

1. Given $y = x^3 + 4x + 1$, find the value of $\frac{dy}{dx}$ when $x = 3$

(4)

$$\frac{dy}{dx} = 3x^2 + 4$$

when $x = 3$

$$\frac{dy}{dx} = 3(3)^2 + 4$$

$$= 3(9) + 4$$

$$= 27 + 4$$

$$= \underline{\underline{31}}$$

Q1

(Total 4 marks)

2. Express $\frac{15}{\sqrt{3}} - \sqrt{27}$ in the form $k\sqrt{3}$, where k is an integer.

(4)

$$\frac{15\sqrt{3}}{\sqrt{3}\sqrt{3}} - \sqrt{9}\sqrt{3}$$

$$\frac{15\sqrt{3} - 3\sqrt{3}}{3}$$

$$5\sqrt{3} - 3\sqrt{3}$$

$$\underline{\underline{2\sqrt{3}}}$$

Q2

(Total 4 marks)

3. Find

$$\int \left(3x^2 - \frac{4}{x^2} \right) dx$$

giving each term in its simplest form.

(4)

$$\int 3x^2 - 4x^{-2} dx$$

$$\frac{3x^3}{3} - \frac{4x^{-1}}{-1} + C$$

$$\underline{x^3 + 4x^{-1} + C}$$

4. The line L_1 has equation $4x + 2y - 3 = 0$

(a) Find the gradient of L_1 .

(2)

The line L_2 is perpendicular to L_1 and passes through the point $(2, 5)$.

(b) Find the equation of L_2 in the form $y = mx + c$, where m and c are constants.

(3)

$$\begin{aligned} \text{a)} \quad 4x + 2y - 3 &= 0 \\ 2y &= -4x + 3 \\ y &= -2x + 3/2 \end{aligned}$$

$$\underline{m = -2}$$

b) perpendicular gradient = $1/2$

$$y = 1/2x + c \quad (2, 5)$$

$$5 = 1/2(2) + c$$

$$5 = 1 + c$$

$$c = 4$$

$$\underline{y = 1/2x + 4}$$

5. Solve

(a) $2^y = 8$

(1)

(b) $2^x \times 4^{x+1} = 8$

(4)

a) $y = 3$

b) $2^x \times 2^{2(x+1)} = 8$
 2^{x+2x+2}

$2^{3x+2} = 8$
 $2^{3x+2} = 2^3$

$3x+2 = 3$

$3x = 1$

$x = \frac{1}{3}$

6. A sequence x_1, x_2, x_3, \dots is defined by

$$x_1 = 1$$

$$x_{n+1} = (x_n)^2 - kx_n, \quad n \geq 1$$

where k is a constant, $k \neq 0$

(a) Find an expression for x_2 in terms of k .

(1)

(b) Show that $x_3 = 1 - 3k + 2k^2$

(2)

Given also that $x_3 = 1$,

(c) calculate the value of k .

(3)

(d) Hence find the value of $\sum_{n=1}^{100} x_n$

(3)

a)
$$x_2 = (x_1)^2 - k(x_1)$$

$$x_2 = (1)^2 - k(1)$$

$$= 1 - k$$

b)
$$x_3 = (x_2)^2 - k(x_2)$$

$$= (1 - k)^2 - k(1 - k)$$

$$= (1 - k)(1 - k) - k(1 - k)$$

$$= 1 - k - k + k^2 - k + k^2$$

$$= 2k^2 - 3k + 1$$

$$= \underline{1 - 3k + 2k^2}$$

c)
$$2k^2 - 3k + 1 = 1$$

$$2k^2 - 3k = 0$$

$$k(2k - 3) = 0$$

$$k = 0 \quad k = \frac{3}{2}$$

k is non zero $\therefore k = \underline{\underline{\frac{3}{2}}}$

d)
$$1, \underbrace{-\frac{1}{2}}_{0.5}, 1, \underbrace{-\frac{1}{2}}_{0.5}, \text{ etc.}$$
~~$$100 \times 0.5 = 50$$~~

Question 6 continued

50 pairs of numbers that add to 0.5

$$50 \times 0.5 = \underline{25}$$

7. Each year, Abbie pays into a savings scheme. In the first year she pays in £500. Her payments then increase by £200 each year so that she pays £700 in the second year, £900 in the third year and so on.

(a) Find out how much Abbie pays into the savings scheme in the tenth year. (2)

Abbie pays into the scheme for n years until she has paid in a total of £67200.

(b) Show that $n^2 + 4n - 24 \times 28 = 0$ (5)

(c) Hence find the number of years that Abbie pays into the savings scheme. (2)

$$a) \quad a = 500 \quad d = 200$$

$$u_n = a + (n-1)d$$

$$u_{10} = 500 + 9(200)$$

$$= 500 + 1800$$

$$= \underline{\underline{2300}}$$

$$b) \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$67200 = \frac{n}{2}(2(500) + (n-1)(200))$$

$$= \frac{n}{2}(1000 + 200n - 200)$$

$$= \frac{n}{2}(800 + 200n)$$

$$67200 = 400n + 100n^2$$

$$672 = 4n + n^2$$

$$0 = n^2 + 4n - 672$$

$$= n^2 + 4n - 24 \times 28$$

$$c) \quad (n + 28)(n - 24)$$

$$n = -28 \quad n = 24$$

$$\underline{\underline{n = 24}} \quad (\text{cannot be negative})$$

8. A rectangular room has a width of x m.

The length of the room is 4 m longer than its width.

Given that the perimeter of the room is greater than 19.2 m,

(a) show that $x > 2.8$

(3)

Given also that the area of the room is less than 21 m^2 ,

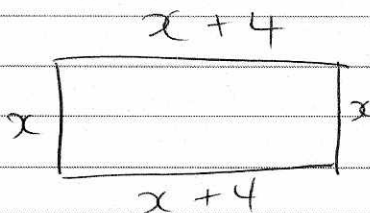
(b) (i) write down an inequality, in terms of x , for the area of the room.

(ii) Solve this inequality.

(4)

(c) Hence find the range of possible values for x .

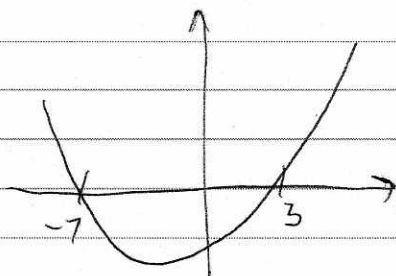
(1)



$$\begin{aligned} \text{a) } 4x + 8 &> 19.2 \\ 4x &> 11.2 \\ x &> \underline{2.8} \end{aligned}$$

$$\text{b) } x(x+4) < 21$$

$$\begin{aligned} \text{ii) } x^2 + 4x &< 21 \\ x^2 + 4x - 21 &< 0 \\ (x+7)(x-3) &< 0 \\ x = -7 \quad x = 3 \end{aligned}$$



$$-7 < x < 3$$

Question 8 continued

c/

$$\underline{\underline{2.8 < x < 3}}$$



9.

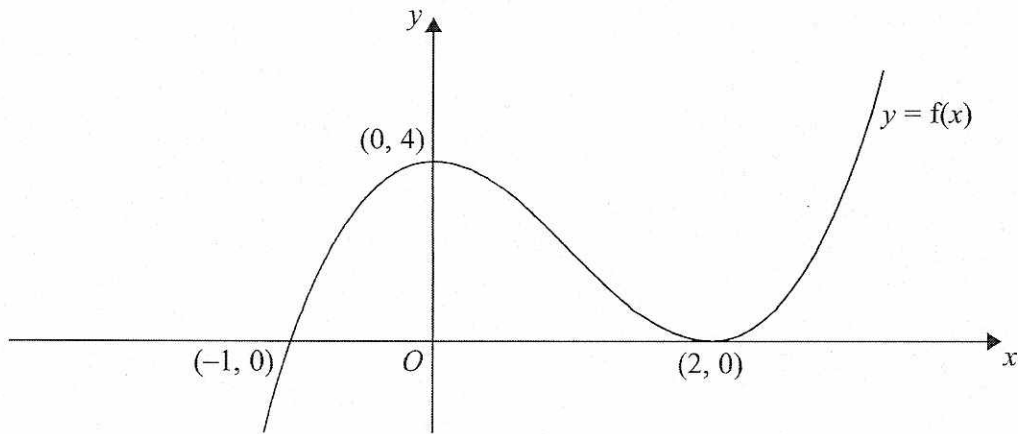


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.

The curve C passes through the point $(-1, 0)$ and touches the x -axis at the point $(2, 0)$.

The curve C has a maximum at the point $(0, 4)$.

(a) The equation of the curve C can be written in the form

$$y = x^3 + ax^2 + bx + c$$

where a , b and c are integers.

Calculate the values of a , b and c .

(5)

(b) Sketch the curve with equation $y = f(\frac{1}{2}x)$ in the space provided on page 24

Show clearly the coordinates of all the points where the curve crosses or meets the coordinate axes.

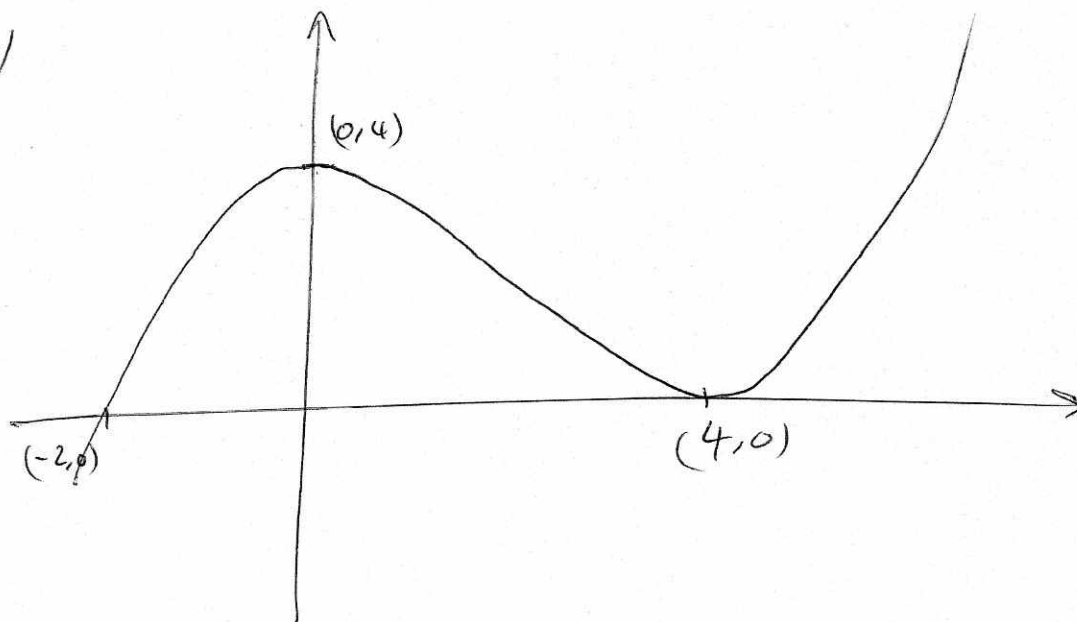
(3)

$$\begin{aligned} a) \quad y &= (x+1)(x-2)(x-2) \\ &= (x+1)(x^2-4x+4) \\ &= x^3+x^2-4x^2-4x+4x+4 \\ &= x^3-3x^2+4 \end{aligned}$$

$$a = -3 \quad b = 0 \quad c = 4$$

Question 9 continued

b/



10. A curve has equation $y = f(x)$. The point P with coordinates $(9, 0)$ lies on the curve.

Given that

$$f'(x) = \frac{x+9}{\sqrt{x}}, \quad x > 0$$

(a) find $f(x)$.

(6)

(b) Find the x -coordinates of the two points on $y = f(x)$ where the gradient of the curve is equal to 10

(4)

$$\begin{aligned} a) \quad f'(x) &= \frac{x}{\sqrt{x}} + \frac{9}{\sqrt{x}} \\ &= x^{1/2} + 9x^{-1/2} \end{aligned}$$

$$\begin{aligned} f(x) &= \frac{x^{3/2}}{3/2} + \frac{9x^{1/2}}{1/2} + C \\ &= \frac{2}{3}x^{3/2} + 18x^{1/2} + C \end{aligned}$$

$$9, 0 \quad 0 = \frac{2}{3}(9)^{3/2} + 18(9)^{1/2} + C$$

$$0 = \frac{2}{3}(27) + 18(3) + C$$

$$0 = 18 + 54 + C$$

$$0 = 72 + C$$

$$C = -72$$

$$f(x) = \frac{2}{3}x^{3/2} + 18x^{1/2} - 72$$

$$b) \quad 10 = \frac{x+9}{\sqrt{x}}$$

$$10\sqrt{x} = x + 9$$

$$0 = x - 10\sqrt{x} + 9$$

$$0 = (x^{1/2} - 9)(x^{1/2} - 1)$$

$$x^{1/2} = 9 \quad x^{1/2} = 1$$

$$(x > 0)$$

$$x = 81 \quad x = 1$$

11.

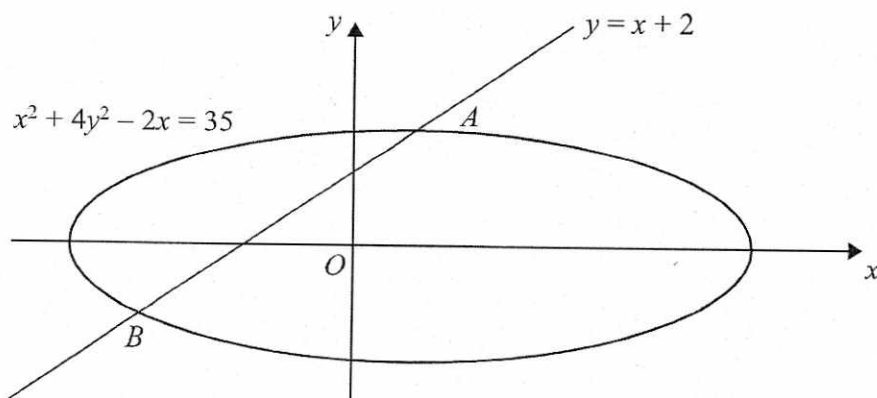


Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

- (a) Find the coordinates of A and the coordinates of B . (6)
- (b) Find the distance AB in the form $r\sqrt{2}$ where r is a rational number. (3)

$$a/ \quad x^2 + 4y^2 - 2x = 35$$

$$y = (x + 2)$$

$$x^2 + 4(x+2)^2 - 2x = 35$$

$$x^2 + 4(x^2 + 4x + 4) - 2x = 35$$

$$x^2 + 4x^2 + 16x + 16 - 2x = 35$$

$$5x^2 + 14x + 16 = 35$$

$$5x^2 + 14x - 19 = 0$$

$$(5x + 19)(x - 1) = 0$$

$$x = \underline{\underline{-\frac{19}{5}}} \quad x = \underline{\underline{1}}$$

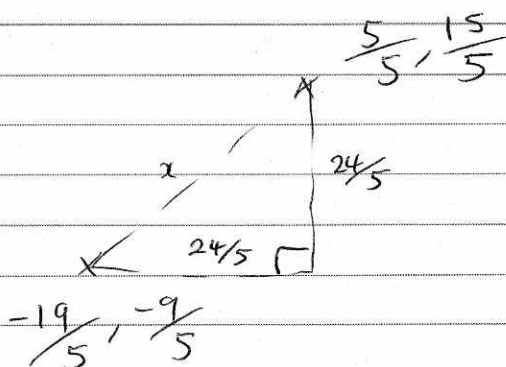
$$y = \underline{\underline{-\frac{19}{5}}} + 2 \quad y = 1 + 2$$

$$= \underline{\underline{-\frac{9}{5}}} \quad = \underline{\underline{3}}$$



Question 11 continued

$$b) \left(-\frac{19}{5}, -\frac{9}{5}\right) \quad (1, 3)$$



$$x^2 = \left(\frac{24}{5}\right)^2 + \left(\frac{24}{5}\right)^2$$

$$x^2 = \frac{24^2}{25} + \frac{24^2}{25}$$

$$x^2 = \frac{2(24^2)}{25}$$

$$x = \sqrt{\frac{2(24)^2}{25}}$$

$$= \frac{\sqrt{2}(24)}{5}$$

$$= \frac{24\sqrt{2}}{5}$$