C3 NUMERICAL METHODS — Worksheet B

1 For each equation, show that it can be rearranged into the given iterative form. Use this and the given value of x_0 to find x_1, x_2 and x_3. Give your value of x_3 correct to 4 decimal places.

a $9 + 4x - 2x^3 = 0$ $\quad x_{n+1} = \sqrt[3]{2x_n + 4.5}$ $\quad x_0 = 2$

b $e^x - 8x + 5 = 0$ $\quad x_{n+1} = \ln(8x_n - 5)$ $\quad x_0 = 3$

c $\tan x - 5x + 13 = 0$ $\quad x_{n+1} = \arctan(5x_n - 13)$ $\quad x_0 = -1.2$

d $\ln x + \sqrt{x} + 1.4 = 0$ $\quad x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$ $\quad x_0 = 0.16$

2 For each equation, show that it can be rearranged into the given iterative form and state the values of the constants a and b. Use this and the given value of x_0 to find x_1, x_2 and x_3. Give your value of x_3 correct to 3 decimal places.

a $e^{2x-1} - 6x = 0$ $\quad x_{n+1} = a(\ln bx_n + 1)$ $\quad x_0 = 1.7$

b $\frac{2}{x} + \cos x - 3 = 0$ $\quad x_{n+1} = \frac{a}{b - \cos x_n}$ $\quad x_0 = 0.8$

c $2x^3 - 6x - 11 = 0$ $\quad x_{n+1} = \sqrt{a + \frac{b}{x_n}}$ $\quad x_0 = 2$

d $15\ln(x + 3) - 4x = 0$ $\quad x_{n+1} = e^{ax_n} + b$ $\quad x_0 = -2.5$

3 In each case, use the given iteration formula and value of x_0 to find a root of the equation $f(x) = 0$ to the stated degree of accuracy. Justify the accuracy of your answers.

a $f(x) = 10^x + 3x - 4$ $\quad x_{n+1} = \log_{10}(4 - 3x_n)$ $\quad x_0 = 0.44$ $\quad$ 3 decimal places

b $f(x) = x^2 + \frac{1}{x-5}$ $\quad x_{n+1} = \sqrt{\frac{x_n^3 + 1}{5}}$ $\quad x_0 = 0.5$ $\quad$ 2 significant figures

c $f(x) = 30 - 5x + \sin 2x$ $\quad x_{n+1} = 6 + 0.2\sin 2x_n$ $\quad x_0 = 6$ $\quad$ 3 significant figures

d $f(x) = e^{4-x} - \ln x$ $\quad x_{n+1} = 4 - \ln(\ln x_n)$ $\quad x_0 = 3.7$ $\quad$ 3 decimal places

4 $$f(x) = x^5 - 10x^3 + 4.$$

The equation $f(x) = 0$ has a root in the interval $-4 < x < -3$.

a Use the iteration formula $x_{n+1} = \sqrt[5]{10x_n^3 - 4}$ and the starting value $x_0 = -3.2$ to find the value of this root correct to 2 decimal places.

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = \sqrt[3]{\frac{a}{b - x_n^2}}$.

b Find the values of the constants a and b in this formula.

The equation $f(x) = 0$ has another root in the interval $0 < x < 1$.

c Using the iteration formula with your values from part **b** and the starting value $x_0 = 1$, find the value of this root correct to 3 decimal places.

5 $$f : x \to \arcsin 2x - 0.5x - 0.7,\ x \in \mathbb{R},\ |x| \le 0.5$$

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = a\sin(bx_n + c)$.

a Find the values of the constants a, b and c in this formula.

The equation $f(x) = 0$ has a solution in the interval (0.3, 0.4).

b Using the iterative formula with your values from part **a** and the starting value $x_0 = 0.4$, find this solution correct to 3 decimal places.