

$$\begin{aligned}
 1 \quad \mathbf{a} &= \sqrt{9}\sqrt{3} + 2\sqrt{25}\sqrt{2} \\
 &= 10\sqrt{2} + 3\sqrt{3} \\
 \mathbf{b} &= \sqrt{18} - \sqrt{48} \\
 &= \sqrt{9}\sqrt{2} - \sqrt{16}\sqrt{3} \\
 &= 3\sqrt{2} - 4\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad x^2 - 2x &= 12 - 2x \\
 x^2 &= 12 \\
 x &= \pm\sqrt{12} = \pm 2\sqrt{3} \\
 x > 0 \quad \therefore x &= 2\sqrt{3}
 \end{aligned}$$

$$\begin{aligned}
 3 \quad 25^x &= (5^2)^x = 5^{4x+1} \\
 5^{2x} &= 5^{4x+1} \\
 2x &= 4x + 1 \\
 x &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 4 \quad \mathbf{a} &= \sqrt[3]{8} \times \sqrt[3]{3} = 2\sqrt[3]{3} \\
 \mathbf{b} \quad \sqrt[3]{81} &= \sqrt[3]{27} \times \sqrt[3]{3} = 3\sqrt[3]{3} \\
 \therefore \sqrt[3]{24} + \sqrt[3]{81} &= 2\sqrt[3]{3} + 3\sqrt[3]{3} = 5\sqrt[3]{3} \\
 &= \sqrt[3]{125 \times 3} = \sqrt[3]{375} \\
 \therefore n &= 375
 \end{aligned}$$

$$\begin{aligned}
 5 \quad \frac{10\sqrt{3}}{\sqrt{15}} &= \frac{10\sqrt{3}}{\sqrt{5}\sqrt{3}} = \frac{10}{\sqrt{5}} = \frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = 2\sqrt{5} \\
 \frac{4}{\sqrt{5}-\sqrt{7}} \times \frac{\sqrt{5}+\sqrt{7}}{\sqrt{5}+\sqrt{7}} &= \frac{4(\sqrt{5}+\sqrt{7})}{5-7} = -2\sqrt{5} - 2\sqrt{7} \\
 \therefore \frac{10\sqrt{3}}{\sqrt{15}} + \frac{4}{\sqrt{5}-\sqrt{7}} &= 2\sqrt{5} - 2\sqrt{5} - 2\sqrt{7} \\
 &= -2\sqrt{7} \quad [k = -2]
 \end{aligned}$$

$$\begin{aligned}
 6 \quad \mathbf{a} &= \sqrt{\frac{75}{2}} = \frac{5\sqrt{3}}{\sqrt{2}} = \frac{5\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5}{2}\sqrt{6} \\
 \mathbf{b} &= \sqrt{\frac{48}{5}} - \sqrt{\frac{20}{3}} = \frac{4\sqrt{3}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}} \\
 &= \frac{4\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} - \frac{2\sqrt{5}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \\
 &= \frac{4}{5}\sqrt{15} - \frac{2}{3}\sqrt{15} \\
 &= \frac{2}{15}\sqrt{15}
 \end{aligned}$$

$$\begin{aligned}
 7 \quad \mathbf{a} \quad \mathbf{i} \quad xy &= 2^{t-1} \times 2^{3t} = 2^{4t-1} \\
 \mathbf{ii} \quad 2y^2 &= 2 \times (2^{3t})^2 = 2 \times 2^{6t} = 2^{6t+1} \\
 \mathbf{b} \quad 2^{6t+1} - 2^{4t-1} &= 0 \\
 2^{6t+1} &= 2^{4t-1} \\
 6t+1 &= 4t-1 \\
 t &= -1
 \end{aligned}$$

$$\begin{aligned}
 8 \quad 3x\sqrt{2} - \sqrt{2} &= 4x + 6 \\
 x(3\sqrt{2} - 4) &= 6 + \sqrt{2} \\
 x &= \frac{6+\sqrt{2}}{3\sqrt{2}-4} = \frac{6+\sqrt{2}}{3\sqrt{2}-4} \times \frac{3\sqrt{2}+4}{3\sqrt{2}+4} = \frac{(6+\sqrt{2})(3\sqrt{2}+4)}{18-16} \\
 &= \frac{1}{2}(18\sqrt{2} + 24 + 6 + 4\sqrt{2}) \\
 &= \frac{1}{2}(30 + 22\sqrt{2}) \\
 &= 15 + 11\sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 9 \quad \mathbf{a} \quad 6^{y+1} &= 36^{x-2} = (6^2)^{x-2} \\
 6^{y+1} &= 6^{2x-4} \\
 y+1 &= 2x-4 \\
 y &= 2x-5 \\
 \mathbf{b} \quad x - \frac{1}{2}y &= x - \frac{1}{2}(2x-5) = x - x + \frac{5}{2} = \frac{5}{2} \\
 \therefore 4^{x-\frac{1}{2}y} &= 4^{\frac{5}{2}} = (\sqrt{4})^5 = 2^5 = 32
 \end{aligned}$$

$$\begin{aligned}
 10 \quad \mathbf{a} &= 3 + 3\sqrt{2} - \sqrt{2} - 2 \\
 &= 1 + 2\sqrt{2} \\
 \mathbf{b} &= \frac{\sqrt{2}}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{\sqrt{2}(\sqrt{2}+1)}{2-1} \\
 &= \sqrt{2}(\sqrt{2}+1) \\
 &= 2 + \sqrt{2}
 \end{aligned}$$

$$\begin{aligned}
 11 \quad (2^4)^{x+1} &= (2^3)^{2x+1} \\
 2^{4x+4} &= 2^{6x+3} \\
 4x+4 &= 6x+3 \\
 x &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 12 \quad a^2 - 4a\sqrt{3} + 12 &= b - 20\sqrt{3} \\
 a \text{ and } b \text{ integers} \quad \therefore \quad &-4a = -20 \\
 &a = 5 \\
 \text{also} \quad &a^2 + 12 = b \\
 &b = 37
 \end{aligned}$$

$$\begin{aligned}
 13 \quad \text{a} \quad (2^{-2})^{t-3} &= 2^3 \\
 2^{6-2t} &= 2^3 \\
 6-2t &= 3 \\
 t &= \frac{3}{2} \\
 \text{b} \quad (3^{-1})^y &= (3^3)^{y+1} \\
 3^{-y} &= 3^{3y+3} \\
 -y &= 3y+3 \\
 y &= -\frac{3}{4}
 \end{aligned}$$

$$\begin{aligned}
 14 \quad \text{a} \quad &= 2\sqrt{5}(\sqrt{5}-3) \\
 &= 10 - 6\sqrt{5} \\
 \text{b} \quad &= 3 + 2\sqrt{5} - 3\sqrt{5} - 10 \\
 &= -7 - \sqrt{5} \\
 \text{c} \quad &= \frac{1+\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{(1+\sqrt{5})(\sqrt{5}+2)}{5-4} \\
 &= (1+\sqrt{5})(\sqrt{5}+2) \\
 &= \sqrt{5} + 2 + 5 + 2\sqrt{5} \\
 &= 7 + 3\sqrt{5}
 \end{aligned}$$

$$\begin{aligned}
 15 \quad \text{a} \quad a &= (b^{\frac{3}{4}})^3 = b^{\frac{9}{4}} \\
 a^{\frac{1}{2}} &= (b^{\frac{9}{4}})^{\frac{1}{2}} = b^{\frac{9}{8}} \\
 \text{b} \quad b &= (a^{\frac{1}{3}})^{\frac{4}{3}} = a^{\frac{4}{9}} \\
 b^{\frac{1}{2}} &= (a^{\frac{4}{9}})^{\frac{1}{2}} = a^{\frac{2}{9}}
 \end{aligned}$$

$$\begin{aligned}
 16 \quad \text{a} \quad \text{area} &= \frac{1}{2}(2\sqrt{3}-1)(\sqrt{3}+2) \\
 &= \frac{1}{2}(6+4\sqrt{3}-\sqrt{3}-2) \\
 &= \frac{1}{2}(4+3\sqrt{3}) \text{ or } 2 + \frac{3}{2}\sqrt{3} \\
 \text{b} \quad AC^2 &= (2\sqrt{3}-1)^2 + (\sqrt{3}+2)^2 \\
 &= 12 - 4\sqrt{3} + 1 + 3 + 4\sqrt{3} + 4 = 20 \\
 \therefore AC &= \sqrt{20} = \sqrt{4}\sqrt{5} = 2\sqrt{5} \\
 \text{c} \quad \tan(\angle ACB) &= \frac{2\sqrt{3}-1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{(2\sqrt{3}-1)(\sqrt{3}-2)}{3-4} \\
 &= -(2\sqrt{3}-1)(\sqrt{3}-2) \\
 &= -(6-4\sqrt{3}-\sqrt{3}+2) \\
 &= -(8-5\sqrt{3}) = 5\sqrt{3}-8
 \end{aligned}$$

$$\begin{aligned}
 17 \quad \text{a} \quad \text{i} \quad 2^{x+2} &= 2^2 \times 2^x = 4y \\
 \text{ii} \quad 4^x &= (2^2)^x = 2^{2x} = (2^x)^2 = y^2 \\
 \text{b} \quad y^2 - 4y &= 0 \\
 y(y-4) &= 0 \\
 y &= 0 \text{ or } 4 \\
 2^x = 0 \text{ (no solutions)} &\text{ or } 2^x = 4 \\
 x &= 2
 \end{aligned}$$

$$\begin{aligned}
 18 \quad 5\sqrt{3} &= 2(1+\sqrt{3})^2 + p(1+\sqrt{3}) + q \\
 5\sqrt{3} &= 2 + 4\sqrt{3} + 6 + p + p\sqrt{3} + q \\
 p, q \text{ rational} \quad \therefore \quad &5\sqrt{3} = 4\sqrt{3} + p\sqrt{3} \\
 &p = 1 \\
 \text{and} \quad &0 = 2 + 6 + p + q \\
 &q = -9
 \end{aligned}$$