

1.

$$f(x) = x^4 + x^3 + 2x^2 + ax + b$$

where a and b are constants.

When $f(x)$ is divided by $(x - 1)$, the remainder is 7.

(a) Show that $a + b = 3$.

(2)

When $f(x)$ is divided by $(x + 2)$, the remainder is -8 .

(b) Find the value of a and the value of b .

(5)

$$a/ \quad f(1) = 7$$

$$(1)^4 + (1)^3 + 2(1)^2 + a(1) + b = 7$$

$$4 + a + b = 7$$

$$\underline{\underline{a + b = 3}}$$

$$b/ \quad f(-2) = -8$$

$$(-2)^4 + (-2)^3 + 2(-2)^2 + a(-2) + b = -8$$

$$16 - 8 + 8 - 2a + b = -8$$

$$16 - 2a + b = -8$$

$$\underline{\underline{-2a + b = -24}}$$

$$a + b = 3$$

$$\underline{\underline{-2a + b = -24}}$$

$$3a = 27$$

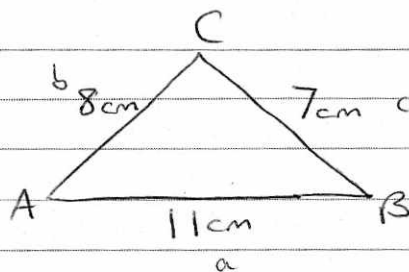
$$a = 9$$

$$\underline{\underline{b = -6}}$$

2. In the triangle ABC , $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(a) Find the size of angle C , giving your answer in radians to 3 significant figures. (3)

(b) Find the area of triangle ABC , giving your answer in cm^2 to 3 significant figures. (3)



$$\begin{aligned} \text{a) } \cos A &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{(8)^2 + (7)^2 - (11)^2}{2(8)(7)} \end{aligned}$$

$$\begin{aligned} \cos A &= \frac{1}{14} \\ A &= 1.64 \text{ (3sf)} \end{aligned}$$

$$\begin{aligned} \text{b) } \text{Area} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} (8)(7) \sin(1.64) \\ &= \cancel{27.91105942} \quad \cancel{27.92} \\ &= \cancel{28.0 \text{ cm}^2 \text{ (3sf)}} \\ &= 27.92848609 \\ &= 27.9 \text{ cm}^2 \text{ (3sf)} \end{aligned}$$



3. The second and fifth terms of a geometric series are 750 and -6 respectively.

Find

- (a) the common ratio of the series, (3)
- (b) the first term of the series, (2)
- (c) the sum to infinity of the series. (2)

$$u_n = ar^{n-1}$$

$$\begin{aligned} \text{a)} \quad u_2 &= ar = 750 \\ u_5 &= ar^4 = -6 \end{aligned}$$

$$r^3 = -\frac{1}{125}$$

$$\underline{\underline{r = -\frac{1}{5}}}$$

$$\begin{aligned} \text{b)} \quad a\left(-\frac{1}{5}\right) &= 750 \\ \underline{\underline{a = -3750}} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad S_{\infty} &= \frac{a}{1-r} \\ &= \frac{-3750}{1 - \left(-\frac{1}{5}\right)} \\ &= \underline{\underline{-3125}} \end{aligned}$$



4.

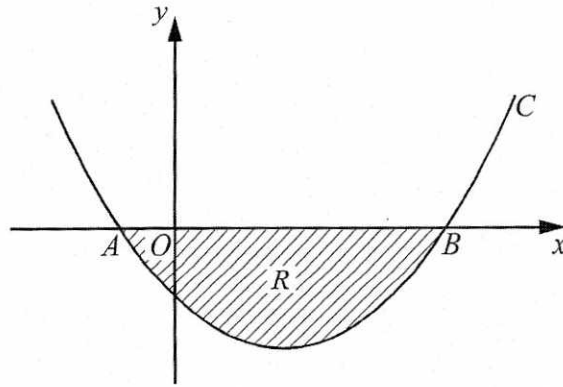


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$y = (x+1)(x-5)$$

The curve crosses the x -axis at the points A and B .

(a) Write down the x -coordinates of A and B .

(1)

The finite region R , shown shaded in Figure 1, is bounded by C and the x -axis.

(b) Use integration to find the area of R .

(6)

a) crosses x when $y = 0$
 $0 = (x+1)(x-5)$
 $x = -1 \quad x = 5$
 $(-1, 0) \quad (5, 0)$

b) $\int_{-1}^5 (x+1)(x-5) dx$

$$\int_{-1}^5 x^2 - 5x + x - 5 dx$$

$$\int_{-1}^5 x^2 - 4x - 5 dx$$

$$\left[\frac{x^3}{3} - \frac{4x^2}{2} - 5x + C \right]_{-1}^5$$

Question 4 continued

$$\left[\frac{x^3}{3} - 2x^2 - 5x + c \right]_{-1}^5$$

$$\left[\frac{(5)^3}{3} - 2(5)^2 - 5(5) \right] - \left[\frac{(-1)^3}{3} - 2(-1)^2 - 5(-1) \right]$$

$$= 36 \text{ units}^2$$

(Total 7 marks)

Q4

5. Given that $\binom{40}{4} = \frac{40!}{4!b!}$,

(a) write down the value of b .

(1)

In the binomial expansion of $(1+x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

a) 36

b/ coefficient of $x^4 = \binom{40}{4} = 91390$

$x^5 = \binom{40}{5} = 658008$

$\frac{658008}{91390} = \underline{\underline{7.2}}$

6.

$$y = \frac{5}{3x^2 - 2}$$

(a) Complete the table below, giving the values of y to 2 decimal places.

x	2	2.25	2.5	2.75	3
y	0.5	0.38	0.30	0.24	0.2

(2)

(b) Use the trapezium rule, with all the values of y from your table, to find an approximate value for

$$\int_2^3 \frac{5}{3x^2 - 2} dx.$$

(4)

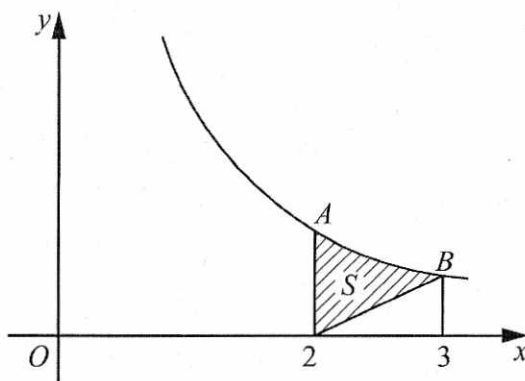


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \frac{5}{3x^2 - 2}$, $x > 1$.

At the points A and B on the curve, $x = 2$ and $x = 3$ respectively.

The region S is bounded by the curve, the straight line through B and $(2, 0)$, and the line through A parallel to the y -axis. The region S is shown shaded in Figure 2.

(c) Use your answer to part (b) to find an approximate value for the area of S .

(3)

$$b/ \quad 0.25 \left(\frac{0.5}{2} + 0.38 + 0.30 + 0.24 + \frac{0.2}{2} \right)$$

$$= 0.3175 \text{ units}^2$$

$$c/ \quad \text{Area of triangle} = \frac{1}{2}(1)(0.2)$$

$$= 0.1$$

$$0.3175 - 0.1 = \underline{\underline{0.2175 \text{ units}^2}}$$



7. (a) Show that the equation

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

can be written in the form

$$4 \sin^2 x + 7 \sin x + 3 = 0$$

(2)

(b) Hence solve, for $0 \leq x < 360^\circ$,

$$3 \sin^2 x + 7 \sin x = \cos^2 x - 4$$

giving your answers to 1 decimal place where appropriate.

(5)

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = 1 - \sin^2 x$$

$$a/ \quad 3 \sin^2 x + 7 \sin x = (1 - \sin^2 x) - 4$$

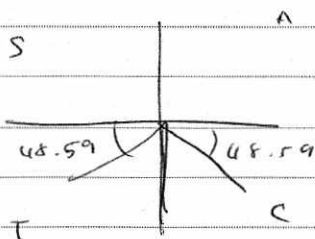
$$3 \sin^2 x + 7 \sin x = -\sin^2 x - 3$$

$$4 \sin^2 x + 7 \sin x + 3 = 0$$

$$b/ \quad (4 \sin x + 3)(\sin x + 1) = 0$$

$$\sin x = -\frac{3}{4} \quad \sin x = -1$$

$$x = -48.59 \quad x = -90$$



$$x = \underline{228.6^\circ, 270^\circ, 311.4^\circ}$$

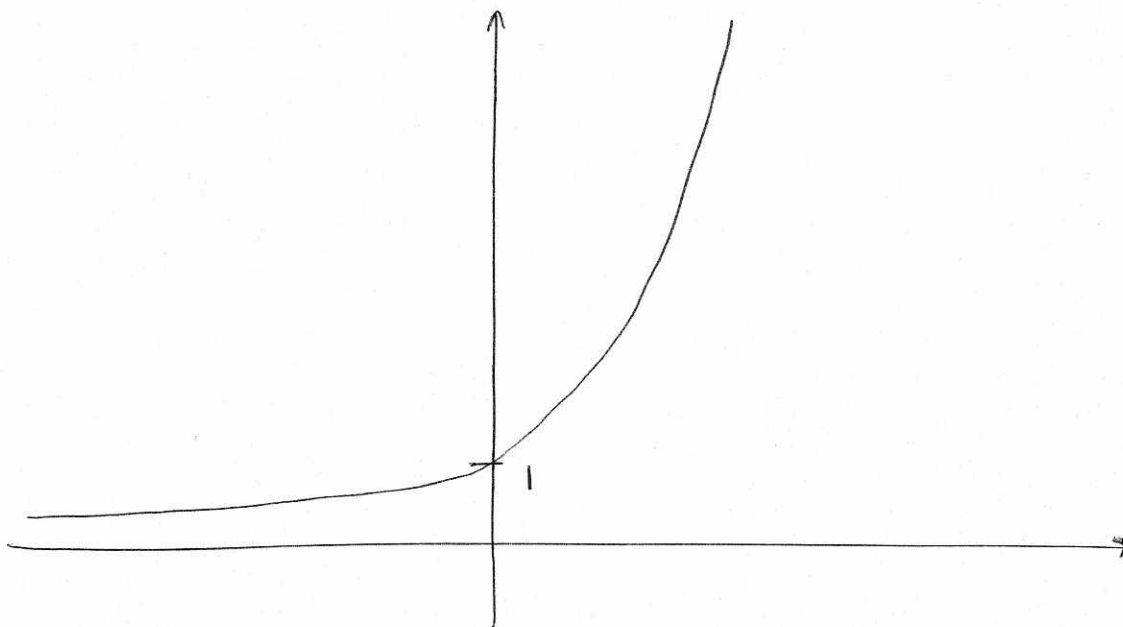


8. (a) Sketch the graph of $y = 7^x$, $x \in \mathbb{R}$, showing the coordinates of any points at which the graph crosses the axes. (2)

- (b) Solve the equation

$$7^{2x} - 4(7^x) + 3 = 0$$

giving your answers to 2 decimal places where appropriate. (6)



Question 8 continued

$$b/ \quad 7^{2x} - 4(7^x) + 3 = 0$$

$$(7^x - 3)(7^x - 1) = 0$$

$$7^x = 3 \quad 7^x = 1$$

$$x = \log_7 3 \quad \underline{\underline{x = 0}}$$

$$\underline{\underline{x = 0.56 \text{ 2dp}}}$$

9. The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C ,

(a) show that the centre of C has coordinates $(3, 6)$, (1)

(b) find an equation for C . (4)

(c) Verify that the point $(10, 7)$ lies on C . (1)

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants. (4)

a/ centre of circle is the diameter's midpoint

$$\left(\frac{-2+8}{2}, \frac{11+1}{2} \right)$$

$$(3, 6)$$

b/ $(x-3)^2 + (y-6)^2 = r^2$ $\begin{matrix} x & y \\ (8, 1) \end{matrix}$

$$(8-3)^2 + (1-6)^2 = r^2$$

$$(5)^2 + (-5)^2 = r^2$$

$$50 = r^2$$

$$r = \sqrt{50}$$

$$\underline{(x-3)^2 + (y-6)^2 = 50}$$

c/ $\begin{matrix} x & y \\ (10, 7) \end{matrix}$

$$(10-3)^2 + (7-6)^2 = 50$$

$$49 + 1 = 50$$

$$50 = 50$$

$\therefore (10, 7)$ lies on C .



Question 9 continued

$$d/ \quad \text{centre } \begin{matrix} x_1 & y_1 \\ (3, 6) \end{matrix} \quad \begin{matrix} x_2 & y_2 \\ (10, 7) \end{matrix}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$= \frac{7 - 6}{10 - 3} = \frac{1}{7}$$

tangent at $(10, 7)$ is perpendicular to radius.

$$\therefore m = -7$$

$$y = -7x + c \quad \begin{matrix} x & y \\ (10, 7) \end{matrix}$$

$$7 = -7(10) + c$$

$$7 = -70 + c$$

$$77 = c$$

$$\underline{y = -7x + 77}$$

10. The volume V cm³ of a box, of height x cm, is given by

$$V = 4x(5-x)^2, \quad 0 < x < 5$$

(a) Find $\frac{dV}{dx}$.

(4)

(b) Hence find the maximum volume of the box.

(4)

(c) Use calculus to justify that the volume that you found in part (b) is a maximum.

(2)

$$\begin{aligned} \text{a/} \quad V &= 4x(5-x)^2 \\ &= 4x(5-x)(5-x) \\ &= 4x(25-5x-5x+x^2) \\ &= 4x(25-10x+x^2) \\ &= 100x - 40x^2 + 4x^3 \end{aligned}$$

$$\frac{dV}{dx} = 100 - 80x + 12x^2$$

b/ Maximum value is where $\frac{dV}{dx} = 0$

$$100 - 80x + 12x^2 = 0$$

$$25 - 20x + 3x^2 = 0$$

$$3x^2 - 20x + 25 = 0$$

$$(3x-5)(x-5) = 0$$

$$x = \frac{5}{3} \quad x = 5$$

$$0 < x < 5 \quad \therefore x = \frac{5}{3}$$

$$V = 4\left(\frac{5}{3}\right)\left(5 - \frac{5}{3}\right)^2$$

$$= 74.074 \text{ cm}^3$$

$$= 74.1 \text{ cm}^3 \text{ (3sf)}$$

$$\begin{aligned} \text{c/} \quad \frac{d^2V}{dx^2} &= -80 + 24x \\ &= -80 + 24\left(\frac{5}{3}\right) \end{aligned}$$

$$= -40$$

$\frac{d^2V}{dx^2}$ is negative \therefore it is a maximum