right) \\ &= 4.5 \text{ cm}^2 \text{ (2sf)} \end{aligned}$$



8.

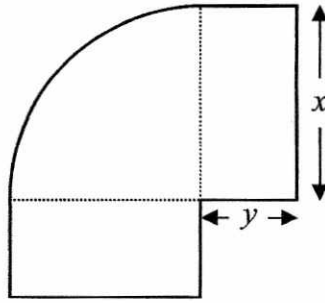


Figure 3

Figure 3 shows a flowerbed. Its shape is a quarter of a circle of radius x metres with two equal rectangles attached to it along its radii. Each rectangle has length equal to x metres and width equal to y metres.

Given that the area of the flowerbed is 4 m^2 ,

(a) show that

$$y = \frac{16 - \pi x^2}{8x} \quad (3)$$

(b) Hence show that the perimeter P metres of the flowerbed is given by the equation

$$P = \frac{8}{x} + 2x \quad (3)$$

(c) Use calculus to find the minimum value of P .

(5)

(d) Find the width of each rectangle when the perimeter is a minimum.
Give your answer to the nearest centimetre.

(2)

$$A = 2xy + \frac{1}{4} \pi x^2$$

$$4 = 2xy + \frac{1}{4} \pi x^2$$

$$16 = 8xy + \pi x^2$$

$$16 - \pi x^2 = 8xy$$

$$y = \frac{16 - \pi x^2}{8x}$$



Question 8 continued

$$P = 2x + 4y + \frac{1}{4} 2\pi x$$

$$P = 2x + 4 \left(\frac{16 - \pi x^2}{8x} \right) + \frac{1}{4} 2\pi x$$

$$= 2x + \frac{16 - \pi x^2}{2x} + \frac{1}{2} \pi x$$

$$= 2x + \frac{8}{x} - \frac{1}{2} \pi x + \frac{1}{2} \pi x$$

$$= 2x + \frac{8}{x}$$

$$= \frac{8}{x} + 2x$$

$$c/ \quad P = 8x^{-1} + 2x$$

$$\frac{dP}{dx} = -8x^{-2} + 2$$

Min value where $\frac{dP}{dx} = 0$

$$-8x^{-2} + 2 = 0$$

$$2 = 8x^{-2}$$

$$2 = \frac{8}{x^2}$$

$$2x^2 = 8$$

$$x^2 = 4$$

$$x = 2$$

(not -2, length cannot be negative)

$$P = 8(2)^{-1} + 2(2)$$

$$= \underline{\underline{8 \text{ m}}}$$

$$d/ \quad y = \frac{16 - \pi(2)^2}{8(2)}$$

$$= 0.2146018366 \text{ m}$$

$$= 21 \text{ cm (nearest cm)}$$



9. (i) Find the solutions of the equation $\sin(3x - 15^\circ) = \frac{1}{2}$, for which $0 \leq x \leq 180^\circ$

(6)

(ii)

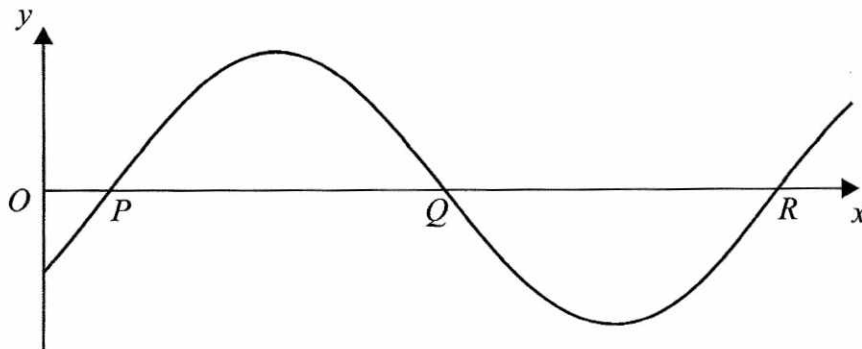


Figure 4

Figure 4 shows part of the curve with equation

$$y = \sin(ax - b), \text{ where } a > 0, 0 < b < \pi$$

The curve cuts the x -axis at the points P , Q and R as shown.

Given that the coordinates of P , Q and R are $(\frac{\pi}{10}, 0)$, $(\frac{3\pi}{5}, 0)$ and $(\frac{11\pi}{10}, 0)$ respectively, find the values of a and b .

(4)

$$\begin{aligned} \text{i/ } \sin(3x - 15) &= \frac{1}{2} \\ 3x - 15 &= \sin^{-1}(\frac{1}{2}) \\ 3x - 15 &= 30, 150, 390, 510 \\ x &= 15^\circ, 55^\circ, 135^\circ, 175^\circ \end{aligned}$$

ii/ original interceptions $0, \pi, 2\pi$

$$\frac{0 + b}{a} = \frac{\pi}{10} \quad (1)$$

$$\frac{\pi + b}{a} = \frac{3\pi}{5} \quad (2)$$

$$\frac{2\pi + b}{a} = \frac{11\pi}{10}$$



Question 9 continued

$$\textcircled{1} \quad b = \frac{\pi}{10} a$$

$$\textcircled{2} \quad \pi + b = \frac{3\pi}{5} a$$

$$\textcircled{2} \quad b = \frac{3\pi}{5} a - \pi$$

$$\frac{\pi}{10} a = \frac{3\pi}{5} a - \pi$$

$$-\frac{1}{2}\pi a = -\pi$$

$$a = 2$$

$$b = \frac{\pi}{5}$$

