

**1. In this question you should show all stages of your working.**

 **Solutions relying on calculator technology are not acceptable.**

Using algebra, solve the inequality

*x*2 – *x* > 20

writing your answer in set notation.

**(3)**

**(Total for Question 1 is 3 marks)**

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**2. In this question you should show all stages of your working.**

 **Solutions relying on calculator technology are not acceptable.**

Given



express *y* in terms of *x*, writing your answer in simplest form.

**(3)**

**(Total for Question 2 is 3 marks)**

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**3.** Find



writing your answer in simplest form.

**(4)**

**(Total for Question 3 is 4 marks)**

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**4.** [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point *A*(–24**i** – 10**j**) m relative to a fixed point *O*.

After 4 seconds the stone is at the point *B*(12**i** + 5**j**) m relative to the fixed point *O*.

The motion of the stone is modelled as that of a particle moving in a straight line at

constant speed.

Using the model,

(*a*)prove that the stone passes through *O*,

**(2)**

(*b*)calculate the speed of the stone.

**(3)**

**(Total for Question 4 is 5 marks)**

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**5.**

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Figure 1 shows part of the curve with equation *y* = 3*x*2 – 2

The point *P*(2, 10) lies on the curve.

(*a*)Find the gradient of the tangent to the curve at *P*.

**(2)**

The point *Q* with *x* coordinate 2 + *h* also lies on the curve.

(*b*)Find the gradient of the line *PQ*, giving your answer in terms of *h* in simplest form.

**(3)**

(*c*)Explain briefly the relationship between part (*b*)and the answer to part (a).

**(1)**

**(Total for Question 5 is 6 marks)**

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**6. In this question you should show all stages of your working.**

 **Solutions relying on calculator technology are not acceptable.**

(*a*)Using algebra, find all solutions of the equation

3*x*3 – 17*x*2 – 6*x* = 0

**(3)**

(*b*)Hence find all real solutions of

3( *y* – 2)6 – 17( *y* – 2)4 – 6( *y* – 2)2 = 0

**(3)**

**(Total for Question 6 is 6 marks)**

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**7.** A parallelogram *PQRS* has area 50 cm2

Given

• *PQ* has length 14 cm

• *QR* has length 7 cm

• angle *SPQ* is obtuse

find

(*a*)the size of angle *SPQ*, in degrees, to 2 decimal places,

**(3)**

(*b*)the length of the diagonal *SQ*, in cm, to one decimal place.

**(2)**

**(Total for Question 7 is 5 marks)**

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**8.** g(*x*) = (2 + *ax*)8 where *a* is a constant

Given that one of the terms in the binomial expansion of g(*x*) is 3402*x*5

(*a*)find the value of *a*.

**(4)**

Using this value of *a*,

(*b*)find the constant term in the expansion of



**(3)**

**(Total for Question 8 is 7 marks)**

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**9.** Find the value of the constant *k*, 0 < *k* < 9, such that



**s(4)**

**(Total for Question 9 is 4 marks)**

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**10.** A student is investigating the following statement about natural numbers.

“ *n*3 – *n* is a multiple of 4 ”

(*a*)Prove, using algebra, that the statement is true for all odd numbers.

**(4)**

(*b*)Use a counterexample to show that the statement is not always true.

**(1)**

**(Total for Question 10 is 5 marks)**

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**11.** The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, *A* km2, is modelled by the equation

*A* = 80 – 45e*ct*

where *c* is a constant and *t* is the number of years after 1st January 2005.

Using the model,

(*a*)find the area of the nature reserve that was covered by trees just before tree planting

started.

**(1)**

On 1st January 2019 an area of 60 km2 of the nature reserve was covered by trees.

(*b*)Use this information to find a complete equation for the model, giving your value

of *c* to 3 significant figures.

**(4)**

On 1st January 2020, the owners of the nature reserve announced a long-term plan to

have 100 km2 of the nature reserve covered by trees.

(*c*)State a reason why the model is not appropriate for this plan.

**(1)**

**(Total for Question 11 is 6 marks)**

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**12. In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(i) Solve, for 0 < *θ* ≤ 450°, the equation

5 cos2 *θ* = 6 sin *θ*

giving your answers to one decimal place.

**(5)**

(ii) (*a*)A student’s attempt to solve the question

“ Solve, for –90° < *x* < 90°, the equation 3 tan *x* – 5 sin *x* = 0 ”

is set out below.



Identify two errors or omissions made by this student, giving a brief explanation

of each.

**(2)**

The first four positive solutions, in order of size, of the equation

cos (5*α* + 40°) = 

are *α*1, *α*2, *α*3 and *α*4

(b) Find, to the nearest degree, the value of *α*4

**(2)**

**(Total for Question 12 is 9 marks)**

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**13.**



The resting heart rate, *h*, of a mammal, measured in beats per minute, is modelled by the

equation

*h* = *pmq*

where *p* and *q* are constants and *m* is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between log10 *h* and log10*m*

The line meets the vertical log10 *h* axis at 2.25 and has a gradient of –0.235

(*a*)Find, to 3 significant figures, the value of *p* and the value of *q*.

**(3)**

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(*b*)Comment on the suitability of the model for this mammal.

**(3)**

(*c*)With reference to the model, interpret the value of the constant *p*.

**(1)**

**(Total for Question 13 is 7 marks)**

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**14.** A curve *C* has equation *y* = f(*x*) where

f(*x*) = –3*x*2 + 12*x* + 8

(*a*)Write f(*x*) in the form

*a*(*x* + *b*)2 + *c*

where *a*, *b* and *c* are constants to be found.

**(3)**

The curve *C* has a maximum turning point at *M*.

(*b*)Find the coordinates of *M*.

**(2)**



Figure 3 shows a sketch of the curve *C*.

The line *l* passes through *M* and is parallel to the *x*-axis.

The region *R*, shown shaded in Figure 3, is bounded by *C*, *l* and the *y*-axis.

(*c*)Using algebraic integration, find the area of *R*.

**(5)**

**(Total for Question 14 is 10 marks)**

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**15.**

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Figure 4 shows a sketch of a circle *C* with centre *N*(7, 4)

The line *l* with equation *y* = *x* is a tangent to *C* at the point *P*.

Find

(*a*)the equation of line *PN* in the form *y* = *mx* + *c*, where *m* and *c* are constants,

**(2)**

(*b*)an equation for *C*.

**(4)**

The line with equation *y* = *x* + *k*, where *k* is a non-zero constant, is also a tangent to *C*.

(*c*)Find the value of *k*.

**(3)**

**(Total for Question 15 is 9 marks)**

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**16.** The curve *C* has equation *y* = f(*x*) where

f(*x*) = *ax*3 + 15*x*2 – 39*x* + *b*

and *a* and *b* are constants.

Given

• the point (2, 10) lies on *C*

• the gradient of the curve at (2, 10) is –3

(*a*)(i) show that the value of *a* is –2

(ii) find the value of *b*.

**(4)**

(*b*)Hence show that *C* has no stationary points.

**(3)**

(*c*)Write f(*x*) in the form (*x* – 4)Q(*x*) where Q(*x*) is a quadratic expression to be found.

**(2)**

(*d*)Hence deduce the coordinates of the points of intersection of the curve with equation

*y* = f(0.2*x*)

and the coordinate axes.

**(2)**

**(Total for Question 16 is 11 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**