



1. Find

$$\int \left( 6x^2 + \frac{2}{x^2} + 5 \right) dx$$

giving each term in its simplest form.

$$6x^2 + 2x^{-2} + 5 \quad (4)$$

$$\frac{6x^3}{3} + \frac{2x^{-1}}{-1} + 5x + C$$

$$2x^3 - 2x^{-1} + 5x + C$$

Q1

(Total 4 marks)



2. (a) Evaluate  $(32)^{\frac{3}{5}}$ , giving your answer as an integer.

(2)

(b) Simplify fully  $\left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$

(2)

$$\begin{aligned} \text{a)} \quad & 32^{\frac{3}{5}} \\ & = 2^3 \\ & = \underline{8} \end{aligned}$$

$$\text{b)} \quad \left(\frac{25x^4}{4}\right)^{-\frac{1}{2}}$$

$$\left(\frac{4}{25x^4}\right)^{\frac{1}{2}}$$

$$\underline{\underline{\frac{2}{5x^2}}}$$

Q2

(Total 4 marks)

3. Show that  $\frac{2}{\sqrt{12}-\sqrt{8}}$  can be written in the form  $\sqrt{a} + \sqrt{b}$ , where  $a$  and  $b$  are integers. (5)

$$\frac{2(\sqrt{12} + \sqrt{8})}{(\sqrt{12} - \sqrt{8})(\sqrt{12} + \sqrt{8})}$$

$$\frac{2\sqrt{12} + 2\sqrt{8}}{\sqrt{144} + \sqrt{96} - \sqrt{96} - 8}$$

$$\frac{2\sqrt{12} + 2\sqrt{8}}{12 - 8}$$

$$\frac{2\sqrt{12} + 2\sqrt{8}}{4}$$

$$\frac{2(\sqrt{4}\sqrt{3}) + 2(\sqrt{4}\sqrt{2})}{4}$$

$$\frac{4\sqrt{3} + 4\sqrt{2}}{4}$$

$$\underline{\underline{\sqrt{3} + \sqrt{2}}}$$

4.

$$y = 5x^3 - 6x^{\frac{4}{3}} + 2x - 3$$

(a) Find  $\frac{dy}{dx}$  giving each term in its simplest form.

(4)

(b) Find  $\frac{d^2y}{dx^2}$

(2)

$$a) \quad \frac{dy}{dx} = 15x^2 - \frac{24}{3}x^{\frac{1}{3}} + 2$$

$$= 15x^2 - 8x^{\frac{1}{3}} + 2$$

$$b) \quad \frac{d^2y}{dx^2} = 30x - \frac{8}{3}x^{-\frac{2}{3}}$$



5. A sequence of numbers  $a_1, a_2, a_3 \dots$  is defined by

$$a_1 = 3$$

$$a_{n+1} = 2a_n - c \quad (n \geq 1)$$

where  $c$  is a constant.

- (a) Write down an expression, in terms of  $c$ , for  $a_2$  (1)

- (b) Show that  $a_3 = 12 - 3c$  (2)

Given that  $\sum_{i=1}^4 a_i \geq 23$

- (c) find the range of values of  $c$ . (4)

$$\begin{aligned} \text{a)} \quad a_2 &= 2(a_1) - c \\ &= 2(3) - c \\ &= \underline{\underline{6 - c}} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad a_3 &= 2(a_2) - c \\ &= 2(6 - c) - c \\ &= 12 - 2c - c \\ &= \underline{\underline{12 - 3c}} \end{aligned}$$

$$\begin{aligned} \text{c)} \quad a_4 &= 2(a_3) - c \\ &= 2(12 - 3c) - c \\ &= 24 - 6c - c \\ &= 24 - 7c \end{aligned}$$

$$3 + 6 - c + 12 - 3c + 24 - 7c \geq 23$$

## Question 5 continued

$$45 - 11C \geq 23$$

$$45 \geq 23 + 11C$$

$$22 \geq 11C$$

$$2 \geq C$$

$$\underline{\underline{C \leq 2}}$$

Q5

(Total 7 marks)

6. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(a) Find how much he saves in week 15 (2)

(b) Calculate the total amount he saves over the 60 week period. (3)

The boy's sister also saves some money each week over a period of  $m$  weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the  $m$  weeks.

(c) Show that

$$m(m+1) = 35 \times 36 \quad (4)$$

(d) Hence write down the value of  $m$ . (1)

$$a) \quad a = 10 \quad d = 5$$

$$u_n = a + (n-1)d$$

$$u_{15} = 10 + (15-1)5$$

$$= 10 + 14(5)$$

$$= \underline{\underline{80p}}$$

$$b) \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_{60} = \frac{60}{2}(2(10) + (60-1)(5))$$

$$= 30(20 + 59(5))$$

$$= 30(315)$$

$$= 9450$$

$$\underline{\underline{£94.50}}$$

$$c) \quad a = 10 \quad d = 10$$

$$6300 = \frac{m}{2}(2(10) + (m-1)(10))$$

$$6300 = \frac{m}{2}(20 + 10m - 10)$$

$$6300 = \frac{m}{2}(10 + 10m)$$



## Question 6 continued

$$6300 = m(5 + 5m)$$

$$6300 = 5m + 5m^2$$

$$6300 = 5m(1 + m)$$

$$1260 = m(m+1)$$

$$\underline{35 \times 36 = m(m+1)}$$

$$d) \quad \underline{\underline{m = 35}}$$

7. The point  $P(4, -1)$  lies on the curve  $C$  with equation  $y = f(x)$ ,  $x > 0$ , and

$$f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

(a) Find the equation of the tangent to  $C$  at the point  $P$ , giving your answer in the form  $y = mx + c$ , where  $m$  and  $c$  are integers. (4)

(b) Find  $f(x)$ . (4)

$$a) \quad f'(x) = \frac{1}{2}x - \frac{6}{\sqrt{x}} + 3$$

when  $x = 4$

$$f'(4) = \frac{1}{2}(4) - \frac{6}{\sqrt{4}} + 3$$

$$= 2 - 3 + 3$$

$$= \underline{\underline{2}}$$

$$m = 2.$$

$$y = 2x + c \quad \begin{matrix} x & y \\ (4, & -1) \end{matrix}$$

$$-1 = 2(4) + c$$

$$-1 = 8 + c$$

$$\underline{\underline{c = -9}}$$

$$\underline{\underline{y = 2x - 9}}$$

$$b) \quad f'(x) = \frac{1}{2}x - 6x^{-\frac{1}{2}} + 3$$

$$f(x) = \frac{\frac{1}{2}x^2}{2} - \frac{6x^{\frac{1}{2}}}{\frac{1}{2}} + 3x + c$$

$$= \frac{1}{4}x^2 - 12x^{\frac{1}{2}} + 3x + c$$

$$(4, -1) \quad -1 = \frac{1}{4}(4)^2 - 12(4)^{\frac{1}{2}} + 3(4) + c$$

$$-1 = 4 - 24 + 12 + c$$

$$-1 = -8 + c$$

$$c = 7$$



Question 7 continued

$$f(x) = \frac{1}{4}x^2 - 12x^{1/2} + 3x + 7$$

Q7

(Total 8 marks)

8.

$$4x - 5 - x^2 = q - (x + p)^2$$

where  $p$  and  $q$  are integers.

(a) Find the value of  $p$  and the value of  $q$ .

(3)

(b) Calculate the discriminant of  $4x - 5 - x^2$

(2)

(c) On the axes on page 17, sketch the curve with equation  $y = 4x - 5 - x^2$  showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3)

$$a) \quad 4x - 5 - x^2 = q - (x + p)^2$$

$$(x + p)^2 - q = x^2 - 4x + 5$$

$$= (x - 2)^2 - (2)^2 + 5$$

$$= (x - 2)^2 - 4 + 5$$

$$= (x - 2)^2 + 1$$

$$\underline{p = -2} \quad \underline{q = -1}$$

$$b) \quad a = -1 \quad b = 4 \quad c = -5$$

$$(4)^2 - 4(-1)(-5)$$

$$16 - 4(5)$$

$$16 - 20$$

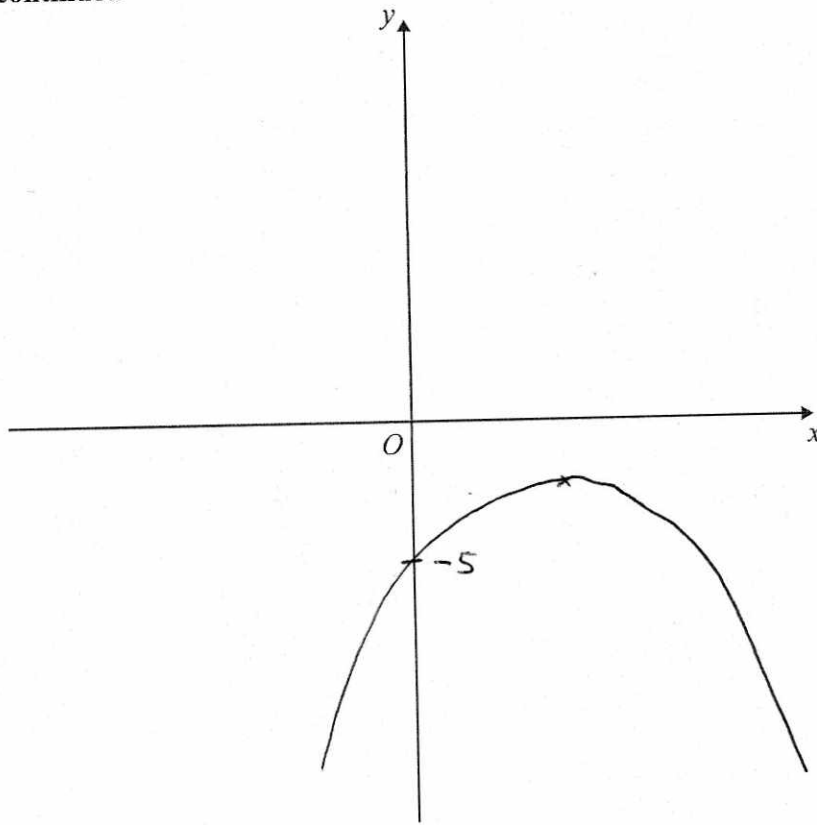
$$\underline{\underline{-4}}$$

c/ crosses y when  $x = 0$

$$y = 4(0) - 5 - (0)^2$$

$$= -5$$

Question 8 continued



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(Total 8 marks)

Q8



9. The line  $L_1$  has equation  $4y + 3 = 2x$

The point  $A(p, 4)$  lies on  $L_1$

- (a) Find the value of the constant  $p$ . (1)

The line  $L_2$  passes through the point  $C(2, 4)$  and is perpendicular to  $L_1$

- (b) Find an equation for  $L_2$  giving your answer in the form  $ax + by + c = 0$ , where  $a, b$  and  $c$  are integers. (5)

The line  $L_1$  and the line  $L_2$  intersect at the point  $D$ .

- (c) Find the coordinates of the point  $D$ . (3)

- (d) Show that the length of  $CD$  is  $\frac{3}{2}\sqrt{5}$  (3)

A point  $B$  lies on  $L_1$  and the length of  $AB = \sqrt{80}$

The point  $E$  lies on  $L_2$  such that the length of the line  $CDE = 3$  times the length of  $CD$ .

- (e) Find the area of the quadrilateral  $ACBE$ . (3)

$$\begin{array}{l} \text{a)} \quad 4y + 3 = 2x \quad \begin{matrix} x & y \\ (p, 4) \end{matrix} \\ \quad 4(4) + 3 = 2p \\ \quad 16 + 3 = 2p \\ \quad 19 = 2p \\ \quad p = 19/2 \end{array}$$

$$\begin{array}{l} \text{b/ } L_1 \quad 4y + 3 = 2x \\ \quad 4y = 2x - 3 \\ \quad y = \frac{1}{2}x - \frac{3}{4} \\ \quad m = \frac{1}{2} \\ \text{perpendicular gradient} = -2 \end{array}$$

$$\begin{array}{l} y = -2x + c \quad (2, 4) \\ 4 = -2(2) + c \\ 4 = -4 + c \\ 8 = c \end{array}$$

## Question 9 continued

$$y = -2x + 8$$

$$\underline{2x + y - 8 = 0}$$

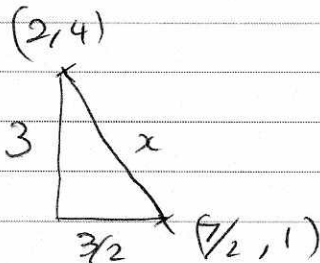
$$\begin{aligned} c/ \quad 2x + y - 8 &= 0 \\ 2x &= 8 - y \\ 4y + 3 &= 2x \end{aligned}$$

$$\begin{aligned} 4y + 3 &= 8 - y \\ 5y + 3 &= 8 \\ 5y &= 5 \\ \underline{y} &= \underline{1} \end{aligned}$$

$$\begin{aligned} 4(1) + 3 &= 2x \\ 7 &= 2x \\ \underline{x} &= \underline{7/2} \end{aligned}$$

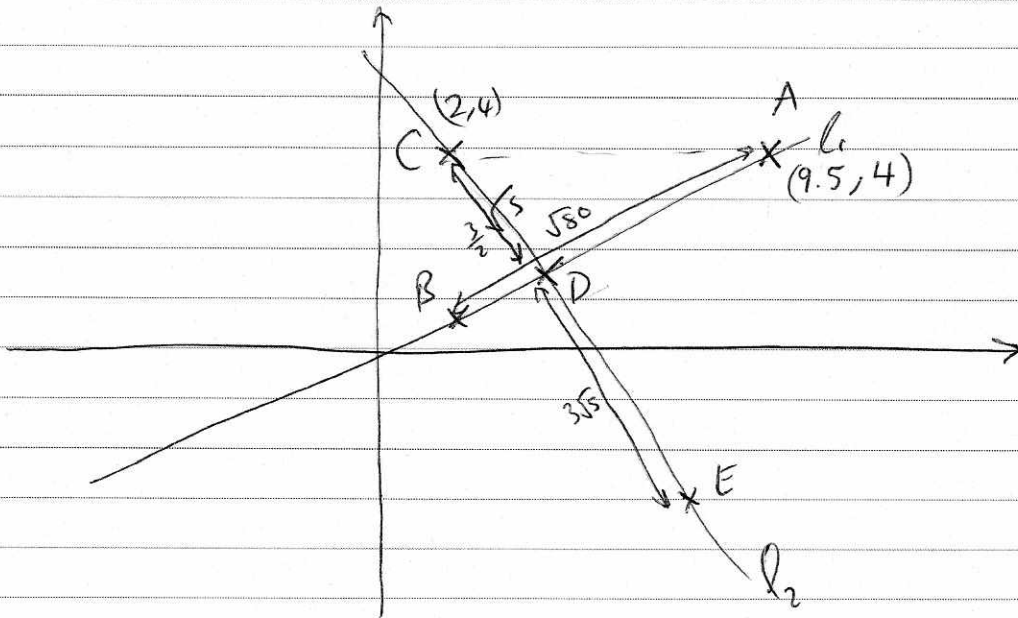
$$\underline{\left(\frac{7}{2}, 1\right)}$$

$$d/ \quad \left(\frac{7}{2}, 1\right) \quad (2, 4)$$



$$\begin{aligned} x^2 &= 3^2 + \left(\frac{3}{2}\right)^2 \\ &= 9 + \frac{9}{4} \\ &= \frac{36}{4} + \frac{9}{4} \\ &= \frac{45}{4} \\ x &= \sqrt{\frac{45}{4}} \\ &= \frac{\sqrt{45}}{2} = \frac{\sqrt{9 \cdot 5}}{2} = \frac{3\sqrt{5}}{2} \end{aligned}$$

Question 9 continued



$$\begin{aligned}
 \text{Area of } ABC &= \frac{1}{2}(\sqrt{80})\left(\frac{3}{2}(\sqrt{5})\right) \\
 &= \frac{1}{2}(4\sqrt{5})\left(\frac{3}{2}\sqrt{5}\right) \\
 &= \frac{3}{4}(4\sqrt{5})(\sqrt{5}) \\
 &= 3(\sqrt{5})(\sqrt{5}) \\
 &= 3(5) \\
 &= 15
 \end{aligned}$$

$$\begin{aligned}
 \text{Area of } ABE &= \frac{1}{2}\sqrt{80} \times 3\sqrt{5} \\
 &= \frac{1}{2}(4\sqrt{5})(3\sqrt{5}) \\
 &= \frac{1}{2}(12(5)) \\
 &= \frac{1}{2}60 \\
 &= 30
 \end{aligned}$$

$$\underline{\text{Total area} = 45 \text{ units}^2}$$





10.

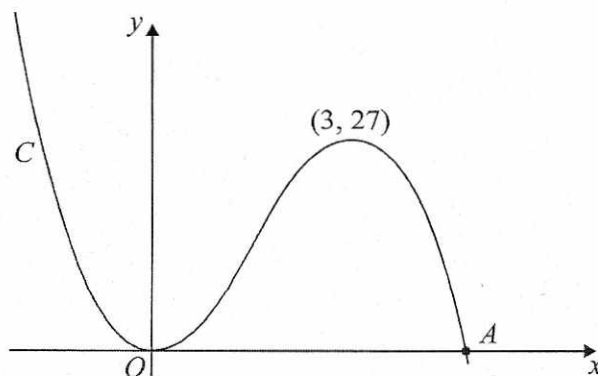


Figure 1

Figure 1 shows a sketch of the curve  $C$  with equation  $y = f(x)$  where

$$f(x) = x^2(9 - 2x)$$

There is a minimum at the origin, a maximum at the point  $(3, 27)$  and  $C$  cuts the  $x$ -axis at the point  $A$ .

(a) Write down the coordinates of the point  $A$ . (1)

(b) On separate diagrams sketch the curve with equation

(i)  $y = f(x + 3)$

(ii)  $y = f(3x)$

On each sketch you should indicate clearly the coordinates of the maximum point and any points where the curves cross or meet the coordinate axes. (6)

The curve with equation  $y = f(x) + k$ , where  $k$  is a constant, has a maximum point at  $(3, 10)$ .

(c) Write down the value of  $k$ . (1)

a/  $(\frac{9}{2}, 0)$

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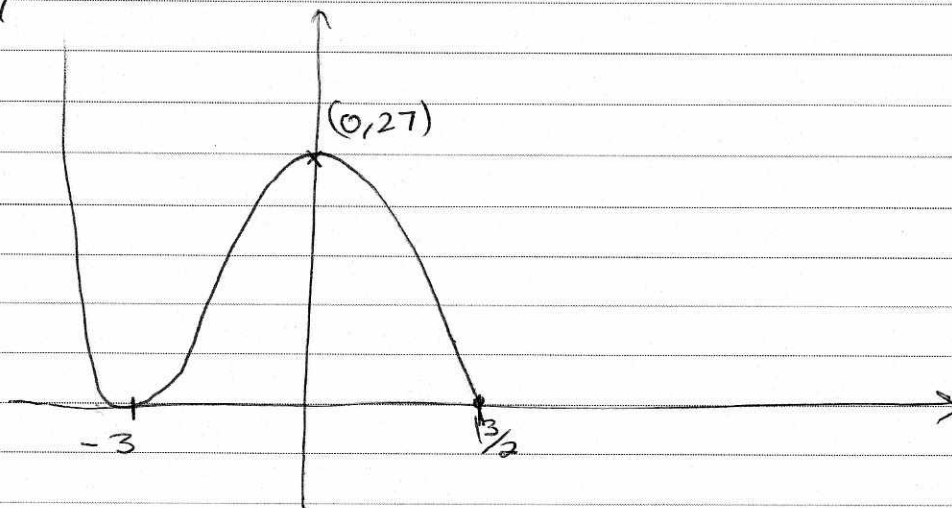


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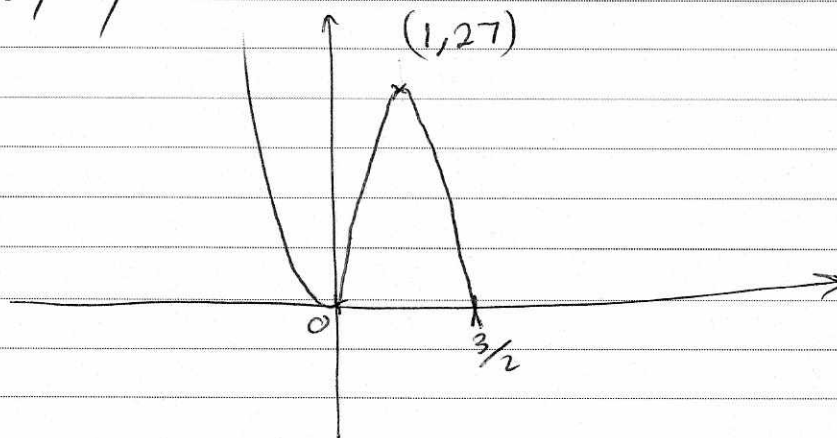


Question 10 continued

b/



or ii/



c/ -17