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1. Find the value of

(a) $25^{\frac{1}{2}}$

(1)

(b) $25^{-\frac{3}{2}}$

(2)

a) $25^{\frac{1}{2}} = \underline{\underline{5}}$

b) $25^{-\frac{3}{2}} = 25^{-3} = 125^{-1}$

$= \underline{\underline{\frac{1}{125}}}$

Q1

(Total 3 marks)



2. Given that $y = 2x^5 + 7 + \frac{1}{x^3}$, $x \neq 0$, find, in their simplest form,

(a) $\frac{dy}{dx}$, (3)

(b) $\int y \, dx$. (4)

2 a)

$$y = 2x^5 + 7 + x^{-3}$$

$$\frac{dy}{dx} = 10x^4 - 3x^{-4}$$

b) $y = 2x^5 + 7 + x^{-3}$

$$\int y \, dx = \frac{2x^6}{6} + 7x + \frac{x^{-2}}{-2} + C$$

$$= \frac{1}{3}x^6 + 7x - \frac{1}{2}x^{-2} + C$$



3. The points P and Q have coordinates $(-1, 6)$ and $(9, 0)$ respectively.

The line l is perpendicular to PQ and passes through the mid-point of PQ .

Find an equation for l , giving your answer in the form $ax + by + c = 0$, where a , b and c are integers.

(5)

$$\begin{aligned} 3) \quad \text{Midpoint} & \left(\frac{-1+9}{2}, \frac{6+0}{2} \right) \\ & (4, 3) \end{aligned}$$

$$M_{PQ} = \frac{0-6}{9-(-1)}$$

$$= \frac{-6}{10}$$

$$= -\frac{3}{5}$$

$$\therefore m = \frac{5}{3}$$

$$y = \frac{5}{3}x + c$$

$$3 = \frac{5}{3}(4) + c$$

$$3 = \frac{20}{3} + c$$

$$\frac{9}{3} = \frac{20}{3} + c$$

$$-\frac{11}{3} = c$$

$$y = \frac{5}{3}x - \frac{11}{3}$$

$$3y = 5x - 11$$



Question 3 continued

$$\underline{5x - 3y - 11 = 0}$$

Q3

(Total 5 marks)



4. Solve the simultaneous equations

$$\begin{aligned}x + y &= 2 \\ 4y^2 - x^2 &= 11\end{aligned}$$

(7)

$$y = 2 - x$$

$$4(2 - x)^2 - x^2 = 11$$

$$4(2 - x)(2 - x) - x^2 = 11$$

$$4(4 - 4x + x^2) - x^2 = 11$$

$$16 - 16x + 4x^2 - x^2 = 11$$

$$3x^2 - 16x + 16 = 11$$

$$3x^2 - 16x + 5 = 0$$

$$(3x - 1)(x - 5) = 0$$

$$x = \frac{1}{3} \quad x = 5$$

$$y = \frac{5}{3} \quad y = -3$$



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Question 4 continued

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(Total 7 marks)

Q4



5. A sequence a_1, a_2, a_3, \dots is defined by

$$a_1 = k,$$

$$a_{n+1} = 5a_n + 3, \quad n \geq 1,$$

where k is a positive integer.

(a) Write down an expression for a_2 in terms of k .

(1)

(b) Show that $a_3 = 25k + 18$.

(2)

(c) (i) Find $\sum_{r=1}^4 a_r$ in terms of k , in its simplest form.

(ii) Show that $\sum_{r=1}^4 a_r$ is divisible by 6.

(4)

$$\begin{aligned} 5a) \quad a_1 &= k \\ a_2 &= 5(a_1) + 3 \\ &= 5k + 3 \end{aligned}$$

$$\begin{aligned} b) \quad a_3 &= 5(a_2) + 3 \\ &= 5(5k + 3) + 3 \\ &= 25k + 15 + 3 \\ &= 25k + 18 \end{aligned}$$

$$\begin{aligned} c) \quad a_4 &= 5(a_3) + 3 \\ &= 5(25k + 18) + 3 \\ &= 125k + 90 + 3 \\ &= 125k + 93 \end{aligned}$$

$$\begin{aligned} \sum_{r=1}^4 a_r &= k + 5k + 3 + 25k + 18 + 125k + 93 \\ &= 156k + 114 \end{aligned}$$

$$d) \quad \underline{6(26k + 19)}$$



6. Given that $\frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$ can be written in the form $6x^p + 3x^q$,

(a) write down the value of p and the value of q .

(2)

Given that $\frac{dy}{dx} = \frac{6x+3x^{\frac{5}{2}}}{\sqrt{x}}$, and that $y = 90$ when $x = 4$,

(b) find y in terms of x , simplifying the coefficient of each term.

(5)

$$6a) \quad \frac{6x + 3x^{\frac{5}{2}}}{x^{1/2}}$$

$$= 6x^{1/2} + 3x^2$$

$$p = 1/2 \quad q = 2$$

$$b) \quad \frac{dy}{dx} = 6x^{1/2} + 3x^2$$

$$y = \frac{6x^{3/2}}{3/2} + \frac{3x^3}{3} + C$$

$$y = 4x^{3/2} + x^3 + C$$

$$(4, 90) \quad 90 = 4(4)^{3/2} + (4)^3 + C$$

$$90 = 32 + 64 + C$$

$$90 = 96 + C$$

$$C = -6$$

$$y = 4x^{3/2} + x^3 - 6$$



Question 6 continued

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Q6

(Total 7 marks)



7. $f(x) = x^2 + (k+3)x + k$

where k is a real constant.

(a) Find the discriminant of $f(x)$ in terms of k . (2)

(b) Show that the discriminant of $f(x)$ can be expressed in the form $(k+a)^2 + b$, where a and b are integers to be found. (2)

(c) Show that, for all values of k , the equation $f(x) = 0$ has real roots. (2)

$$7a) \quad a = 1 \quad b = k + 3 \quad c = k$$

$$b^2 - 4ac$$

$$(k+3)^2 - 4(k)$$

$$k^2 + 6k + 9 - 4k$$

$$\underline{k^2 + 2k + 9}$$

$$b) \quad (k+1)^2 - 1 + 9$$

$$\underline{(k+1)^2 + 8}$$

c) Minimum point $(-1, 8)$



$b^2 - 4ac$ is always positive \therefore 2 real roots



8.

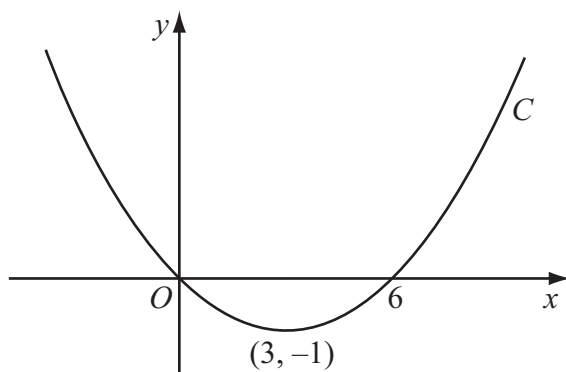


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = f(x)$.
 The curve C passes through the origin and through $(6, 0)$.
 The curve C has a minimum at the point $(3, -1)$.

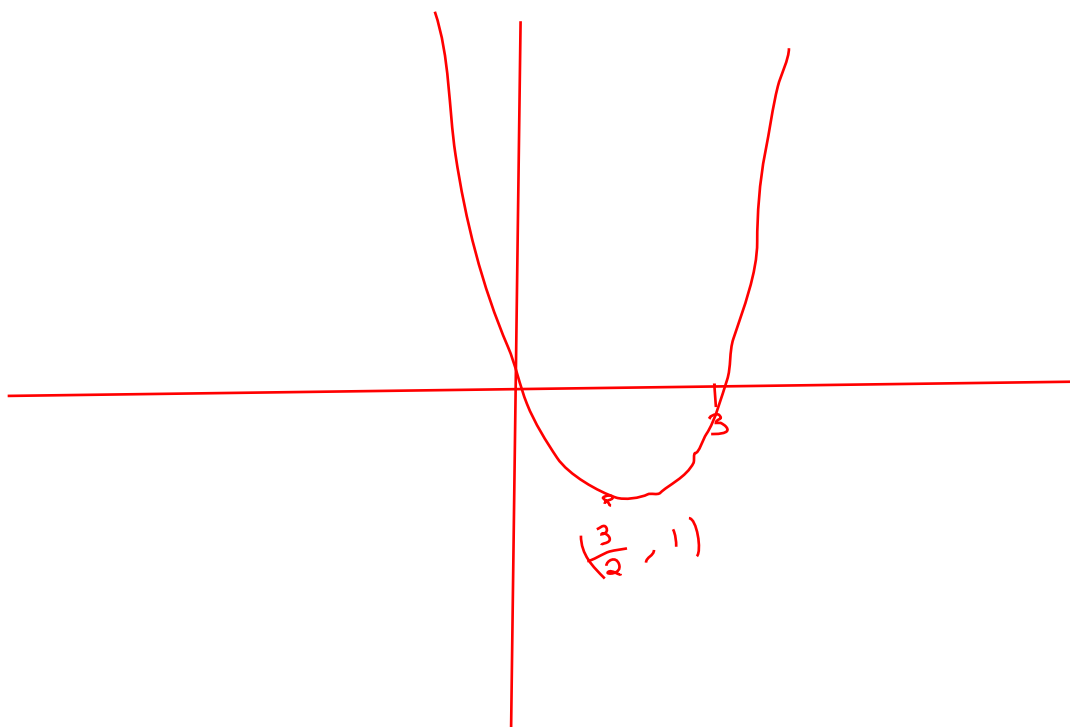
On separate diagrams, sketch the curve with equation

(a) $y = f(2x)$, (3)

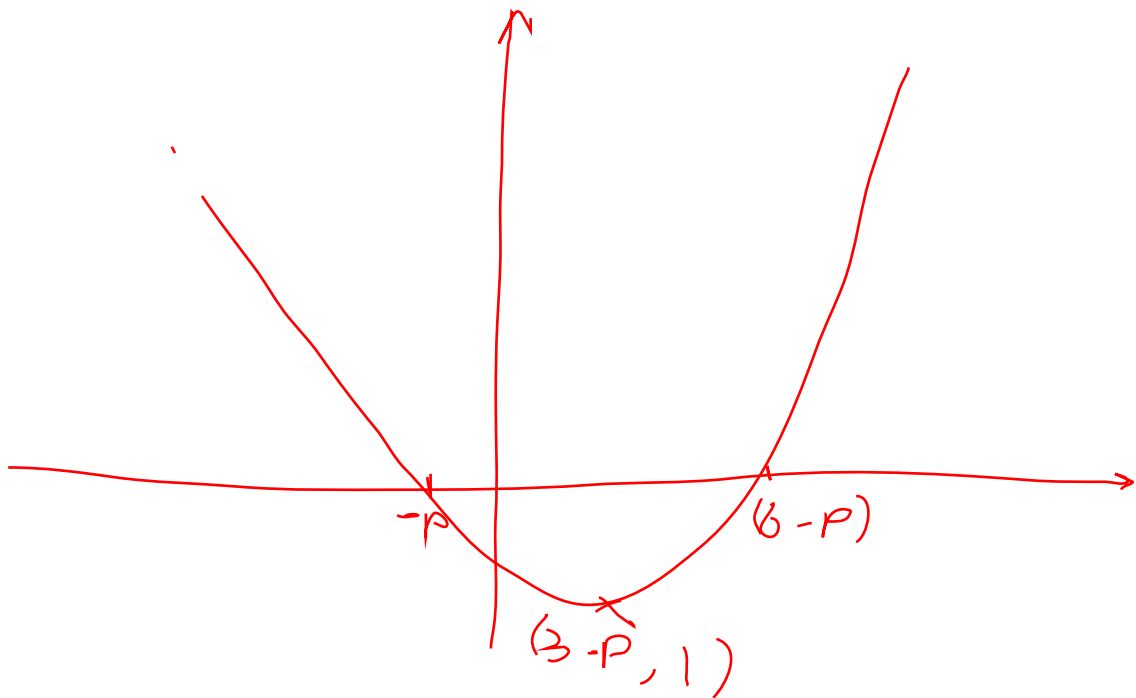
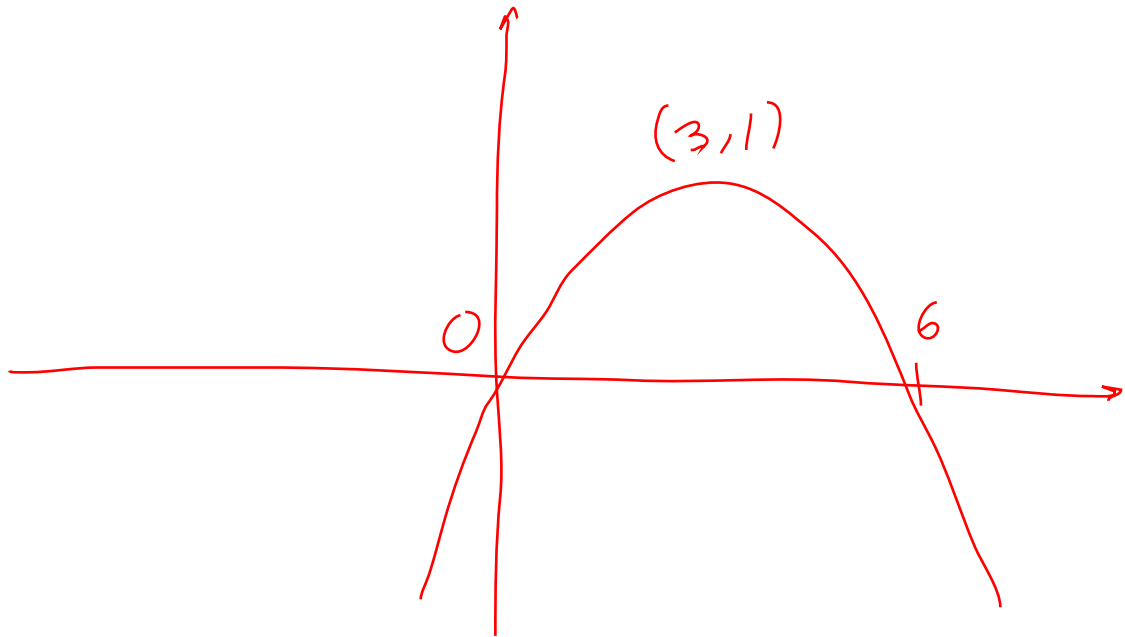
(b) $y = -f(x)$, (3)

(c) $y = f(x + p)$, where p is a constant and $0 < p < 3$. (4)

On each diagram show the coordinates of any points where the curve intersects the x -axis and of any minimum or maximum points.



Question 8 continued



Question 8 continued



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Question 8 continued

Q8

(Total 10 marks)



9. (a) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \dots + 100$$

(3)

- (b) In the arithmetic series

$$k + 2k + 3k + \dots + 100$$

k is a positive integer and k is a factor of 100.

- (i) Find, in terms of k , an expression for the number of terms in this series.

- (ii) Show that the sum of this series is

$$50 + \frac{5000}{k}$$

(4)

- (c) Find, in terms of k , the 50th term of the arithmetic sequence

$$(2k + 1), (4k + 4), (6k + 7), \dots,$$

giving your answer in its simplest form.

(2)

9 a)

$$S_n = \frac{n}{2} (a + L)$$

$$S_{50} = \frac{50}{2} (2 + 100)$$

$$= 25(102)$$

$$= \underline{\underline{2550}}$$

$$b) \frac{100}{k}$$

$$c) \quad a = k \quad d = k \quad n = \frac{100}{k}$$

$$L = 100$$

$$S_n = \frac{n}{2} (a + L)$$

$$= \frac{100}{2k} (k + 100)$$



Question 9 continued

$$= \frac{50}{k} (k + 100)$$

$$= 50 + \frac{5000}{k}$$

$$c) \quad a = 2k + 1$$

$$d = 2k + 3$$

$$U_{50} = ?$$

$$d) \quad U_{50} = 2k + 1 + (50 - 1)(2k + 3)$$

$$= 2k + 1 + 49(2k + 3)$$

$$= 2k + 1 + 98k + 147$$

$$= 100k + 148$$



10. The curve C has equation

$$y = (x+1)(x+3)^2$$

(a) Sketch C , showing the coordinates of the points at which C meets the axes. (4)

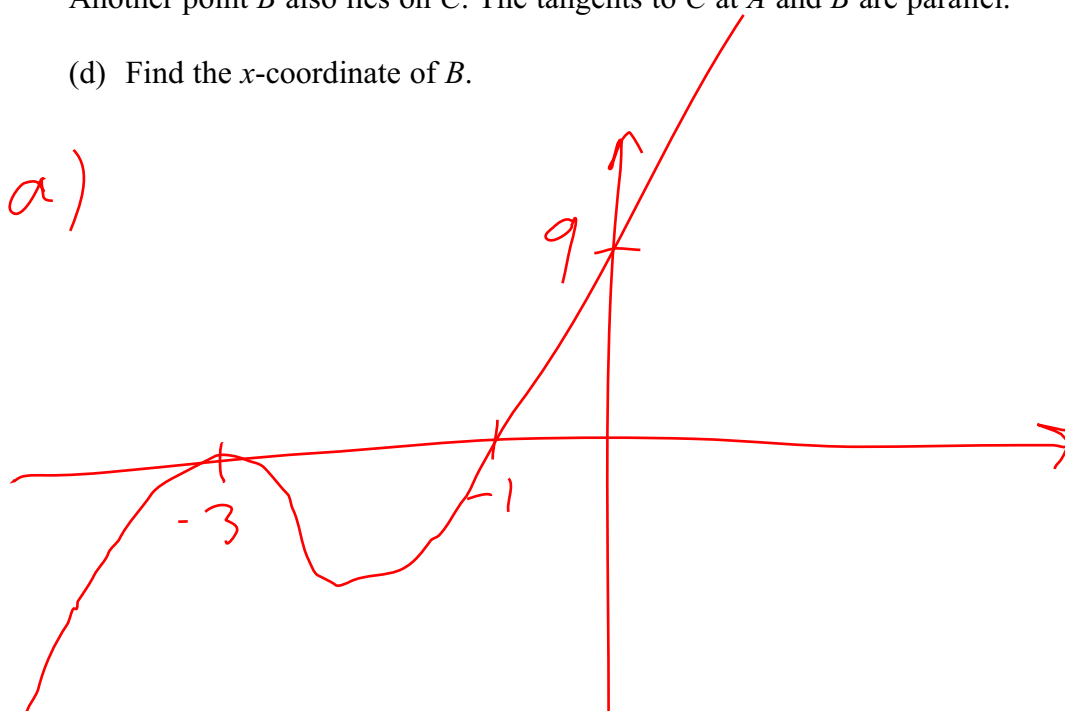
(b) Show that $\frac{dy}{dx} = 3x^2 + 14x + 15$. (3)

The point A , with x -coordinate -5 , lies on C .

(c) Find the equation of the tangent to C at A , giving your answer in the form $y = mx + c$, where m and c are constants. (4)

Another point B also lies on C . The tangents to C at A and B are parallel.

(d) Find the x -coordinate of B . (3)



Question 10 continued

$$\begin{aligned}
 y &= (x+1)(x+3)^2 \\
 &= (x+1)(x^2+6x+9) \\
 &= x^3+6x^2+9x+x^2+6x+9 \\
 &= x^3+7x^2+15x+9
 \end{aligned}$$

$$\frac{dy}{dx} = 3x^2 + 14x + 15$$

b) when $x = -5$

$$\begin{aligned}
 \frac{dy}{dx} &= 3(-5)^2 + 14(-5) + 15 \\
 &= 75 - 70 + 15 \\
 &= 20
 \end{aligned}$$

$$\begin{aligned}
 y &= (x+1)(x+3)^2 \\
 &= (-5+1)(-5+3)^2 \\
 &= (-4)(4) \\
 &= -16
 \end{aligned}$$

$(-5, -16)$

$$y = mx + c$$

$$-16 = 20(-5) + c$$

$$-16 = -100 + c$$

$$c = 84$$



Question 10 continued

$$y = 20x + 84$$

$$d) \quad \frac{dy}{dx} = 20$$

$$3x^2 + 14x + 19 = 20$$

$$3x^2 + 14x - 5 = 0$$

$$(3x - 1)(x + 5) = 0$$

$$\underline{\underline{x = 1/3}} \quad x = -5$$



Question 10 continued

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Q10

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

